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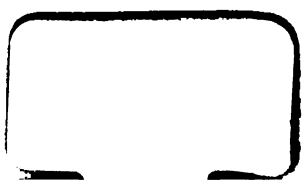

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TRIGONOMETRY

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PREFACE

THIS book has had its origin in the desire of the authors to meet the mutual demands of mathematicians and engineers for a treatment that shall more completely supply the needs of the technological student. It is believed that this has been done by enriching the subject with applications to physics and engineering, in such a way as to increase its value at the same time to the general student. The present volume is, moreover, based upon a preliminary edition actually used for several terms in the classroom.

In view of the peculiar situation of trigonometry in the curriculum, the course has been kept of the usual length. The topics have been arranged, however, in the order of increasing difficulty, by postponing the more abstract but no less essential study of the functions of the general angle, until after the arithmetical solution of triangles. The abundance of exercises and problems will give the teacher large opportunity for selection.

The discussion of the slide rule is inserted because of the increasing employment of this useful instrument.

The authors gratefully acknowledge their indebtedness to Professor E. J. Townsend and Professor H. L. Rietz, of the University of Illinois, and Professor A. Ziwet and Professor J. L. Markley, of the University of Michigan, for valuable criticisms and suggestions.

ARTHUR G. HALL.
FRED G. FRINK.

ANN ARBOR, January, 1909.

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TRIGONOMETRY

GREEK ALPHABET

Letters	Names	Letters	Names	Letters	Names
A α	Alpha	I ι	Iota	P ρ	Rho
B β	Beta	K κ	Kappa	Σ σ ς	Sigma
Γ γ	Gamma	Λ λ	Lambda	T τ	Tau
Δ δ	Delta	M μ	Mu	T υ	Upsilon
E ϵ	Epsilon	N ν	Nu	Φ ϕ	Phi
Z ζ	Zeta	Ξ ξ	Xi	X χ	Chi
H η	Eta	O o	Omicron	Ψ ψ	Psi
Θ θ	Theta	Π π	Pi	Ω ω	Omega

TRIGONOMETRY

PART I

PLANE TRIGONOMETRY

CHAPTER I

GEOMETRIC NOTIONS

1. General statement. It is assumed that the student is well versed in those theorems of elementary geometry concerning angles, arcs, and triangles. It is particularly desirable that he be familiar with the measurement of angles and with the properties of similar triangles.

While the review thus suggested is left to the student, certain more advanced geometric ideas are treated in the remaining articles of this chapter.

Throughout the course the student should make continual, careful, and intelligent use of such drawing instruments as are included in the equipment at technical schools. In case such sets are not available, as in more general classes, there should be provided at least a straightedge, with graduated scale, a protractor, and a pair of compasses or dividers.

2. Directed line segments. A point which moves from one position to a second, without changing its direction of motion, traces a directed line segment. Directed line segments are always read with regard to their direction, *from* the initial extremity *to* the terminal extremity.

Two line segments are equal if they have the same length and direction, whether their lines are coincident or parallel. Either of two line segments having the same length but opposite directions is said to be the negative of the other. If one direction is taken as positive, the opposite direction is negative.

Thus, in Fig. 1,

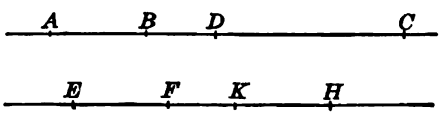

FIG. 1.

$$EF = AB,$$
$$HK = BA = -AB,$$
$$CD = 2BA = -2AB.$$

If A is the initial point and B the terminal point, the line segment is read AB, and in this notation the positive or negative direction of the segment is expressed without the aid of a prefixed + or −. In case the segment is represented by a single symbol, as the letter a, the direction must be indicated in some further manner, as by a prefixed + or −, or by an arrowhead in the figure.

Two line segments lying in the same line are added by placing the initial point of the second upon the terminal point of the first, each retaining its proper direction. The sum is the segment extending from the initial point of the first to the terminal point of the second. Subtraction is performed by reversing the direction of the subtrahend and adding. Line segments having the same or opposite directions may all be transferred to a common line. Their addition and subtraction thus correspond exactly to the algebraic addition and subtraction of positive and negative numbers.

If A, B, C denote three points arranged in any order along a straight line, then

$$AB + BC = AC,$$

and

$$AB + BC + CA = 0.$$

3. Positive and negative angles. If a line rotates (in a plane) about one of its points, an angle is generated, of which the original position of the line is the initial side and the final position the terminal side. A distinction may be made according as the rotation is clockwise or counter-clockwise about the vertex. The counter-clockwise direction is chosen as positive. Angles are always read with regard to their direction of rotation; thus if OA is the initial side and OB the terminal side, the angle is read AOB. This notation includes the direction or sign of the angle, and the + or − should not be prefixed. In case the angle is represented by a single symbol, as by the Greek letter α, the direction must be indicated in some further manner, as by a prefixed + or −, or by a curved arrow in the figure.

Just as with line segments, reversing the direction multiplies the angle by -1; thus $BOA = -AOB$.

Two angles are added by placing them in the same plane with a common vertex, the initial side of the second coincident with the terminal side of the first, each retaining its own direction. The sum is the angle from the initial side of the first to the terminal side of the second. Subtraction is performed by reversing the subtrahend and adding.

In Fig. 2,

$$AOB + BOC = AOC,$$

$$AOC - BOC = AOB.$$

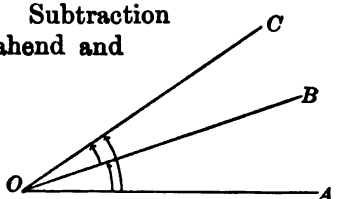

FIG. 2.

EXERCISE I

Solve the following problems graphically:

1. On a train running 40 miles an hour, a man walks 4 miles an hour. Find the speed of the man with reference to the ground, (*a*) if he walks toward the front; (*b*) if he walks toward the rear of the train.

2. The man's speed with reference to the ground is 10 miles an hour. What is the speed of the train (*a*) if he is walking 5 miles an hour toward the front; (*b*) if he is running 8 miles an hour toward the rear?

3. On June 1 the price of corn was 50 cents, and during the succeeding ten days it fluctuated as follows: rose 2 cts., rose 3, fell 1, fell 2, fell 5, fell 3, rose 2, rose 2, rose 3, rose 1. Find the price on June 11.

4. During a football game the progress of the ball from the middle of the field was north 40 yards, south 25, south 5, south 10, south 30, north 50, north 10, north 20. Find the resulting position of the ball.

Combine graphically, using a protractor:

5. $45° + 30°$; $90° + 45°$; $40° + 35° + 50°$.

6. $60° - 45°$; $90° - 50°$; $180° - 120°$.

7. $30° + 80° + 55°$; $40° + 60° - 30°$; $60° - 20° + 70° - 90°$.

8. $40° - 70° + 15°$; $65° + 15° - 90°$; $75° - 180°$.

4. Rectangular coördinates. If two mutually perpendicular straight lines are chosen, and a positive direction on each, the position of any point in their plane is determined by giving its perpendicular distances from these fixed lines. The two lines are called the axes of coördinates, and are usually taken so that one

is horizontal and the other vertical. The point of intersection of the axes is called the origin. The two determining data for any point are called its coördinates. The horizontal distance from the axis OY to the point is the abscissa of the point, and the vertical distance from the axis OX to the point is the ordinate of the point. The point whose abscissa is x and ordinate y is denoted by the notation (x, y). Because it is convenient to lay off the abscissa of a point upon the axis OX and the ordinate upon the axis OY, these axes are referred to as the axes of abscissas and ordinates respectively. When x denotes the abscissa and y the ordinate of the point, the axes may be referred to as the X-axis and the Y-axis respectively.

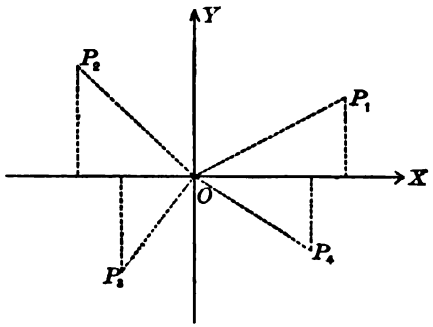

FIG. 3.

The distance from the origin to the point is called the radius vector of the point. It is known whenever the abscissa and the ordinate are given, since the three form, respectively, the hypotenuse, base, and altitude of a right triangle.

The abscissa of a point should always be read *from* the Y-axis *to* the point. The direction from left to right is chosen as positive. Therefore all points at the right of the Y-axis have positive abscissas, and all points at the left, negative abscissas. The ordinate of a point is always read *from* the X-axis *to* the point. The upward direction is chosen as positive. Hence all points above the X-axis have positive ordinates, and all points below, negative ordinates. The radius vector is always read *from* the origin *to* the point, and is always considered positive.

It will be noticed that the abscissa and the ordinate are equal to the projections of the radius vector on the X-axis and Y-axis, respectively; these projections will henceforth be used interchangeably for the coördinates themselves.*

* The foot of the perpendicular dropped from a point upon a given line is said to be the orthogonal or orthographic projection of the point on the line. The projection of a line segment on a given line is the segment from the projection of the initial point of the given segment to that of the terminal point. This kind of projection will be used exclusively throughout this book, unless otherwise expressly stated.

The two axes divide the whole plane into four portions, known as the first, second, third, and fourth quadrants, beginning with the upper right-hand quadrant and numbering counter-clockwise about the origin.

If two points, P and Q, lie in a line through the origin, their coördinates, with the radii vectores, form two similar triangles. If the abscissa, ordinate, and radius vector of P are x, y, v, respectively, those of Q are kx, ky, kv.

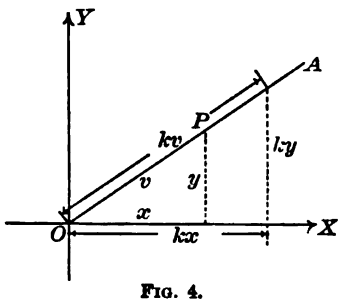

FIG. 4.

EXERCISE II

1. Plot the points (2, 3), (−3, 5), (−2, −4), (1, −3), (3, 0), (0, 4), (−5, 0), (0, −2), (0, 0).

2. Plot the points (3, 2), (6, 4), (12, 8).

3. Plot the points (0, 5), (4, 3), (−3, 4), (0, −5).

4. Find both graphically and by computation the radius vector of each point in examples 1, 2, 3. In what quadrant does each point lie?

5. Describe the location of all points whose abscissas equal 3; whose ordinates equal 5; whose radii vectores equal 6.

6. Write the coördinates of the vertices of a square of side a, if the axes of coördinates are taken along two sides; along its diagonals.

7. The hour hand of a clock is 5 inches long. Find the coördinates of its tip referred to the horizontal and vertical diameters of its face at three o'clock; at six; at eight; at half-past ten.

8. Compare the location of the points (2, 3), (3, 2), (−2, 3), (−2, −3), (3, −2). Describe the directions of their radii vectores.

9. Find the distance from (2, 5) to (6, 9); from (−3, 2) to (4, 5).

10. Find the direction of the line joining (6, 1) to (8, 3); (4, 1) to (1, 4); (6, 3) to (1, 3); (−2, 4) to (1, 1).

11. A man starts from the southwest corner of a government township and goes in turn to the northwest corner of section 36; northwest corner section 22; southeast corner section 3; northeast corner section 5; thence to the point of beginning. Show by sketch the shortest route along section lines, and compute the cross-country distances.

12. A city is laid out in checker-board fashion. The streets are eight to the mile and look to the cardinal points of the compass. It is proposed to introduce two diagonal (45°) streets extending through the busiest corner to the outskirts. What distances will be saved thereby in driving from this corner to each of the points specified below? Nine blocks east and six blocks north; 5 W. and 7 S.; 10 W. and 10 N.; 1 E. and 14 S.

CHAPTER II

THE ACUTE ANGLE

5. Purpose of trigonometry. One of the principal objects of trigonometry is the computation of triangles. We have learned from elementary geometry that a triangle is determined when we know any three of its parts (sides and angles), at least one of them being a side. These data enable us to construct the triangle; but elementary geometry does not teach us how to calculate the remaining parts. The reason is that sides and angles are expressed in different units. It is, therefore, desirable to measure angles not only in degrees, but also by means of lines, or rather by means of ratios of lines. This can be done as shown in the following articles.

6. Definitions of the trigonometric functions. Suppose the acute angle $AOB\ (=\alpha)$ to be placed on a system of axes of coördinates with its vertex at the origin and its initial side OA extending along the X-axis toward the right. Its terminal side OB will fall in the first quadrant. (See Fig. 5.)

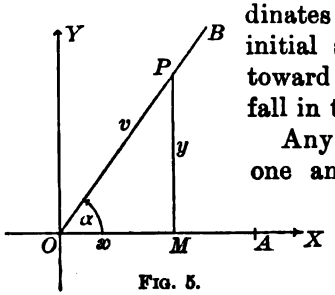

FIG. 5.

Any point P in its terminal side possesses one and only one set of related distances (two coördinates and radius vector). Its abscissa $x\ (=OM)$, its ordinate $y\ (=MP)$, and its radius vector $v\ (=OP)$ are all positive and connected by the relation

$$x^2 + y^2 = v^2.$$

The six ratios between these three distances are of frequent occurrence and prime importance. They serve, indeed, the purpose mentioned in Art. 5, and are given the following names and accompanying abbreviations:

$\frac{y}{v}$ = sine of α = **sin** α, $\qquad$ $\frac{v}{y}$ = cosecant of α = **csc** α,

$\frac{x}{v}$ = cosine of α = **cos** α, $\qquad$ $\frac{v}{x}$ = secant of α = **sec** α,

$\frac{y}{x}$ = tangent of α = **tan** α, $\qquad$ $\frac{x}{y}$ = cotangent of α = **cot** α.

7. Relations between the ratios and the angle. The three related distances of any other point P' in the terminal side of the angle α are connected with the determining distances of P by the relation

$$\frac{x'}{x} = \frac{y'}{y} = \frac{v'}{v} = k.$$

It follows that the values of the six ratios defined in Art. 6 do not depend at all upon the particular choice of the point in the terminal side of the angle, but are determined solely and definitely by the size of the angle. A number that is determined in value by the value of some other number is said to be a function of that other number. The six ratios are therefore called *trigonometric functions* of the angle.

This relation between the ratios and the angle is, moreover, a mutual one, such that a knowledge of one of the ratios leads to a knowledge of the angle.* Thus if we have given $\tan \alpha = \frac{2}{3}$, we can construct the angle α as follows: On the system of axes OX and OY plot the point P whose coördinates are (3, 2), using any convenient scale. Draw the line OA from the origin through the point P; then is XOA the required angle α, in consequence of the definitions of Art. 6.

It appears still further that from the knowledge of any one of the six trigonometric functions the remaining five can be found. Thus in the foregoing example we know by the Pythagorean proposition that on the scale employed the hypotenuse or radius vector is $\sqrt{9+4} = \sqrt{13}$. We have then at once

$$\sin \alpha = \frac{2}{\sqrt{13}}, \quad \tan \alpha = \frac{2}{3}, \quad \sec \alpha = \frac{\sqrt{13}}{3},$$

$$\cos \alpha = \frac{3}{\sqrt{13}}, \quad \cot \alpha = \frac{3}{2}, \quad \csc \alpha = \frac{\sqrt{13}}{2}.$$

The properties and relations of these functions and their more immediate applications in pure and applied mathematics constitute the subject-matter of trigonometry. The science takes its name from its origin in the attempts of the ancient peoples to measure triangles.

8. Signs and limitations in value. When any acute angle is placed on the axes of coördinates in the manner prescribed in

*This statement, as well as some others in the present chapter, will require some modification when we extend the consideration to angles greater than 90°.

Art. 6, its terminal side will always fall in the first quadrant. The abscissa and ordinate, as well as the radius vector, of every point in the terminal side will all be positive. It follows that their ratios, comprising the six trigonometric functions named in Art. 6, are all positive.

In no case can the abscissa or the ordinate of a point be greater than the radius vector. Indeed, save in the exceptional cases considered in Art. 12, they are less than the radius vector. Consequently, $\sin \alpha$ and $\cos \alpha$ cannot be greater than unity and $\sec \alpha$ and $\csc \alpha$ cannot be less than unity.

EXERCISE III

By careful construction and measurement find the approximate values of the following functions:

1. $\cos 60°$. **2.** $\tan 30°$. **3.** $\csc 45°$.

4. $\cot 35°$. **5.** $\sin 20°$. **6.** $\sec 50°$.

Construct each of the following angles and find by measurement the values of all its functions, given

7. $\sin \alpha = \frac{3}{5}$. **8.** $\cos \alpha = \frac{5}{13}$. **9.** $\tan \alpha = \frac{7}{24}$.

10. $\cot \alpha = \frac{8}{15}$. **11.** $\sec \alpha = 4\frac{1}{3}$. **12.** $\csc \alpha = 2\frac{1}{11}$.

13. $\cos \alpha = \frac{1}{3}\frac{2}{7}$. **14.** $\sin \alpha = \frac{2}{1}\frac{1}{3}$.

15. For what angle is the tangent equal to the cotangent? For what angle is the sine equal to the cosine?

16. Show that the direct functions ($\sin \alpha$, $\tan \alpha$, $\sec \alpha$) are greater or less than the corresponding complementary functions ($\cos \alpha$, $\cot \alpha$, $\csc \alpha$) respectively, according as the angle α is greater or less than 45°.

17. Can $\sin \alpha$ and $\tan \alpha$ be equal? When do they approach equality?

18. Show that $\tan \alpha \cdot \cot \alpha$ does not depend on α. Show that the same is true of $\sin \alpha \cdot \csc \alpha$.

19. Show that $\cos \alpha \cdot \sec \alpha$ does not depend on α. Show the same for $\csc^2 \alpha - \cot^2 \alpha$.

20. Does $\sin^2 \alpha + \cos^2 \alpha$ depend on α? Does $\sec^2 \alpha - \tan^2 \alpha$?

21. Before reading Art. 11, find the values of the sine, cosine, and tangent of 30°, 45°, and 60°.

9. Fundamental relations. The Pythagorean theorem

$$x^2 + y^2 = r^2,$$

and the definitions of Art. 6 yield certain fundamental relations between the six trigonometric functions of a single angle. Thus, we have directly from the definitions

$$\csc a = \frac{1}{\sin a}, \tag{1}$$

$$\sec a = \frac{1}{\cos a}, \tag{2}$$

$$\cot a = \frac{1}{\tan a}. \tag{3}$$

Again, by division,

$$\tan a = \frac{\sin a}{\cos a}, \tag{4}$$

and

$$\cot a = \frac{\cos a}{\sin a}. \tag{5}$$

Dividing by v^2 both members of the equation

$$y^2 + x^2 = v^2,$$

we have

$$\left(\frac{y}{v}\right)^2 + \left(\frac{x}{v}\right)^2 = 1,$$

whence

$$\sin^2 a + \cos^2 a = 1. \tag{6}$$

Dividing, in like manner, by x^2 and by y^2 respectively, we obtain

$$\tan^2 a + 1 = \sec^2 a, \tag{7}$$

$$\cot^2 a + 1 = \csc^2 a. \tag{8}$$

These eight equations constitute the fundamental relations of trigonometry. Of these the fifth, seventh, and eighth may be derived algebraically from the other five. By means of these equations it is possible to express all six functions in terms of any one of them. If the value of any one is given, the values of the others can be found. Simple numerical examples of the latter kind are more quickly solved by the geometrical method of Art. 7.

EXAMPLE 1. To find the remaining functions of the acute angle whose tangent is $\frac{5}{12}$.

(1) *Geometric Method.* The right triangle *OMP* (Art. 6, Fig. 5) has sides of relative length $y = 5$, $x = 12$, whence on the same scale $v = 13$. Thus the sine is $\frac{5}{13}$, etc.

(2) *Analytic Method.* Given $\tan\alpha = \frac{5}{12}$. Then by the formulas of Art. 9,

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{12}{5}; \quad \sec\alpha = \sqrt{1+\tan^2\alpha} = \frac{13}{12};$$

$$\csc\alpha = \sqrt{1+\cot^2\alpha} = \frac{13}{5};$$

$$\cos\alpha = \frac{1}{\sec\alpha} = \frac{12}{13}; \quad \sin\alpha = \frac{1}{\csc\alpha} = \frac{5}{13}.$$

EXAMPLE 2. To express all the functions of α in terms of $\sec\alpha$. By the use of the appropriate formulas of Art. 9, we obtain

$$\cos\alpha = \frac{1}{\sec\alpha}; \quad \sin\alpha = \sqrt{1-\cos^2\alpha} = \frac{\sqrt{\sec^2\alpha - 1}}{\sec\alpha};$$

$$\csc\alpha = \frac{1}{\sin\alpha} = \frac{\sec\alpha}{\sqrt{\sec^2\alpha - 1}}; \quad \tan\alpha = \sqrt{\sec^2\alpha - 1};$$

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{\sqrt{\sec^2\alpha - 1}}.$$

EXAMPLE 3. Verify the following relation by reducing the first member to the second:

$$\frac{\tan^2\beta}{\sec\beta - 1} - 1 = \sec\beta.$$

By means of the formulas of Art. 9, we have

$$\frac{\tan^2\beta}{\sec\beta - 1} - 1 = \frac{\sec^2\beta - 1}{\sec\beta - 1} - 1 = \sec\beta + 1 - 1 = \sec\beta.$$

EXERCISE IV

By means of the formulas of Art. 9, find the values of the remaining functions of each of the following angles, given

1. $\sin\alpha = \frac{17}{145}$. 2. $\cot\beta = \frac{44}{117}$. 3. $\cos\gamma = \frac{44}{117}$. 4. $\tan\gamma = \frac{12}{119}$.

Express all the functions of α in terms of

5. $\tan\alpha$. 6. $\cos\alpha$. 7 $\cot\alpha$. 8. $\sin\alpha$.

Find the values of the following expressions:

9. $\dfrac{\tan\alpha + \cot\alpha}{\tan\alpha - \cot\alpha}$, when $\cos\alpha = \dfrac{9}{41}$.

10. $\dfrac{\sec\alpha - \cos\alpha}{\tan\alpha - \sin\alpha}$, when $\sin\alpha = \dfrac{12}{13}$.

11. $\dfrac{\sin\beta}{1+\cos\beta}+\cot\beta$, when $\tan\beta=\dfrac{20}{21}$.

12. $\cos\beta\cdot\tan\beta+\sin\beta\cdot\cot\beta$, when $\sec\beta=\frac{13}{12}$.

Express the following in terms of a single function:

13. $\dfrac{\csc\alpha}{\cot\alpha+\tan\alpha}$ in terms of $\cos\alpha$.

14. $\dfrac{\sin\beta}{1+\cos\beta}+\dfrac{1+\cos\beta}{\sin\beta}$ in terms of $\csc\beta$.

15. $\sec\gamma-\tan\gamma$ in terms of $\sin\gamma$.

16. $\dfrac{1}{1-\sin\gamma}+\dfrac{1}{1+\sin\gamma}$ in terms of $\tan\gamma$.

Verify the following identities:

17. $\cos^4\beta-\sin^4\beta=2\cos^2\beta-1$.

18. $\cos\alpha\cdot\tan\alpha=\sin\alpha$.

19. $\dfrac{\cot^2\alpha}{1+\cot^2\alpha}=\cos^2\alpha$.

20. $\dfrac{1}{\tan^2\beta+1}+\dfrac{1}{\cot^2\beta+1}=1$.

10. Functions of complementary angles. If, in Fig. 6, $\angle XOB$ is constructed equal to $\angle AOY$, XOB $(=\beta)$ and XOA $(=\alpha)$ are complementary. The triangles $OM'P'$ and OMP are similar, the pairs of corresponding sides being v' and v, x' and y, y' and x.

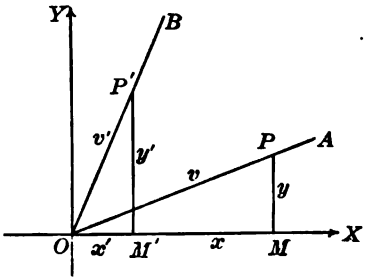

FIG. 6.

We have then

$$\sin(90^\circ-\alpha)=\sin\beta=\frac{y'}{v'}=\frac{x}{v}=\cos\alpha,$$

$$\cos(90^\circ-\alpha)=\cos\beta=\frac{x'}{v'}=\frac{y}{v}=\sin\alpha,$$

$$\tan(90^\circ-\alpha)=\tan\beta=\frac{y'}{x'}=\frac{x}{y}=\cot\alpha,$$

$$\cot(90^\circ-\alpha)=\cot\beta=\frac{x'}{y'}=\frac{y}{x}=\tan\alpha,$$

$$\sec(90° - \alpha) = \sec\beta = \frac{v'}{x'} = \frac{v}{y} = \csc\alpha,$$

$$\csc(90° - \alpha) = \csc\beta = \frac{v'}{y'} = \frac{v}{x} = \sec\alpha.$$

The prefix "co" is thus seen to be the abbreviation of the word "complement's." The general theorem may be stated as follows:

Any trigonometric function of an acute angle is equal to the corresponding co-function of its complementary angle.

Thus, $\sin 72° = \cos 18°$, $\cot 54° = \tan 36°$, etc.

11. Functions of 45°, 30°, 60°. The exact values of the functions of certain angles are readily found.

(1) If, in Fig. 7, $\angle XOA = 45°$, the triangle OMP is isosceles, so that $x = y = \frac{1}{\sqrt{2}}v$. We have at once

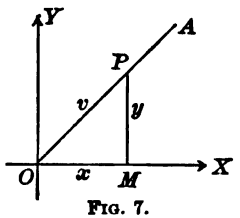

FIG. 7.

$$\sin 45° = \cos 45° = \tfrac{1}{2}\sqrt{2},$$

$$\tan 45° = \cot 45° = 1,$$

$$\sec 45° = \csc 45° = \sqrt{2}.$$

(2) If, in Fig. 8, $\angle XOA = 30°$, and $\angle XOC$ is constructed equal to $-30°$, the triangle QOP is equilateral, so that $y = \frac{1}{2}v$, $x = \frac{1}{2}\sqrt{3}\,v$.

We have, at once,

$$\sin 30° = \tfrac{1}{2}, \quad \cos 30° = \tfrac{1}{2}\sqrt{3},$$

$$\tan 30° = \tfrac{1}{3}\sqrt{3}, \quad \cot 30° = \sqrt{3},$$

$$\sec 30° = \tfrac{2}{3}\sqrt{3}, \quad \csc 30° = 2.$$

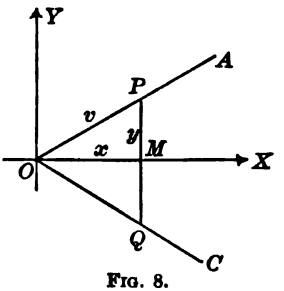

FIG. 8.

(3) In like manner to (2), or by Art. 10, we obtain

$$\sin 60° = \tfrac{1}{2}\sqrt{3}, \quad \cos 60° = \tfrac{1}{2},$$

$$\tan 60° = \sqrt{3}, \quad \cot 60° = \tfrac{1}{3}\sqrt{3},$$

$$\sec 60° = 2, \quad \csc 60° = \tfrac{2}{3}\sqrt{3}.$$

12. Functions of 0° and 90°.

(1) As the $\angle XOA$ (see Fig. 9) decreases so as to approach the limit zero, the abscissa x will approach equality to the radius

vector v. If, at the same time, the radius vector remains finite, the ordinate y will approach the limit zero.

It should be noticed that the cosecant varies in such a manner that its denominator approaches the limit zero while its numerator remains constantly equal to the finite number v, so that the value of the cosecant increases without limit as the angle approaches zero. It is then said to become infinite and is represented by the symbol ∞. The cotangent also becomes infinite as the angle approaches zero, since its numerator, although changing, never exceeds v.

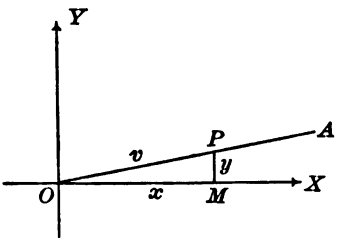

FIG. 9.

We have, then,

$$\sin 0° = 0, \quad \cos 0° = 1,$$
$$\tan 0° = 0, \quad \cot 0° = \infty,$$
$$\sec 0° = 1, \quad \csc 0° = \infty.$$

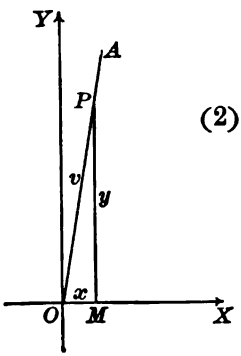

FIG. 10.

(2) In like manner, we obtain

$$\sin 90° = 1, \quad \cos 90° = 0,$$
$$\tan 90° = \infty, \quad \cot 90° = 0,$$
$$\sec 90° = \infty, \quad \csc 90° = 1.$$

EXAMPLE. Solve the equation

$$\sec^2\gamma - \sqrt{3}\tan\gamma = 1.$$

Expressing wholly in terms of $\tan\gamma$,

we have $\tan^2\gamma + 1 - \sqrt{3}\tan\gamma - 1 = 0,$

or $\tan^2\gamma - \sqrt{3}\tan\gamma = 0,$

whence $\tan\gamma = 0$ and $\sqrt{3}.$

Then, by Arts. 11 and 12,

$$\gamma = 0° \text{ and } 60°.$$

EXERCISE V

1. Express in terms of an angle less than 45° the functions of 75°.

2. Express in terms of an angle less than 45° the functions of 65°.

Verify the following for $\alpha = 30°$; also for $\alpha = 45°$:

3. $\sin 2\alpha = 2 \sin \alpha \cos \alpha$.

4. $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$.

5. $\tan 2\alpha = \dfrac{2 \tan \alpha}{1 - \tan^2 \alpha}$.

6. $\cot 2\alpha = \dfrac{\cot^2 \alpha - 1}{2 \cot \alpha}$.

Notice that $\sin 2\alpha$ does *not* equal $2 \sin \alpha$, $\cos 3\alpha$ does *not* equal $3 \cos \alpha$, etc.

Verify for $\alpha = 30°$:

7. $\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$.

8. $\cos 3\alpha = 4 \cos^3 \alpha + 3 \cos \alpha$.

Evaluate the following expressions:

9. $\sin 60° \cos 30° - \cos 60° \sin 30°$.

10. $\cos 60° \cos 30° + \sin 60° \sin 30°$.

11. $\csc^2 45° + \sin 60° \tan 30°$.

12. $\sin 60° \tan 45° - \sec 30° \sin^2 45° \cot 60°$.

Solve each of the following equations for some one function of α and find the angle in degrees. Verify the results by substitution in the given equation.

13. $\tan \alpha - 3 \cot \alpha = 0$.

14. $\sec \alpha + 2 \cos \alpha = 3$.

15. $4 \sin^2 \alpha + 3 \cot^2 \alpha = 4$.

16. $\sin \alpha + 3 \cos \alpha = 2\sqrt{2}$.

17. A ladder 22 feet long leans against a wall, its foot being 11 feet away from the wall. Find (*a*) the angle formed by the ladder with the ground; (*b*) the height of the top of the ladder above the ground.

18. The diagonal of a rectangle makes an angle of 30° with the long side. If the length of this side is 14 inches, what is the length of the short side of the rectangle? of the diagonal?

19. The side of a conical pile of sand makes an angle of 45° with the floor. If the height from the floor is 12 feet, what is the area of the base?

20. A guy rope (assumed to be straight) has a length of 60 feet and extends from the top of a mast 30 feet high to the ground. Find the angle between the rope and the mast.

13. Variation of the trigonometric functions as the angle varies. Suppose the angle θ to vary continuously from 0° to 90°. The

terminal side revolves about the origin from the position OX to the position OY. If v remains constant, y will increase from 0 to v, while x will decrease from v to 0. Consequently, the numerator of the fraction $\frac{y}{v}(=\sin\theta)$ increases from 0 to v, while the denominator remains constant. Hence, while θ increases from 0° to 90°, $\sin\theta$ increases from 0 to 1.

The numerator of the fraction $\frac{x}{v}(=\cos\theta)$ decreases from v to 0, while the denominator remains constantly equal to v. Hence, while θ increases from 0° to 90°, $\cos\theta$ decreases from 1 to 0.

The numerator of the fraction $\frac{y}{x}(=\tan\theta)$ increases from 0 to v, while the denominator decreases from v to 0. Hence, while θ increases from 0° to 90°, $\tan\theta$, beginning with zero, increases without limit as θ approaches 90°. We express this by saying that $\tan\theta$ varies from 0 to ∞.

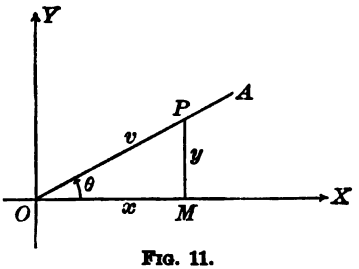

FIG. 11.

The student should trace carefully the variation of the other trigonometric functions and compare the results with the values found in Arts. 11 and 12. Article 7 should be read again at this point.

14. Inverse trigonometric functions. The same functional relation is expressed by the two statements, "m is the sine of the acute angle α" and "α is the acute angle whose sine is m." The corresponding symbolic notations are

$$m = \sin\alpha, \qquad \alpha = \arcsin m,^*$$

with the understanding that α is an acute angle and that m is a positive number not greater than unity. A similar symbolic relation holds for the other trigonometric functions. It is frequently read "arc-sine m," or "anti-sine m," since two mutually inverse functions are said each to be the anti-function of the other.

* This notation is universally used in Europe and is fast gaining ground in this country. A less desirable symbol,

$$\alpha = \sin^{-1} m,$$

is still found in English and American texts.

The notation $\alpha = \text{inv}\sin m$ is perhaps better still on account of its general applicability. (See Art. 80.)

The inverse notation is convenient for the statement of problems. The purposes of interpretation and manipulation are better served by transforming to the corresponding direct notation.

EXAMPLE. Find the value of $\sin(90° - \operatorname{arccot} \frac{5}{12})$.

In the direct notation the example reads: Given $\cot \alpha = \frac{5}{12}$, find $\sin(90° - \alpha)$. Then, by Arts. 10 and 9,

$$\sin(90° - \alpha) = \cos \alpha = \tfrac{5}{13}.$$

EXERCISE VI

1. Trace the variation in value of $\sec \theta$.
2. Trace the variation in value of $\csc \theta$.

Find the values of the following:

3. $\tan(\cos^{-1} \frac{8}{17})$.
4. $\sin(\operatorname{arccot} \frac{4}{3})$.
5. $\cos(90° - \arctan \frac{7}{24})$.
6. $\cot(90° - \sin^{-1} \frac{4}{65})$.
7. $\sec(90° - \operatorname{arcsec} 2)$.
8. $\csc(90° - \operatorname{arccsc} \sqrt{2})$.
9. $\sin(2 \tan^{-1} 1)$.
10. $\cos(2 \sin^{-1} \frac{1}{2})$.

Solve the following equations:

11. $2 \sin^2 \beta + 3 \cos \beta - 3 = 0$.
12. $\sec \beta - 2 \tan \beta = 0$.
13. $\tan \beta (2 \sin \beta - 1)(\sec \beta - \sqrt{2}) = 0$.
14. $\sin \beta (2 \cos \beta - \sqrt{3})(\tan \beta - 1) = 0$.

Verify the identities:

15. $\sin^4 \alpha + \cos^4 \alpha + \sin^2 \alpha \cos^2 \alpha = \sin^4 \alpha - \sin^2 \alpha + 1$.
16. $(\csc \alpha - \cot \alpha)(\csc \alpha + \cot \alpha) = 1$.
17. $(\tan \alpha + \cot \alpha)(\sin \alpha \cdot \cos \alpha) = 1$.
18. $1 - \tan^4 \alpha = 2 \sec^2 \alpha - \sec^4 \alpha$.
19. $\sin^6 \alpha + \cos^6 \alpha = 1 - 3 \sin^2 \alpha \cos^2 \alpha$.
20. $\cos^4 \alpha - \sin^4 \alpha = 1 - 2 \sin^2 \alpha$.

15. Orthogonal projection. In accordance with the definitions of Art. 4 (see note, page 4) it follows that the projection of a line segment on any line is equal to the length of the line segment multiplied by the cosine of the angle formed by the line segment with the line of projection. Thus, in Fig. 12, the projection of AB on RS is

$$MN = AB \cos \alpha.$$

In like manner, the projection of AB on a line perpendicular to RS (*i.e.* making $+90°$ with RS) has the value $AB \sin \alpha$. These projections are called the *components* of the line segment AB along and at right angles to the direction RS.

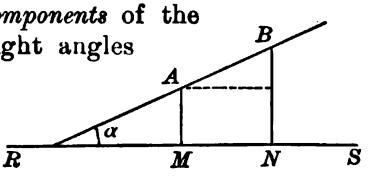

FIG. 12.

In physics, line segments are often used to represent quantities that have direction as well as magnitude; for example, forces, velocities, accelerations. The components of the line segment used to represent a force represent components of the force; likewise for a line segment representing a velocity, acceleration, or moment. Suppose, for example, that the line segment AB, Fig. 13, represents a force applied to the block m resting on a horizontal plane. This segment has the component EB parallel to the plane and the component FB perpendicular to the plane. Segment EB represents a force component F_x parallel to the plane, which tends to move the block along the plane; segment FB represents a force component F_y perpendicular to the plane and tending to produce pressure between the block and plane. Denoting by F the force represented by AB, we have

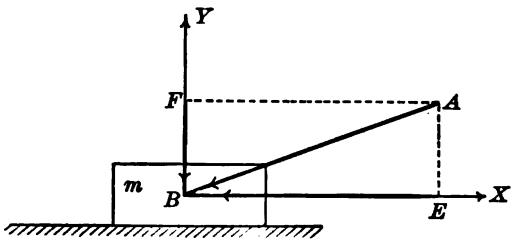

FIG. 13.

$$F_x = F \cos \alpha, \qquad F_y = F \sin \alpha.$$

EXAMPLE. At a given instant a point is moving in a direction at an angle of 30° with a given horizontal line with a velocity of 20 feet per second. Find the component of the velocity along the line.

Taking the given line as the X-axis, we have for the component v_x

$$v_x = v \cos 30° = 20 \times \tfrac{1}{2}\sqrt{3} = 17.321 \text{ feet per second.}$$

The component along a line perpendicular to the given horizontal line in the plane of motion is

$$v_y = v \sin 30° = 20 \times \tfrac{1}{2} = 10 \text{ feet per second.}$$

EXERCISE VII

The student should draw appropriate figures for each of the following exercises.

1. Find the projections of a line segment 8.5 inches in length on the X- and Y-axes, (*a*) when the segment makes an angle of 45° with the X-axis; (*b*) when it makes an angle of 60° with the Y-axis.

2. A crank 16 inches long rotates in a vertical plane. When the crank makes an angle of 30° with the horizontal diameter of the circle described by the moving end, what is the distance of the moving end, (*a*) from the horizontal diameter? (*b*) from the vertical diameter?

3. If in Fig. 13 the force F denoted by AB is 40 pounds, find the components F_x and F_y, (*a*) when $\alpha = 30°$; (*b*) when $\alpha = 45°$. Discuss the cases $\alpha = 0°$ and $\alpha = 90°$.

4. A steamer is moving at a speed of 18 miles per hour in a direction north of east, making an angle of 30° with an east and west line. At what rate is the steamer sailing eastward? at what rate northward?

5. A guy wire exerts a pull of 3000 pounds on its anchorage and makes an angle of 30° with the ground. Find the component of this force, (*a*) along the ground; (*b*) vertical.

6. The eastward and northward components of the velocity of a moving body are found to be $v_x = 12$ miles per hour and $v_y = 12\sqrt{3}$ miles per hour, respectively. Find (*a*) the magnitude and (*b*) the direction of the body's velocity.

CHAPTER III

RIGHT TRIANGLES

16. Laws for solution. If, in a right triangle, two independent parts are known, in addition to the right angle, the three remaining parts can be found. Thus two given parts, at least one of which is a side, determine a right triangle. The formulas needed in all cases to effect this solution are five in number. Two are the statements of well-known geometric theorems, while the other three are the immediate consequences of the definitions of the trigonometric ratios contained in Art. 6.

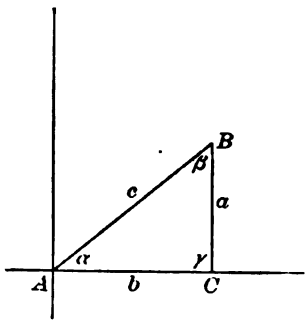

FIG. 14.

In Fig. 14 let ACB be a right triangle, right-angled at C. We shall denote the interior angles at the vertices by α, β, γ, and the lengths of the sides opposite them by a, b, c, respectively. Note that $\gamma = 90°$, and c is the hypotenuse.

The five formulas are the following:

$$a^2 + b^2 = c^2, \tag{1}$$

$$\alpha + \beta = 90°, \tag{2}$$

$$\frac{a}{c} = \sin\alpha = \cos\beta, \tag{3}$$

$$\frac{b}{c} = \cos\alpha = \sin\beta, \tag{4}$$

$$\frac{a}{b} = \tan\alpha = \cot\beta. \tag{5}$$

Equation (1) follows from the Pythagorean theorem, and (2) from the fact that the sum of the angles of a triangle is equal to two right angles. In order to establish the last three, place the triangle on the axes of coördinates described in Art. 4, the side AC extending from the origin to the right along the X-axis, and the hypotenuse lying in the first quadrant, as in Fig. 14.

Then b, a, c, are respectively the abscissa, ordinate, and radius vector of B, a point on the terminal side of the angle α, which is conventionally placed.

It follows at once from Art. 6 that

$$\frac{a}{c} = \sin \alpha,$$

$$\frac{b}{c} = \cos \alpha,$$

$$\frac{a}{b} = \tan \alpha.$$

The corresponding values of the functions of the angle β result from Art. 10.

17. Area of right triangles. The formulas for the area of a right triangle follow from the familiar geometric theorem

$$\text{Area} = \tfrac{1}{2} \times \text{base} \times \text{altitude},$$

or expressed symbolically,

$$A = \tfrac{1}{2}\, ab. \tag{1}$$

The substitution for a of its value from the preceding article gives

$$A = \tfrac{1}{2}\, bc \sin \alpha. \tag{2}$$

Again, introducing the values of both a and b,

$$A = \tfrac{1}{2}\, c^2 \sin \alpha \cdot \cos \alpha. \tag{2}$$

Other formulas for the area may also be obtained.

18. Method of solution. The solution of any problem consists of four parts: the analysis, the algebraic solution, the arithmetical computation, and the interpretation of the results.

(1) The student should read and analyze the problem, noting which parts are known and which are desired. The construction of a neat and sufficiently accurate figure is helpful and advisable.

(2) The student should select from the five formulas of Art. 16 those containing a single unknown part each, in addition to the known parts, and should solve them for these unknown parts while still in the literal form.

Experience has led to the adoption of the following two rules of procedure:

(*A*) The use of the Pythagorean formula, $a^2 + b^2 = c^2$, is to be avoided save when the data are very simple or a table of squares and square roots is at hand.

(*B*) So far as is consistent with rule *A*, each unknown part should be found in terms of those parts originally given in the problem, in order to avoid accumulation of errors.

In conformity with these rules, the angle relation $\alpha + \beta = 90°$, and two of the three trigonometric formulas serve to effect the solution, while the remaining trigonometric formula affords a check on the work.

(3) The solution is now effected by introducing the numerical data and performing the necessary computations. The correctness and accuracy of the results are greatly enhanced by extreme orderliness of arrangement and neatness of detail.

The use of the trigonometric tables and the employment of suitable checks will be discussed in subsequent articles.

(4) The geometric or physical significance of the results obtained should be fully considered and interpreted.

EXAMPLE 1. Given $c = 254$, $\alpha = 30°$, to find a, b, β.

In this instance the analysis and construction are obvious.

The three appropriate formulas yield at once the forms

$$\beta = 90° - \alpha,$$

$$a = c \sin \alpha,$$

$$b = c \cos \alpha.$$

The formula $b = a \tan \beta$ affords the check.

On introducing the numerical data, we obtain

$$\beta = 90° - 30° = 60°,$$

$$a = 254 \times \tfrac{1}{2} = 254 \times .5 = 127,$$

$$b = 254 \times \tfrac{1}{2}\sqrt{3} = 254 \times .86605 = 219.9767.$$

The check formula gives

$$b = 127 \times 1.7321 = 219.9767.$$

EXAMPLE 2. Given $a = 39.00$, $b = 80.00$, to find c, α, β, and the area.

As before, we may pass immediately to the second stage. Now c is given directly in terms of a and b by the formula $c = \sqrt{a^2 + b^2}$.

If we are to avoid the use of this formula, we must first find α and β, and then get c by means of one of these angles. We use the forms:

$$\tan \alpha = \frac{a}{b},$$

$$\beta = 90° - \alpha,$$

$$c = \frac{a}{\sin \alpha},$$

$$A = \tfrac{1}{2}\, ab,$$

and for the check $$c = \frac{b}{\sin \beta}.$$

We obtain, then $\tan \alpha = 39 \div 80 = .4875,$

$\alpha = 25° 59'$, as found from Table III,

$\beta = 90° - 26° 59' = 64° 01',$

$c = 39 \div .4381 = 89.01,$

$A = \tfrac{1}{2} \times 39 \times 80 = 1560,$

and for the check $c = 80 \div .8989 = 89.00,$

showing a difference of .01.

On account of the simplicity of the numbers, we may, by using the formula $c^2 = a^2 + b^2$, find, exactly, $c = 89$.

Explain the accumulation of errors and, hence, the reason for rule of procedure (*B*).

EXAMPLES. 1. Given $c = 42$, $\beta = \arcsin .28$; find a and b.

2. Given $b = 27$, $\alpha = \tan^{-1} .75$; find a and c.

3. Given $a = 300$, $\alpha = \cos^{-1} .45$; find c and b.

4. Given $c = 200$, $\alpha = \operatorname{arccot} 1.12$; find a and b.

19. Trigonometric tables. In the first example worked in the preceding article, the functions of 30° had been determined in Art. 11. In the second example, however, the value of $\tan \alpha$ was not one of those previously ascertained, and the value of α was not recognizable from its tangent. For convenience of reference the numerical values of the sines, cosines, tangents, and cotangents of all angles differing by intervals of one minute from 0° to 90° have been collected in Table III, on pages 71–89. The arrangement is simple and plain. The degree numbers from 0° to 44° occur at the top of the page, with the minutes running down the left margin.

The numerical values of the functions, computed to four decimal places, are placed in columns under the names of the functions.

Since $\sin(90° - \alpha) = \cos\alpha$, and $\tan(90° - \alpha) = \cot\alpha$, the space required may be reduced one half. The degree numbers of angles from 45° to 90° are printed at the bottom of the pages in reversed order, the minutes run up the right margin, and the names of the functions are in reversed order at the bottom.

For the present the student need not concern himself with smaller divisions of the angle than the minute. Further refinement is attained by a method to be described in Art. 26.

Table IV, on pages 91–93, contains the squares of numbers less than 1000 and, by interpolation, of numbers up to 9999. The first page gives directly the squares of numbers from 1 to 100. On the second and third pages the tens and units digits of the number to be squared are in the left margin, while the hundreds digits are at the tops of the several columns. The last two figures of the square are in the column at the right under U., opposite the tens and units digits; the first three, or four, figures of the square are in the same line in the column under the hundreds digit. In the right margin are the last two figures of the tabular difference used in interpolation, to which must be prefixed the remainder obtained by subtracting the first three, or four, figures of the square from those in the same column immediately beneath, or that remainder diminished by 1 when the asterisk (*) is present. The use of the table is best shown by illustration.

EXAMPLES. 1. $328^2 = 107{,}584.$

2. $475.3 = 475^2 + .3 \times 951 = 225{,}625 + 285 = 225{,}910.$*

3. $28.37^2 = 28.3^2 + .07 \times 567 = 800.89 + 3.97 = 804.56.$

Square roots are extracted by reversing the process; thus,

4. $\sqrt{27556} = 166.$

5. $\sqrt{658{,}037} = \sqrt{657{,}721} + 316 + 1623 = 811 + .2 = 811.2.$

20. Errors and checks. The results obtained are not always, nor even usually, exactly correct. The deviations from the true values are of two sorts, mistakes and errors, and a sharp distinction must be made between them.

* This result is, of course, only approximately correct. The true result may be obtained as follows:

$$475.3^2 = 475^2 + .3 \times (475.3 + 475) = 225{,}625 + 285.09 = 225{,}910.09.$$

The data for problems arising in actual practice are derived from observations made with instruments for measuring lengths, angles, etc.

Mistakes may arise from a false reading of the observing instrument, a misapprehension of the problem, the employment of the wrong formula, faulty addition, etc. They are never allowable or excusable.

On the other hand, instruments are so constructed as to yield results only to a certain degree of precision, which should be ascertained for each instrument. Moreover, observation is performed by the human apparatus, eyes, ears, etc., and a certain personal equation, an anticipation or lagging in sight or hearing, is always present, varying with personal fitness and experience. Methods of eliminating instrumental errors, so as to obtain the maximum precision possible with the instruments used, are given in standard works on engineering instruments. Again, the arithmetical calculation involves the trigonometric ratios, which are, in general, non-terminating decimal fractions, while their values in the mathematical tables are computed only to a certain number of decimal places. Errors, therefore, will always be present; but every precaution should be taken to keep the errors due to computation well within the limits of error of the observed data and desired results fixed by the nature of the problem.

In both observation and solution, certain additional processes are employed to avoid, or to reveal, mistakes. These processes are known as checks and vary with the nature of the problem.

While no general rules for checks can be laid down, a frequent practice in the solution of triangles is to make use of a formula connecting the required parts, just found, noting if the results are within the range of allowable error. The size of this allowable error should be known for each table.

As a check to arithmetical computation, graphical construction is well understood and strongly advised. As a means of avoiding the grosser mistakes, a free-hand sketch will frequently suffice by guiding the student to a reasonable interpretation of data, and indicating possible results.

A drawing constructed to scale will further aid by yielding values more or less approximate, approaching those obtained by computation.

Carried a step farther as regards accuracy, by the use of precise instruments, the graphical construction often attains to the

dignity of an independent solution, with results falling within the limits prescribed by the physical conditions of the problem.

There is no better evidence of careful work than the recording of a reasonable error obtained by the comparison of two methods. In practical work the allowable per cent of error becomes an important consideration.

EXERCISE VIII

Find the missing parts of the following triangles, using the natural trigonometric functions, Table III.

	α	β	a	b	c	A
1.	25° 10′				34	
2.	52° 20′				73	
3.		61° 15′			243	
4.		78° 35′			521	
5.	21° 25′		235			
6.	72° 45′		720			
7.		80° 30′	1200			
8.		17° 30′		1500		
9.			240		360	
10.			381		715	
11.				521	630	
12.				840	1400	
13.			648	864		
14.			595	600		
15.			215	385		
16.			2111	1234		
17.			95			.7980
18.				264		30360
19.	74° 20′					1225
20.		24° 50′				843

21. In the same vertical plane the distances shown in Fig. 15 were measured in feet along the surface of the ground. The distances of the different points below the instrument, as measured by a rod, are given also in feet. The vertical scale is exaggerated for clearness. What is the horizontal distance from B to G? (Check by a table of squares and square roots.)

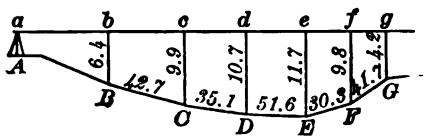

FIG. 15.

22. A line surveyed across a ridge is 1500 feet in horizontal length. Stakes are set 100 feet apart horizontally by level chaining. By leveling, the elevations of the surface at the different stakes is obtained as follows: 730.2, 735.9, 739.7, 743.4, 750.1, 751.8, 760.7, 764.1, 764.3, 765.8, 765.0, 763.2, 758.3, 750.2, 743.1, 740.2. What length of wire will be required for fencing along this line? (Check by a table of squares and square roots.)

23. If a gravel roof slopes one half inch to the horizontal foot, what angle does it make with the horizon?

24. If the face of a wall has a batter or inclination of one inch in one vertical foot, what is its angle with the vertical?

25. What is the angle of ascent of a railway built on a 2 per cent grade (*i.e.* 2 vertical feet to 100 horizontal feet)?

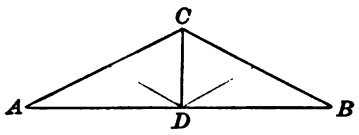

FIG. 16.

26. The pitch of a roof is the ratio $\frac{CD}{AB}$. (See Fig. 16.) What is the inclination to the horizon of a roof with $\frac{1}{2}$ pitch, $\frac{1}{4}$ pitch, $\frac{1}{8}$ pitch?

27. What is the pitch of a roof sloping to the horizon at 15°, 30°, 45°?

28. What is the inclination to the horizon of the corner or hip rafter of a pyramidal roof whose pitches are $\frac{1}{2}$?

29. What is the inclination from the vertical of the corner edge of a wall, both of its faces having a batter of $\frac{1}{10}$?

30. At what angle does a railway slope if it has a grade of 0.25%, 0.5%, 2.5%?

31. At what angle must a cog railway ascend in order to rise 2640 feet in one horizontal mile?

32. A battleship known to be 341 feet long subtends an angle of 3° 20′ when presenting its broadside to a fort on shore. For what distance should guns be sighted when trained upon it? (Note that the isosceles triangle having the length of the ship for its base is separable into two right triangles.)

33. In planning the stairway for a house it is desired that the riser, or vertical distance between steps, shall be 7 inches, and the treads, or horizontal distances between faces, 11 inches. What will be the angle of inclination of the hand rail?

34. Taking the data of the preceding problem, what will be the length of the hand rail if straight, provided the height between floors is 11 feet 8 inches?

35. A cylindrical water tower whose external diameter is 25 feet subtends a horizontal angle of 5° 30′ as viewed from a distance. How far is its center from the instrument?

(Note that we have a triangle that is right-angled when the line of sight is tangent. The base is the radius of the tower and the opposite angle is half of the one observed.)

36. What horizontal angle would be subtended, at a distance of 2 miles, by a vertical cylindrical gas receiver 60 feet in diameter?

(See note to problem 35.)

37. The end of a pendulum 34 inches long swings through an arc of $3\frac{1}{2}$ inches. Find the angle through which the pendulum swings.

38. When vertically over a village, a balloon's angle of inclination, as viewed from 9 miles distant, was 15° 20′. Assuming the surface of the country to be fairly level, what was the height of the balloon?

39. A flagstaff 110 feet high is covered by a vertical angle of 12° 30′ at a point approximately on a level with its center. How far is the observer from the staff?

40. The data of a preliminary survey are as follows:

$AB = 240.9$ feet.	Angle at $B = 62° 11'$ left.
$BC = 310.7$ feet.	Angle at $C = 55° 50'$ left.
$CD = 611.5$ feet.	Angle at $D = 43° 42'$ right.
$DE = 237.2$ feet.	Angle at $E = 51° 23'$ right.
$EF = 528.0$ feet.	

Considering A, Fig. 17, as the origin of coördinates and AB as the axis of abscissas, it is required to compute coördinates for all points given, thus providing for the accurate mapping of the survey.

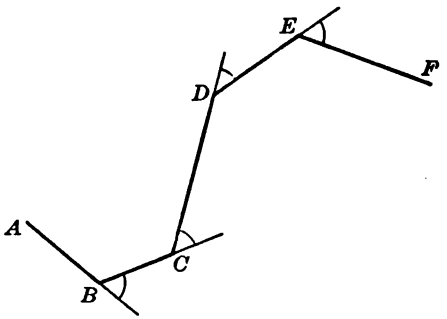

FIG. 17.

41. Find the missing parts and area of the following isosceles triangles (see Fig. 18 for lettering):

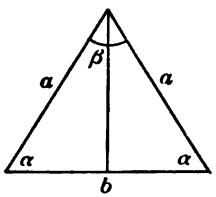

FIG. 18.

$\alpha = 35°$, $a = 42$; $\alpha = 72°$, $b = 125$;

$a = 350$, $b = 180$; $\beta = 54°$, $a = 360$;

$\beta = 51° 26'$, $b = 480$; $a = 640$, $b = 840$.

42. Find the lengths of the chords of the following arcs in terms of the radius: 30°, 36°, 40°, 45°, 60°, 75°, 90°, 120°. Compute, given $R = 100$.

43. Express in terms of the sine and radius the relation between the chord of an arc and the chord of half the arc.

44. Express in trigonometric form the most important relations between the radius R of the circumscribed circle, the radius r of the inscribed circle, and the side s of a regular polygon of n sides.

45. Compute and tabulate the perimeter and the circumferences of the circum- and in-circles of a regular polygon of n sides for $n = 4, 8, 16, 32$, given $R = 10$.

46. Compute and tabulate the area of a regular polygon of n sides and of its circum- and in-circles for $n = 4, 8, 16, 32$, given $R = 10$.

47. Repeat example 45 for $n = 6, 12, 24, 48$.

48. Repeat example 46 for $n = 6, 12, 24, 48$.

49. A body is acted upon by three forces of magnitudes 20, 40, 60, parallel to the sides of an equilateral triangle. Resolve these forces along two perpendicular axes, then combine, and thus find the magnitude and direction of the resultant.

50. A body situated at one vertex of a regular hexagon is acted upon by five forces represented in magnitude and direction by the vectors drawn to the five other vertices. Resolve along and perpendicular to the diameter through the point and find the magnitude and direction of the resultant.

51. A point describes a circle with uniform speed. Determine the position of its projection upon a diameter in terms of its angular displacement from that diameter.

52. A point describes a circle of radius 30 feet at a rate of 3 revolutions per minute. Find the position of its projection upon a diameter at the end of 15 seconds after passing the extremity of that diameter.

53. Determine the components of the vertical acceleration g along and perpendicular to a plane inclined at an angle α to the horizon.

54. If $g = 32$, find the acceleration along and perpendicular to a plane whose inclination is 30°, 15°, 10°, 5°.

55. A man weighing 150 pounds stands midway on a 30-foot ladder whose foot is 10 feet horizontally from the vertical wall against which it leans. Find the normal (perpendicular) pressure on the ladder and the force tending to cause him to slide along the ladder.

56. Find the components along the X- and Y-axes of a force of 65 pounds making an angle of 28° 13′ with the X-axis.

57. A steamer is sailing in such a way that its speed due east is 12 miles per hour and its speed due south is 14 miles per hour. Find the direction of the steamer's course and the speed in that course.

58. In an oblique triangle, angle $B = 45°$, angle $C = 32°$, and side $b = 16$. Find side c. (SUGGESTION. Draw the perpendicular from the vertex A upon the opposite side.) Attempt to deduce a general relation between the functions of the acute angles of an oblique triangle and the opposite sides.

CHAPTER IV

LOGARITHMS

21. Definition of a logarithm. If we have given

$$56 = 10^{0.74819},\ 79 = 10^{0.89763},$$

we can find the product of 56 and 79 without performing the operation of multiplication, provided we know in advance the powers of 10. For, we have from the general laws governing exponents,

$$\begin{aligned} 56 \times 79 &= 10^{0.74819} \times 10^{0.89763} \\ &= 10^{0.74819 + 0.89763} \\ &= 10^{1.64582} = 4424. \end{aligned}$$

It will be seen that the process of multiplication has been replaced by the simpler one of addition.

Many other processes in computation can be simplified in a similar manner; for example, if we wish to find the cube root of a number, say 89.1, we have

$$89.1 = 10^{1.94988},$$

and consequently

$$\sqrt[3]{89.1} = (10^{1.94988})^{\frac{1}{3}} = 10^{0.64996} = 4.466^{+}.$$

In this case the extraction of a root has been accomplished by the simple process of division. In order to extend this method we must know all of the powers of some convenient number. The exponents involved are called logarithms, and the number raised to a power is referred to as the base of the logarithmic system. We may define a logarithm more exactly as follows:

If a is any number and x and n are so related that $a^x = n$, then x is called the *logarithm* of n to the base a; that is, a logarithm is the index of the power to which the base must be raised to obtain the given number.

This relation is denoted symbolically by writing

$$x = \log_a n,$$

and is read "x is equal to the logarithm of n to the base a."

Thus 3 is the logarithm of 8 to the base 2, since $2^3 = 8$; and in the illustrations given above, 0.74819 is the logarithm of 56 to the base 10, etc.

The two statements

$$a^x = n, \quad x = \log_a n$$

are inverse to each other, just as are the relations $\sin x$ and $\arcsin x$, etc., of Art. 14.

EXERCISE. Find by inspection $\log_3 27$, $\log_5 .625$, $\log_8 32$, $\log_5 .04$.

The logarithm of a number to itself as base is unity, since $n^1 = n$.

The logarithm of 1 to any base other than zero is zero, since $a^0 = 1$.

In conformity with the definition just laid down, it follows that, if two numbers are equal, their logarithms to the same base are equal. It is also true conversely, that if the logarithms of two numbers to the same base are equal, the numbers are equal.*

If the base is real and positive, real logarithms produce only positive numbers. If the base is real and negative, even logarithms produce positive numbers; odd logarithms, negative numbers. For this reason only real positive bases are chosen in practice, and only positive numbers are combined by the aid of their logarithms. The sign of the result is ascertained entirely apart from the numerical computation.

22. Laws of combination. Logarithms are important in trigonometry and elsewhere as labor-saving devices in calculations with numbers containing many digits. Only so much of the theory of logarithms as is necessary for this purpose will be developed in the present chapter.

The laws of combination of numbers by the aid of their logarithms follow at once from the definition of the preceding article.

I. *The logarithm of the product of two factors is equal to the sum of their logarithms, all to the same base.*

For, if $x = \log_a n$ and $y = \log_a m$ we may write

$$n = a^x \text{ and } m = a^y.$$

* In the theory of analytic functions a broader definition of the logarithm is laid down, and the statement just made requires modification.

Multiplying, we have, by the exponential law,

$$nm = a^{x+y},$$

whence, $$\log_a nm = x + y \equiv \log_a n + \log_a m. \tag{1}$$

This law may evidently be extended to any finite number of factors.

II. *The logarithm of the quotient is equal to the logarithm of the dividend minus the logarithm of the divisor, all to the same base.*

For, if $x = \log_a n$ and $y = \log_a m$, we may write as before,

$$n = a^x, \quad m = a^y.$$

Dividing, we have $\frac{n}{m} = a^{x-y}$,

whence, $$\log_a \left(\frac{n}{m}\right) = x - y = \log_a n - \log_a m. \tag{2}$$

Manifestly $\log_a \left(\frac{1}{m}\right) = -\log_a m$.

III. *The logarithm of the power of a number is equal to the logarithm of the number multiplied by the index of the power.*

For, if $x = \log_a n$, then $n = a^x$.

Hence, $$n^p = (a^x)^p = a^{px}$$

or, $$\log_a (n^p) = px = p \log_a n. \tag{3}$$

IV. *The logarithm of the root of a number is equal to the logarithm of the number divided by the index of the root.*

For, if $x = \log_a n$, then $n = a^x$. Extracting the qth root of both members, we get

$$\sqrt[q]{n} = a^{\frac{x}{q}},$$

whence, $$\log_a \sqrt[q]{n} = \frac{x}{q} = \frac{1}{q} \log_a n. \tag{4}$$

23. Common logarithms. Any number may be used as a base of a system of logarithms. For certain purposes the so-called natural system of logarithms, which has for its base the number $e = 2.71828183\cdots$, has advantages. For the purposes of ordinary numerical computation, however, it is most convenient to employ for the base of the system of logarithms, 10, the base of the universally adopted system of numeration.

The common logarithms of all exact integral powers of 10 are positive integers; for instance

$$\begin{aligned}\log_{10}(1000000) &= \log_{10}(10^6)\\ &= 6\log_{10}10\\ &= 6.\end{aligned}$$

The logarithms of reciprocals of integral powers of 10 are negative integers; thus

$$\begin{aligned}\log_{10}(.00001) &= \log_{10}(10^{-5})\\ &= -5\log_{10}10\\ &= -5.\end{aligned}$$

The logarithms of numbers situated between two consecutive integral powers of 10, say between 10^k and 10^{k+1}, lie between k and $k+1$, where k is any integer, positive or negative. Thus

$$10^3 < 2417 < 10^4,$$

whence, $$3 < \log_{10} 2417 < 4,$$

or, $$\log_{10} 2417 = 3 + \text{a number lying between 0 and 1.}$$

The logarithms of numbers greater than the base consist of an integer plus a proper fraction. The fractional part is written decimally, calculated to a number of decimal places, depending on the degree of accuracy desired in the use of the table. The integral part of the logarithm is called the *characteristic;* the decimal fraction, its *mantissa.*

Hereafter, in this book, except in Chapter IX, we shall have to do only with common logarithms and, unless otherwise expressly stated, $\log n$ will denote $\log_{10} n$.

24. Characteristic. If one number is equal to another number multiplied by a factor which is a power of 10, the logarithms of the two numbers differ by an integer. For

$$\begin{aligned}\log(10^k \times n) &= \log(10^k) + \log n\\ &= k + \log n.\end{aligned}$$

EXAMPLE. $$\begin{aligned}\log 34000 &= 3 + \log 34\\ &= 4 + \log 3.4, \text{ etc.}\end{aligned}$$

Every number containing one digit at the left of the decimal point lies between 10^0 and 10^1. The characteristic of its logarithm

is therefore 0. The cipher should always be written to indicate that the characteristic has not been overlooked.

Every number containing k digits at the left of the decimal point is 10^{k-1} times a number with one digit at the left. The characteristic is therefore $k-1$. We have then the following rule for the characteristic:

The characteristic of the logarithm of any number greater than unity is one less than the number of digits at the left of the decimal point.

Should the number be less than unity, move the decimal point ten places to the right (thus multiplying by 10^{10}) and apply the same rule as before, then write -10 after the logarithm for correction. Thus

$$\log 7.12 = 0.85248,$$

$$\log 71200 = \log (10^4 \times 7.12)$$
$$= 4.85248,$$

$$\log .00712 = \log (10^{-10} \times 71200000)$$
$$= \log (10^{-10} \times 10^7 \times 7.12)$$
$$= 7.85248 - 10.$$

The positive part of the last characteristic is seen to be the difference found by subtracting from 9 the number of ciphers immediately following the decimal point in the number.

The characteristic of the logarithm of any number less than unity is found by subtracting from 9 the number of ciphers between the decimal point and the first significant digit, then affixing -10.

25. Mantissa. We have seen that moving the decimal point in the number merely changes the characteristic of the logarithm, leaving its mantissa unaltered. The mantissa depends solely upon the sequence of significant digits.

In the tables given, the logarithms are computed to five decimal places (see pp. 1–21), and the mantissas alone for all numbers from 100 to 9999 are given, arranged in the following manner: Running down the left margin, under N, are to be found the first three digits of the number. In the next (open) column occur the first two figures of the mantissa. In the next ten columns are the remaining three figures of the mantissa arranged under the fourth digit of the number at the top of the columns.

Thus to find the mantissa of log 3814, we select the row having 381 in the left margin. The first two figures of the mantissa, 58, are found in the first column. The three remaining figures, 138, are found in the column headed 4, the fourth digit of the number, giving the mantissa .58138.

To avoid repetition, the first two figures, 58, are not printed in every line, but are to be used from 3802 to 3890, inclusive. The prefixed asterisk, *006, denotes that the mantissa of 3891 is .59006, not .58006.

EXERCISE IX

1. Find by inspection $\log_2 16$, $\log_9 27$, $\log_4 \frac{1}{16}$.

2. Find by inspection $\log_3 81$, $\log_8 32$, $\log_{27} 9$.

3. What numbers correspond to the following logarithms to base 4: 0, 1, 2, 2.5, 3, -2, -3?

4. What numbers correspond to the following logarithms to base 8: 0, 1, $1\frac{1}{3}$, $-\frac{2}{3}$, -2?

5. Find by logarithms: (*a*) $\dfrac{693}{1467}$; (*b*) $\dfrac{.0872 \times 144}{778}$.

6. Find (*a*) $\sqrt{793}$; (*b*) $\sqrt[3]{.007}$; (*c*) $\sqrt[5]{91}$.

7. Find $\sqrt{\dfrac{18 \times \sqrt{240} \times 75^3}{72 \times \sqrt{640} \times 200}}$.

8. Find $\left(\dfrac{32 \times \sqrt[3]{720} \times 15^4}{72 \times \sqrt{480} \times 24^3}\right)^{\frac{2}{3}}$.

9. Find $\left(\dfrac{14.7}{65}\right)^{\frac{k-1}{k}}$, where $k = 1.41$.

10. Solve for x: $4^x = 24$.

11. Solve for x: $6^x = 25$.

The amount A attained by a principal P at interest at the rate r compounded annually for n years is

$$A = P(1 + r)^n.$$

12. Find the amount of \$3680 at 4 per cent in 6 years.

13. Find the principal which, in 7 years at 5 per cent, amounts to \$5820.

14. At what rate will \$5000 amount to \$7500 in 8 years?

15. In how many years will \$86,500 amount to \$129,600 at $3\frac{1}{2}$ per cent?

16. If a city increases its population $\frac{1}{3}$ each year, in how many years will it double its size?

26. Interpolation. It will be shown in Art. 79 that the difference in the logarithms of two numbers is approximately proportional to the difference in the numbers provided these differences are small. Thus, approximately,

$$\frac{\log 51473 - \log 51470}{\log 51480 - \log 51470} = \frac{51473 - 51470}{51480 - 51470} = \frac{3}{10}.$$

We have, then,

$$\log 51473 = \log 51470 + \tfrac{3}{10}(\log 51480 - \log 51470).$$

Introducing the values from Table I,

$$\begin{aligned}\log 51473 &= 4.71155 + \tfrac{3}{10}(4.71164 - 4.71155)\\ &= 4.71155 + .3 \times .00009\\ &= 4.71155 + .00003\\ &= 4.71158.\end{aligned}$$

The difference .00009, or omitting the denominator, the 9 is called the tabular difference corresponding to the logarithm of 5147. Note that the added difference is computed to the nearest fifth decimal place.

This process is called interpolation by the principle of proportional parts. To facilitate interpolation, tables of proportional parts are inserted in the logarithmic tables in the column headed P.P. At the top of each of the P.P. tables is the tabular difference and under this is the number to be added corresponding to the digit at the left. For example

$$\log 38.25 = 1.58263$$

$$\log 38.26 = 1.58274.$$

The difference is .00011 and in the P.P. column is a table headed 11. Suppose now that log 38.257 is required. Opposite 7 under 11 is found 7.7 ; hence 8 is to be added in the fifth decimal place, giving

$$\log 38.257 = 1.58271.$$

27. Numbers from given logarithms. The inverse process of finding the number corresponding to a given logarithm is best explained by an illustration. Given the logarithm 3.84235. Only the mantissa need be considered at first, as the characteristic merely determines the position of the decimal point in the number.

Looking for 84 in the first column after the margin, we find it corresponding to numbers from 692 to 707. The nearest tabular number (mantissa) smaller than 235 is 230, corresponding to the number 6955. The difference is 5, while the tabular difference, found by subtracting 230 from 236, is 6. We have now the proportion for the next digit,

$$\frac{n}{10} = \frac{5}{6};$$

so that the next digit is found by dividing 50 by 6. It is inadvisable to carry the interpolation beyond one additional digit. Since $50 \div 6 = 8 \cdot + \cdots$, we have found the desired number to be 6955.8. The decimal point is placed after the fourth digit according to the rule for the characteristic, the given characteristic being 3. Should the logarithm be followed by -10, the decimal point must finally be moved ten places to the left.

28. Cologarithms. The logarithms of divisors have to be subtracted. Subtraction, however, can be avoided and the logarithmic computation of a succession of multiplications and divisions effected by a single addition process. There is no advantage in using cologarithms when but two factors are involved. When, however, more than two are involved, instead of dividing by the denominator or divisor factors, we may multiply by their reciprocals, obviously a legitimate substitution. Now

$$\log \frac{1}{m} = -\log m = (10 - \log m) - 10.$$

This logarithm, $(10 - \log m) - 10$, is called the cologarithm of m, written $\operatorname{colog} m$. It may be written down immediately from the table by beginning at the left and subtracting each figure from 9, until the last figure, which must be subtracted from 10. Thus

$$\log 28.24 = 1.45086$$

and

$$\operatorname{colog} 28.24 = 8.54914 - 10.$$

29. Logarithms of trigonometric functions. Logarithms of the trigonometric functions are arranged in Table II in the same manner as are the natural functions, or true numerical values of the functions. Logarithmic secants and cosecants need not be printed, since they are the cologarithms of the cosines and sines.

The sines and cosines of angles and the tangents of angles less

than 45° are numerically less than unity. In conformity with Art. 24, therefore, their logarithms are written in the augmented form,

$$\log \sin 65° 21' = 9.95850 - 10.$$

The -10 is not printed in the table but it is always understood. The positive portion of the characteristic is printed in the table. Usage differs with respect to printing the logarithmic tangents of angles greater than 45°. Engineering and physical instruments are usually graduated to minutes or larger divisions of the angle, so that it is not feasible to carry the interpolation farther than to tenths of minutes. The tables of functions and of proportional parts printed in connection with this book are arranged with this in view.

Astronomic observations justify carrying the interpolation to seconds, and astronomers use for this purpose tables computed to seven or more decimal places.

For example,

$$\log \sin 29° 37' = 9.69390 - 10,$$

$$\log \sin 29° 38' = 9.69412 - 10.$$

The difference is .00022, and in the P.P. column is a table headed 22. Suppose now that log sin 29° 37.4′ is required. Opposite 4 under 22 is found 8.8; hence 9 is to be added in the fifth decimal place, giving

$$\log \sin 39° 37.4' = 9.69399 - 10.$$

EXERCISE X

1. Find from the table the logarithms of 72484, 619.25, 695×10^7, .00064375, 3×10^{21}.

2. Find from the table the logarithms of 91386, 14.295, 321×10^9, .000078541, 2×10^{24}.

3. Find the numbers whose logarithms are 3.71295, 12.61242, 8.21312 − 10.

4. Find the numbers whose logarithms are 4.21382, 11.75153, 6.13579 − 10.

5. Find Young's modulus of elasticity from the formula $Y = \frac{mgl}{\pi r^2 s}$, if $m = 4932.5$, $g = 980$, $l = 110.5$, $\pi = 3.1416$, $r = .25$, $s = .3$.

6. Find the radius of the sun if its mass is 2.03×10^{33} grams, and its average density is 1.41, knowing that mass = volume × density.

7. The radius r of each of two equal, tangent, iron spheres which attract each other with a force of 1 gram's weight, is given by the formula

$$\frac{(\frac{4}{3}\pi\rho r^3)^2}{4r^2} = \frac{M}{R^2},$$

in which the density of iron $\rho = 7.5$, the mass of the earth $M = 6.14 \times 10^{27}$ grams, and the radius of the earth $R = 6.37 \times 10^{8}$ cm., while $\pi = 3.1416$. Solve for r and compute by logarithms.

8. Solve example 7 for spheres of lead with density $\rho = 11.3$.

9. The population of a county increases each year by 12.5 per cent of the number at the beginning of the year. If its population Jan. 1, 1776, was 2.5×10^{6}, what will it be Dec. 31, 1926?

10. If the number of births and deaths per annum are 3.5 per cent and 1.2 per cent respectively of the population at the beginning of each year, and the population on Jan. 1, 1830, was 5×10^{5}, find the population Jan. 1, 1905.

11. Find from the tables log sin 25° 32.3′, log cot 71° 18.6′, colog cos 16° 29.2′.

12. Find from the tables log cos 19° 25.7′, log tan 31° 16.2′, colog sin 65° 12.8′.

13. Find the angles corresponding to log cos $\alpha = 9.31723$, log cot $\beta = 9.16251$, log tan $\gamma = 0.61253$.

14. Find the angles corresponding to log sin $\alpha = 9.63152$, log tan $\beta = 9.71728$, log cot $\gamma = 0.15382$.

15. Francis deduces the following formula for the discharge over a weir, $q = 3.01\, bH^{1.5}$, in which q is the discharge in cubic feet per second, b the breadth of the crest, and H the head of water. Find by logarithms the discharge when $b = 3.5$ and $H = 1.2$.

16. A common formula for finding the diameter of a water pipe is

$$d = 0.479 \left[\frac{flq^2}{h}\right]^{\frac{1}{5}}$$

in which f is a friction factor, l the length of the pipe, q the discharge, and h the head. Find d when $f = 0.02$, $l = 500$, $q = 5$, $h = 10$.

17. The discharge from a triangular weir is given as $q = c\,\frac{4}{15}\sqrt{2g}\,H^{\frac{5}{2}}$, in which c is a constant, g the acceleration of gravity, and H the head. Find q when $g = 32.2$, $H = 1.2$, $c = 0.592$.

18. The formula for velocity head is $h = 0.01555\, V^2$. Find h when $V = 5$.

19. The elevation of the outer rail on what is known as a one-degree railway curve to resist centrifugal force is sometimes given by the formula $e = 0.00066\, V^2$, e being in inches and V the speed of the train in miles per hour. When $V = 45$, compute e.

20. Another expression for the relation of the preceding problem is $e = \frac{gV^2}{32.2\,R}$. Here e is in feet, g is the gauge of the track, V is the speed in feet per second, and R is the radius of the curve. Given $g = 4.71$, $V = 66$, $R = 5730$, compute e.

21. The difference between the base and hypotenuse of a right triangle is given by $c - a = \frac{b^2}{c + a}$, and when a and c are nearly equal, approximately by $c - a = \frac{b^2}{2c}$.

Find the per cent of error introduced by the second method when the angle between a and c is 15°.

22. If a = length of a short circular arc and c = its chord, then approximately $a - c = \frac{a^3}{24R^2}$. Given $a = \frac{2\pi R}{24}$ and $R = 100$, compute the value of this difference.

23. The relation between the pressure and volume of air expanding under certain conditions is $p_1 v_1^{1.41} = pv^{1.41}$, where p_1 and v_1 are initial values. If $p_1 = 40$, $v_1 = 5.5$, find v when $p = 24$; also when $p = 16$.

24. The relation between the initial and final temperatures and pressures is given by the equation

$$\frac{t + 460}{t_1 + 460} = \left(\frac{p}{p_1}\right)^{\frac{.41}{1.41}}.$$

With $t_1 = 60$ and the other data as in Ex. 23, find the final temperatures for $p = 24$ and $p = 16$, respectively.

30. The slide rule. The principles of logarithmic computation are conveniently illustrated by means of the slide rule, now widely used in performing mechanically such operations as admit of the use of logarithms. A brief description of this instrument will be found profitable at this stage, and its use by the student as a ready check upon the numerical solution of problems is strongly recommended. As will be seen by an inspection of the simplified diagram of Fig. 19, the rule is essentially a device for adding and

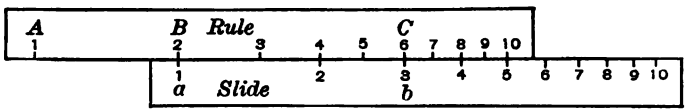

FIG. 19.

subtracting logarithms, thereby giving a wide range of computations. In the figure the point a on the "slide" is set opposite the point B on the "rule." If both scales, which are alike, are so divided that AB is equal, or proportional, to log 2 and ab to log 3, then C on the rule opposite b on the slide gives the distance AC equal, or proportional, to log 6. That is, log 2 + log 3 = log (2×3) = log 6.

Similarly by subtraction, $AC - ab = AB$;

that is $$\log 6 - \log 3 = \log 2.$$

The point a of the slide is called the *index*, hence we have the following rules for simple operations.

1. To multiply two numbers, set the index opposite one number on the rule and opposite the other number on the slide read the product on the rule.

2. To divide one number by another, set the divisor on the slide opposite the dividend on the rule and read the quotient on the rule opposite the index.

In the instrument as actually constructed, * Fig. 20, there are four scales denoted respectively by A, B, C, and D, of which scales B and C are on the

FIG. 20.

slide. For convenience in compound operations the rule is provided with a runner r by means of which a setting of the slide may be preserved while the slide is moved to a new position. The following example will illustrate the manipulation of slide and runner.

EXAMPLE 1. Find $\dfrac{63 \times 115 \times 27}{14.6 \times 342}$.

Set 14.6 on C scale opposite 63 on D scale; move runner to 115 on C scale; move 342 on C scale to runner, and opposite 27 on C scale read result on D scale.

In this, as in all slide-rule computations, the decimal point must be located by inspection.

On the lower side of the slide are three scales, the outer of which are marked S and T respectively. The following examples illustrate the use of these scales.

EXAMPLE 2. Find 36 sin 22°.

Set 22 on the S scale opposite the mark on the slot in the right-end of the rule; then opposite the end of the A scale can be read on the B scale the natural sine of 22°. Now opposite 36 on the A scale read the result on the B scale.

EXAMPLE 3. Find 26.5 tan 13° 15′.

Reverse the slide and set 13° 15′ on the T scale opposite the mark on the slot; then opposite the end of the B scale can be read on the D scale the natural tangent of 13° 15′. Set the runner at this point and replace the slide with the index at the runner. Opposite 26.5 on the C scale read the required product on the D scale.

EXAMPLE 4. Find $56^{1.8}$.

Set the index of C scale opposite 56 on D scale and opposite the mark on the under side of the right-hand end of the rule read 748 on the middle scale

* A more detailed description of the slide rule is not within the scope of this book. A manual describing fully the use of the instrument can be had of any firm selling slide rules.

of the lower side of the slide. This reading is the mantissa of the logarithm of 56. The characteristic 1 must be supplied as usual. Now in the usual way find 1.3×1.748; that is, put index to 1.748 on D scale and opposite 1.3 on C scale read the product 2.272. This is the logarithm of $56^{1.3}$. Set the mantissa 272 on the logarithm scale opposite the mark on the rule and read 118.7 on the D scale opposite the index.

EXERCISE XI

1. Find (*a*) $\frac{64 \times 37}{163}$; (*b*) $\frac{193}{67 \times 2.1}$; (*c*) $\frac{0.05 \times 137 \times 62}{14 \times 28 \times 6.5}$.

2. Find (*a*) $127 \sin 24°$, (*b*) $0.32 \sin 72°$, (*c*) $16.5 \cos 35°$.

3. Find (*a*) $37 \tan 8° 20'$, (*b*) $1.35 \tan 40° 10'$.

4. Find (*a*) $\frac{17 \cot 32°}{64}$, (*b*) $35.5 \frac{\sin 32°}{\sin 47°}$.

5. Find (*a*) $28\frac{1}{2}$, (*b*) $\sqrt[3]{3.65}$, (*c*) $7.31^{1.27}$.

31. Right triangles solved by logarithms. — It is now possible, with the aid of the logarithmic tables, to solve right triangles the numerical values of whose parts contain more digits than those given in Chapter III, without entailing laborious multiplications and divisions.

EXAMPLE 1. Given $a = 51.72$, $\beta = 73° 46'$.

Solving the proper formulas for the unknown parts, we have

$$\alpha = 90° - \beta,$$

$$c = \frac{a}{\cos \beta},$$

$$b = a \tan \beta,$$

$$A = \tfrac{1}{2} a^2 \tan \beta,$$

$$b = c \cos \alpha, \text{ check.}$$

$$\text{Sum of angles} = 90° \; 00'$$

$$\beta = 73° \; 46'$$

$$\alpha = 16° \; 14'.$$

$$\log a = 1.71366$$

$$\log \cos \beta = 9.44646 - 10$$

$$\log c = 2.26720$$

$$\therefore \; c = 185.01$$

$$\log a = 1.71366$$
$$\log \tan \beta = \underline{0.53587}$$
$$\log b = 2.24953$$
$$\therefore b = 177.64$$

. . . . . .

$$2 \log a = 3.42732$$
$$\log \tan \beta = \underline{0.53587}$$
$$\operatorname{colog} 2 = \underline{9.69897 - 10}$$
$$\log A = 13.66216 - 10$$
$$\therefore A = 4593.67$$

Check

$$\log c = 2.26720$$
$$\log \cos \alpha = \underline{9.98233 - 10}$$
$$\log b = 12.24953 - 10$$
$$\therefore b = 177.64$$

Note that $\log a^2 = 2 \log a$. In solving, first write all the forms needed for the complete solution; secondly, look up and write in all the needed logarithms of the data from the tables; thirdly, perform the additions and subtractions; lastly, from the logarithmic results find the numbers. Then $\log \cos \beta$, $\log \tan \beta$, and $\log \cos \alpha$ ($= \log \sin \beta$) can all be found from once turning to the angle 73° 46′.

A form of computation sometimes used is given below. It has the advantage of being more compact than the usual form, and furthermore the logarithms of the data stand close to the data, thus permitting easy verification of results or correction of mistakes.

			Check
$a = 51.72$	$\log 1.71366$	$\log 1.71366$	
$\beta = 73° 46'$	$\log \cos \underline{9.44646 - 10}$	$\log \tan 0.53587$	
$c = \underline{185.01}$	$\log 2.26720$		$\log 2.26720$
$b = \overline{177.64}$			
$\alpha = \underline{16° 14'}$		$\log \overline{2.24953}$	$\log \cos \underline{9.98233 - 10}$
$b = \overline{177.64}$			$\log \overline{2.24953}$

a^2	$\log 3.42732$	
β	$\log \tan 0.53587$	
2	$\operatorname{colog} \underline{0.69897 - 10}$	
$A = 4593.7$	$\log 3.66216$	

EXAMPLE 2. Given $b = 7124.5$, $c = 9365.4$.

We have,

$$\cos \alpha = \frac{b}{c},$$
$$\beta = 90° - \alpha,$$
$$a = c \sin \alpha,$$
$$A = \tfrac{1}{2} bc \sin \alpha,$$
$$a = b \tan \alpha, \text{ check.}$$

$$\log b = 3.85275$$
$$\log c = 3.97153$$
$$\log \cos \alpha = 9.88122 - 10$$
$$\alpha = 40° 28.4'$$
$$\beta = 49° 31.6'$$

$$\log c = 3.97153$$
$$\log \sin \alpha = 9.81231 - 10$$
$$\log a = 13.78384 - 10$$
$$a = 6079.2$$

Check

$$\log b = 3.85275$$
$$\log \tan \alpha = 9.93109 - 10$$
$$\log a = 13.78384 - 10$$
$$a = 6079.2$$

The following is the compact arrangement of the computation:

			Check
$b = 7124.5$	$\log 3.85275$		$\log 3.85275$
$c = 9365.4$	$\log 3.97153$	$\log 3.97153$	
$\alpha = 40° 28.4'$	$\log \cos 9.88122 - 10$	$\log \sin 9.81231 - 10$	$\log \tan 9.93109 - 10$
$\beta = 49° 31.6'$			
$a = 6079.2$		$\log 3.78384$	
$a = 6079.2$			$\log 3.78384$

It appears that the Pythagorean proposition, $a^2 + b^2 = c^2$, is not used because it is not adapted to the use of logarithms. It might be used in this case, however, in the form

$$a = \sqrt{(c + b)(c - b)}.$$

EXERCISE XII

Find the missing parts of the following triangles, using logarithms. (The work may be checked with a slide rule.)

	α	β	a	b	c	A
1.	63°				2584	
2.			7531		8642	
3.	75° 15.2′		965.24			
4.			47.193			3972.6
5.			7.3298	6.1032		
6.	18° 25.5′				32.96	
7.			132.97			985.27
8.			53.215	13.712		
9.			65983		72916	
10.	29° 50.2′			10.207		
11.	25° 17.4′					382.97
12.			.00020	.00037		
13.		63° 12.7′			7.1436	
14.				.07154	.09127	
15.		35° 16.4′	.62961			
16.		35° 16.8′				41658
17.			.00615	.00415		
18.		80° 12.5′		5.2108		
19.				.00729	.01625	
20.		25° 18.2′			1729.3	

The examples 1–20 of Exercise VIII may also be solved by logarithms and the results compared with those there obtained.

21. Find the radius of the circle inscribed in a regular pentagon whose side is 12 feet.

22. Find the side of a regular pentagon inscribed in a circle whose radius is 15 feet 7 inches.

23. Find the area of a regular octagon whose circumscribed circle has a diameter of 10 feet.

24. A tower 120 feet high throws a shadow 69.2 feet long upon the plane of its base. What is the angle of inclination of the sun?

25. The top of a certain lighthouse is known to be 73 feet above the water. From a boat the angle between the top and its reflection is measured as 6° 45′. How far is the boat from the light?

26. Two trains leave a station at the same time, one going north at the rate of 30 miles per hour, and the other east at the rate of 40 miles per hour.

How far apart will they be in 20 minutes, and what is the direction of the line joining them?

27. Show that if a is the side of a regular polygon of n sides, the area of the polygon is given by $A = \frac{1}{4}a^2 n \cot \frac{180°}{n}$.

28. Show that if r is the radius of a circle, then the area of a regular circumscribed polygon of n sides is $A = r^2 n \tan \frac{180°}{n}$.

29. Find a value for the area of an inscribed polygon corresponding to that given above.

30. Taking the moon's diameter as 31′ 20″ and its distance from the earth as 239,000 miles, what is its diameter in miles?

31. At what distance may a mountain 4 miles high be seen across a plain, the earth being taken as a sphere of 4000 miles radius?

32. If the sun's diameter is taken at 866,000 miles and its distance from the earth as 93,000,000 miles, what angle should it subtend at the center of the earth.

33. An approximate formula for an ordinate at the center of a chord to a circle is $m = \frac{l^2}{4}\frac{\sin \alpha}{100}$, in which l is the length of the chord in feet and α the deflection or circumferential angle subtended by a base or chord of 100 feet. Find m for $l = 30$, $\alpha = 2°$.

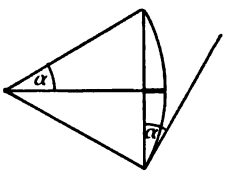

FIG. 21.

34. If I is the angle of intersection between two tangents to a circle of radius R, the distance T from a point of tangency to the point of intersection is given by $T = R \tan \frac{I}{2}$. Find T for $R = 3000$ feet, and $I = 22° 52'$.

35. The length of a chord is given by $2R \sin \frac{1}{2} I$, in which I is the central angle. Find the chord length for $R = 2000$, $I = 12° 13'$.

36. A river which obstructs chaining on a survey is passed by triangulation. The line AB, Fig. 22, is measured 200 feet perpendicular to AC, and the angle ABC found to be 35° 27′. What is the distance AC?

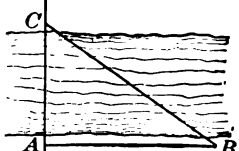

FIG. 22.

37. With an instrument at A, Fig. 23, a level line of sight passes 6 ft. above the top of a wall as measured on a rod. The angles of depression* to the top and bottom of the vertical face are respectively, 2° 31′ and 42° 16′. What is the height of the wall?

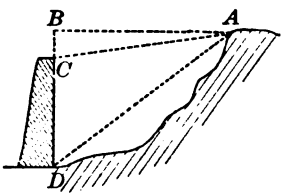

FIG. 23.

* The angles of elevation and depression of an object measure respectively its angular distance above or below the horizon of the observer.

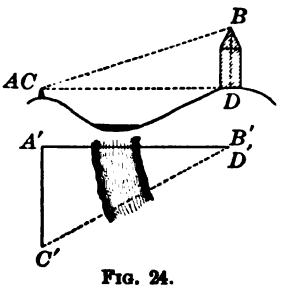

FIG. 24.

38. In order to obtain both the horizontal and vertical distances to an inaccessible point, the solution of two triangles may be necessary. Fig. 24 represents two views of the problem. Wishing the distances AD and BD, first lay out the base line AC of any convenient length perpendicular to AB. Measure the angle ACD and compute AD.

Next from AD and the angle DAB, the angles of elevation, compute DB.

Having $AC = 300$ ft., $ACB = 61° 34'$, and $DAB = 14° 41'$, find AD and DB.

CHAPTER V

THE OBTUSE ANGLE

32. Definitions of the trigonometric functions of obtuse angles. If an obtuse angle (*i.e.* an angle greater than 90° and less than 180°) is placed on the axes of coördinates in the same manner as was the acute angle in Art. 6, the terminal line will extend into the second quadrant. The trigonometric functions of such angles are defined exactly as in Art. 6. Thus in Fig. 25,

$$\sin \alpha = \frac{y}{v},$$

$$\cos \alpha = \frac{x}{v},$$

$$\tan \alpha = \frac{y}{x}, \text{ etc.}$$

33. Signs and limitations in value. The abscissas of all points in OA (Fig. 25) are negative, while their ordinates and radii vectores are positive. It is evident, therefore, that some of the defining ratios are negative. In accordance with the law of signs in algebraic division, we find that the sines and cosecants of all obtuse angles are positive, while their cosines, secants, tangents, and cotangents are negative.

The student should verify each of these statements in detail and become unhesitatingly familiar with these fundamental facts.

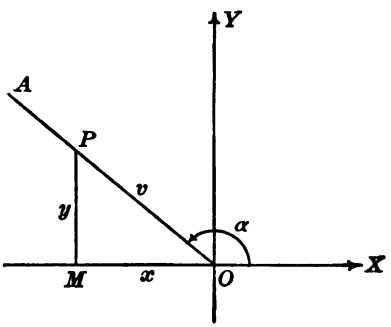

FIG. 25.

Furthermore, the sine and cosine cannot be numerically greater than unity and the secant and cosecant cannot be numerically less than unity.

QUERY. What are the limitations in value of the tangent and cotangent?

34. Fundamental relations. If the effects of the law of signs are traced, it will be seen that all the relations of Art. 9 hold also for functions of an obtuse angle without any modifications.

35. Variation. As the angle θ varies from 90° to 180°, while v remains constant, x is always negative and varies from 0 to $-v$, and y is positive and varies from v to 0. Consequently, as θ increases from 90° to 180°, $\sin\theta$ decreases from 1 to 0, $\cos\theta$ decreases (algebraically) from 0 to -1, $\tan\theta$ increases from $-\infty$ to 0, $\cot\theta$ decreases from 0 to $-\infty$, $\sec\theta$ increases from $-\infty$ to -1, $\csc\theta$ increases from 1 to ∞.

The terms positive infinity and negative infinity require careful consideration. If θ varies continuously from 89° to 90°, $\tan\theta$ varies in such a way as to exceed in magnitude any previously assigned definite value, however large. As it is positive for all values of θ in the first quadrant, it is consequently said to become positively infinite ($+\infty$). If θ varies continuously from 91° to 90°, $\tan\theta$ varies so as to exceed numerically any previously assigned definite value. As it is, however, always negative for values of θ in the second quadrant, it is said to become negatively infinite ($-\infty$). The plus or minus sign written before the symbol ∞ merely indicates whether the trigonometric function increases numerically without limit through a positive or a negative set of values.

36. Functions of 180°. As θ approaches 180°, v remaining constant, x approaches $-v$ and y approaches 0. We have, then,

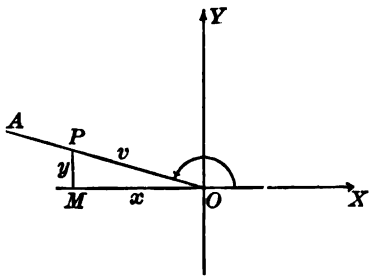

FIG. 26.

$$\sin 180° = 0,$$
$$\cos 180° = -1,$$
$$\tan 180° = 0,$$
$$\cot 180° = \infty,$$
$$\sec 180° = -1,$$
$$\csc 180° = \infty.$$

37. Functions of supplementary angles. Two angles are called supplementary if their sum is 180°. Thus, in Fig. 27, α and β are

supplementary, and $\beta = 180 - \alpha$, α being acute. The triangles OMP and ONQ are similar, but ON is negative. The pairs of corresponding sides are v and v', x and x', y and y'. Hence we have

$$\sin (180° - \alpha) = \sin \beta = \frac{y'}{v'} = \frac{y}{v} = \sin \alpha,$$

$$\cos (180° - \alpha) = \cos \beta = \frac{x'}{v'} = -\frac{x}{v} = -\cos \alpha,$$

$$\tan (180° - \alpha) = \tan \beta = \frac{y'}{x'} = -\frac{y}{x} = -\tan \alpha.$$

Similarly:

$$\cot (180° - \alpha) = -\cot \alpha,$$

$$\sec (180° - \alpha) = -\sec \alpha,$$

$$\csc (180° - \alpha) = \csc \alpha.$$

As a consequence of the relation $\sin (180° - \alpha) = \sin \alpha$, two values exist for $\arcsin m$, the one acute, the other obtuse, and supplemental to each other.

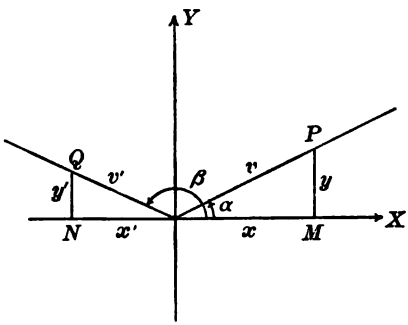

FIG. 27.

In case $m = 1$, the two values are identical.

QUERY. Is this also true of $\arccos m$, $\arctan m$, etc.?

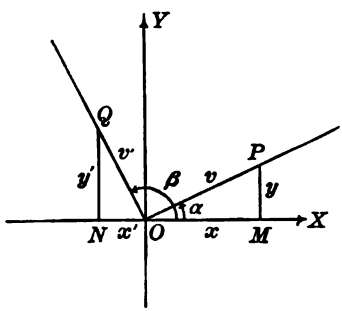

FIG. 28.

38. Functions of $(90° + \alpha)$. In Fig. 28, $\beta = 90° + \alpha$, α being acute. The triangles OMP and ONQ are similar, but the pairs of homologous sides are v and v', x and y', y and x', while x' is negative. We thus obtain

$$\sin (90° + \alpha) = \sin \beta = \frac{y'}{v'} = \frac{x}{v} = \cos \alpha,$$

$$\cos (90° + \alpha) = \cos \beta = \frac{x'}{v'} = -\frac{y}{v} = -\sin \alpha,$$

$$\tan (90° + \alpha) = \tan \beta = \frac{y'}{x'} = -\frac{x}{y} = -\cot \alpha.$$

In like manner,

$$\cot(90° + \alpha) = -\tan\alpha,$$
$$\sec(90° + \alpha) = -\csc\alpha,$$
$$\csc(90° + \alpha) = \sec\alpha.$$

EXERCISE XIII

1. Find the values of the functions of 135°. (See Art. 11.)

2. Find the values of the functions of 150°. (See Art. 11.)

3. Find the value of $\sin[\cos^{-1}(-\frac{8}{17})]$, $\tan(\csc^{-1}\frac{41}{9})$, $\cos[\arctan(-\frac{5}{12})]$, the angles being of the second quadrant.

4. Find the value of $\cos(\arccos-\frac{5}{13})$, $\sin[\tan^{-1}(-\frac{15}{112})]$, $\cot(\arcsin\frac{24}{25})$, the angles being of the second quadrant.

5. Express in terms of an angle less than 45°, cos 160°, tan 130°, sec 150°.

6. Express in terms of an angle less than 45°, sin 170°, csc 95°, cot 140°.

7. Verify for $\alpha = 60°$, the equations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha,\quad \cos 2\alpha = 2\cos^2\alpha - 1.$$

8. Verify for $\alpha = 45°$, the equations

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha,$$
$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha.$$

9. Verify for $\alpha = 120°$, the equations

$$\cos\frac{1}{2}\alpha = \sqrt{\frac{1+\cos\alpha}{2}},$$

$$\tan\frac{1}{2}\alpha = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha}.$$

10. Verify for $\alpha = 120°$, the equations

$$\sin\frac{1}{2}\alpha = \sqrt{\frac{1-\cos\alpha}{2}},$$

$$\cot\frac{1}{2}\alpha = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \frac{1+\cos\alpha}{\sin\alpha}.$$

11. Verify for $\alpha = 120°$, $\beta = 30°$, the equations

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta,$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$$

12. Verify for $\alpha = 120°$, $\beta = 60°$, the equations

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta,$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

13. Fill in the proper values in the following table for handy reference:

α	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$	$\sec\alpha$	$\csc\alpha$	
0°	0						
30°	$\frac{1}{2}$						
45°	$\frac{1}{2}\sqrt{2}$						
60°	$\frac{1}{2}\sqrt{3}$						
90°	1						
120°	$\frac{1}{2}\sqrt{3}$						
135°	$\frac{1}{2}\sqrt{2}$						
150°	$\frac{1}{2}$						
180°	0						

CHAPTER VI

OBLIQUE TRIANGLES

39. Formulas for solution. In the oblique triangle ABC, Fig. 29, let the angles be denoted by α, β, γ, and the lengths of the opposite sides by a, b, c, as in the figure.

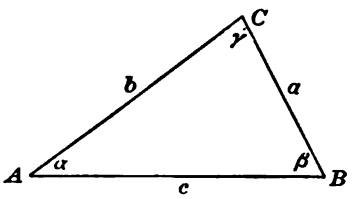

FIG. 29.

The relation $\alpha + \beta + \gamma = 180°$ always exists, and consequently when two of the angles are known, the third is determined. Five of the six parts of the triangle still remain to be found; namely, the three sides and two angles. It has been shown in elementary geometry that if any three independent parts are given, the triangle is determined and the remaining parts can be found. Then two formulas, in addition to the one just stated, are sufficient for the complete solution.

It is, nevertheless, convenient to express the relations between the sides and angles in a variety of forms. Those given in the following pages are selected on the score of utility. They fall into sets of three each. From any one of each set the other two may be written by cyclic advance of the letters involved; *i.e.* by changing a into b, b into c, c into a, and at the same time α into β, β into γ, γ into α. The legitimacy of this process and the truth of the resulting formulas appear from the consideration that no distinction is made as to any one side or any one angle. Any side and its opposite angle can be exchanged for any other pair. The cyclic advance affords a convenient systematic method of writing all possible forms.

From any one of these sets, as for instance that of Art. 40, or that of Art. 42, all the other sets may be derived by purely analytical processes. An independent geometric proof is given of each, however. The derivation by the analytic method suggested will afford a valuable review exercise after Chapter VIII has been studied.

40. Law of projections. If, in Fig. 30, the perpendicular CD is drawn from C to AB, the portions AD and DB are respectively the projections on the side AB of the other two sides AC and CB. Consequently, by Art. 15, we have

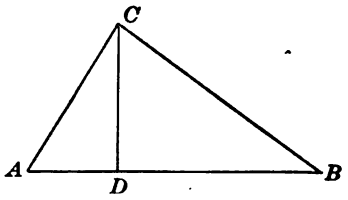

FIG. 30.

$$AB = AC\cos\alpha + CB\cos\beta,$$

or

$$c = b\cos\alpha + a\cos\beta.$$

By drawing the perpendicular from A and B in turn, we get

$$a = c\cos\beta + b\cos\gamma,$$

$$b = a\cos\gamma + c\cos\alpha.$$

By cyclic advance of the letters the first formula is transformed into the second, the second into the third, and the third into the first.

41. Law of sines. Connect the circumcenter K in Fig. 31 with the vertices, A, B, C, and the midpoints, L, M, N, of the sides.

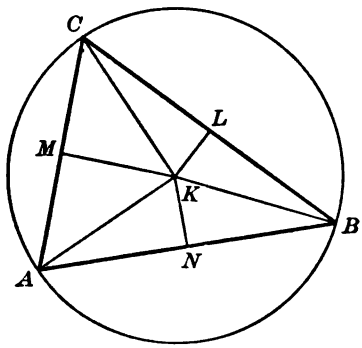

FIG. 31.

Then is $\angle BKC = 2\alpha$, $\angle CKA = 2\beta$, $\angle AKB = 2\gamma$. (Why?) In the right triangle KLC, $\angle LKC = \alpha$, and $LC = \frac{1}{2}a$. Denoting the circumradius by R, Art. 16 gives

$$\tfrac{1}{2}a = R\sin\alpha.$$

The other right triangles give likewise

$$\tfrac{1}{2}b = R\sin\beta,$$

$$\tfrac{1}{2}c = R\sin\gamma.$$

Equating the values of $2R$, we obtain the law of sines; namely,

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}.$$

The cyclic symmetry is apparent.

42. Law of cosines. In Fig. 32 the perpendicular p drawn from C divides the opposite side c into two portions m and n, and the

whole triangle into two right triangles ADC and BDC. In the latter triangles, we have, by Art. 16,

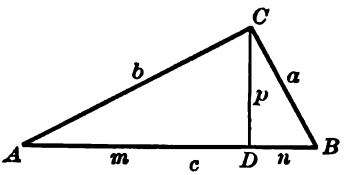

Fig. 32.

$$a^2 = n^2 + p^2$$
$$= (c - m)^2 + p^2$$
$$= (c - b\cos\alpha)^2 + b^2\sin^2\alpha;$$

or $$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos\alpha}.$$

Proper changes in the figure yield

$$\boldsymbol{b^2 = c^2 + a^2 - 2ac\cos\beta},$$
$$\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos\gamma}.$$

These again may be written by cyclic advance of the letters. Useful forms for writing these laws are:

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc},$$

$$\cos\beta = \frac{c^2 + a^2 - b^2}{2ac},$$

$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}.$$

43. Law of tangents. In Fig. 33 draw AE the bisector of the angle at A, and BF and CD perpendicular to it from the other vertices.

Then

$$\angle BAF = \angle DAC = \tfrac{1}{2}\alpha,$$

while

$$\angle DCE = \angle EBF = 90° - \angle BEF$$
$$= 90° - (\angle ABE + \angle BAE)$$
$$= \tfrac{1}{2}(\alpha + \beta + \gamma) - (\beta + \tfrac{1}{2}\alpha)$$
$$= \tfrac{1}{2}(\gamma - \beta).$$

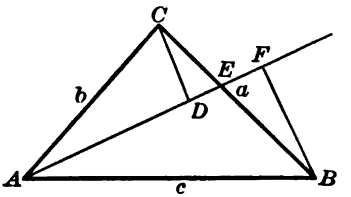

Fig. 33.

Again,

$$DF = EF + DE = AF - AD.$$

From the right triangles in the figure we get

$$\tan\tfrac{1}{2}(\gamma - \beta) = \frac{EF}{BF} = \frac{DE}{CD} = \frac{EF + DE}{BF + CD} = \frac{AF - AD}{BF + CD} = \frac{(c - b)\cos\tfrac{1}{2}\alpha}{(c + b)\sin\tfrac{1}{2}\alpha}$$

or $$\boldsymbol{\tan\tfrac{1}{2}(\gamma - \beta) = \frac{c - b}{c + b}\cot\tfrac{1}{2}\alpha}.$$

The forms $$\tan \tfrac{1}{2}(\alpha - \gamma) = \frac{a-c}{a+c} \cot \tfrac{1}{2}\beta,$$

$$\tan \tfrac{1}{2}(\beta - \alpha) = \frac{b-a}{b+a} \cot \tfrac{1}{2}\gamma,$$

may be obtained from suitably altered figures or by cyclic advance.

If $b > c$, the first formula will stand

$$\tan \tfrac{1}{2}(\beta - \gamma) = \frac{b-c}{b+c} \cot \tfrac{1}{2}\alpha.$$

Similar changes may occur in the other two.

44. Angles in terms of the sides. Construct the inscribed circle, Fig. 34, and denote its radius by r. Denoting the perimeter $a+b+c$ by $2s$, we have

$$AE = AF = s - a,$$

$$BD = BF = s - b,$$

$$CD = CE = s - c.$$

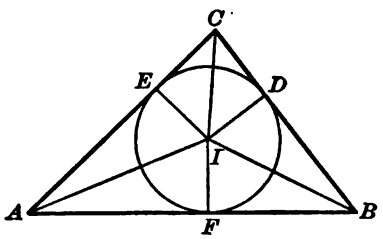

FIG. 34.

Consequently, by Art. 16,

$$\tan \tfrac{1}{2}\alpha = \frac{r}{s-a},$$

$$\tan \tfrac{1}{2}\beta = \frac{r}{s-b},$$

$$\tan \tfrac{1}{2}\gamma = \frac{r}{s-c}.$$

The value of r in terms of the three sides is derived in the corollary of Art. 45, thus completing this theorem.

45. Area of oblique triangles.

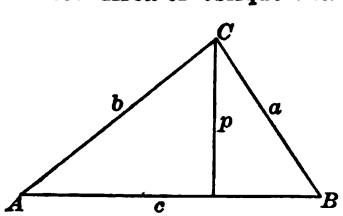

FIG. 35.

(1) By elementary geometry, we have (see Fig. 35)

$$A = \tfrac{1}{2} pc.$$

Introducing the value of p found by Art. 16, we get the formula

$$A = \tfrac{1}{2} bc \sin \alpha,$$

with the cognate forms

$$A = \tfrac{1}{2} ca \sin \beta, \quad A = \tfrac{1}{2} ab \sin \gamma.$$

(2) Squaring both members of the formula just derived, we obtain, with the aid of readily justifiable transformations and substitutions,

$$\begin{aligned} A^2 &= \tfrac{1}{4} b^2 c^2 \sin^2 \alpha \\ &= \tfrac{1}{4} b^2 c^2 (1 - \cos^2 \alpha) \\ &= \frac{bc}{2}(1 + \cos \alpha) \cdot \frac{bc}{2}(1 - \cos \alpha) \\ &= \frac{bc}{2}\left(1 + \frac{b^2 + c^2 - a^2}{2bc}\right) \cdot \frac{bc}{2}\left(1 - \frac{b^2 + c^2 - a^2}{2bc}\right) \\ &= \frac{2bc + b^2 + c^2 - a^2}{4} \cdot \frac{2bc - b^2 - c^2 + a^2}{4} \\ &= \frac{b + c + a}{2} \cdot \frac{b + c - a}{2} \cdot \frac{a - b + c}{2} \cdot \frac{a + b - c}{2} \\ &= s(s - a)(s - b)(s - c). \end{aligned}$$

Whence we have the desired formula

$$A = \sqrt{s(s - a)(s - b)(s - c)}.$$

(3) If r is the radius of the inscribed circle, we have, by elementary geometry,

$$A = rs.$$

COROLLARY. Equating the values of A found in (2) and (3), and solving for r, we get

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}},$$

the result needed to complete the theorem of Art. 44.

46. Numerical solution. The formulas of Arts. 40 and 42 are not adapted to the employment of logarithms. They are useful, however, in case the numerical values of the sides contain few digits.

The solution of oblique triangles falls into four well-defined cases, according as the three given parts consist of

I. Two angles and one side.

II. Two sides and an angle opposite one of them.

III. Two sides and the included angle.

IV. Three sides.

Each of these three cases with a model solution is discussed in detail in the following articles.

47. Case I. Given two angles and one side. Let the given parts be α, β, a.

The solution is effected by means of the formulas of Arts. 39 and 41. Solving for the unknown parts, we have

$$\gamma = 180° - (\alpha + \beta),$$

$$b = \frac{a \sin \beta}{\sin \alpha},$$

$$c = \frac{a \sin \gamma}{\sin \alpha},$$

with the check formula $$c = \frac{b \sin \gamma}{\sin \beta}.$$

Example. Given $\alpha = 47° 13.2'$

$\beta = 65° 24.5'$

$a = 43.176$

$$\begin{aligned} \text{sum of angles} &= 180° \\ \alpha + \beta &= 112° 37.7' \\ \therefore\ \gamma &= 67° 22.3' \end{aligned}$$

$$\begin{aligned} \log a &= 1.63524 \\ \log \sin \beta &= 9.95871 - 10 \\ \operatorname{colog} \sin \alpha &= 0.13433 \\ \log b &= 11.72828 - 10 \\ \therefore\ b &= 53.491 \end{aligned}$$

$$\begin{aligned} \log a &= 1.63524 \\ \log \sin \gamma &= 9.96522 - 10 \\ \operatorname{colog} \sin \alpha &= 0.13433 \\ \log c &= 11.73479 - 10 \\ \therefore\ c &= 54.299 \end{aligned}$$

Check

$$\begin{aligned} \log b &= 1.72828 \\ \log \sin \gamma &= 9.96522 - 10 \\ \operatorname{colog} \sin \beta &= 0.04129 \\ \log c &= 11.73479 - 10 \\ \therefore\ c &= 54.299 \end{aligned}$$

The compact form of computation is as follows:

$a = 43.176$	log 1.63524	log 1.63524	*Check*
$\beta = 65° 24.5'$	log sin 9.95871 − 10		colog sin 0.04129
$\alpha = 47° 13.2'$	colog sin 0.13433	colog sin 0.13433	
$b = 53.491$	log b 1.72828		log 1.72828
$\gamma = 67° 22.3'$		log sin 9.96522 − 10	log sin 9.96522 − 10
$c = 54.299$		log c 1.73479	log c 1.73479
$c = 54.299$			

Examples

Find the remaining three parts, given

1. $\beta = 65° 15.5'$, $\gamma = 81° 24.6'$, $b = 724.32$.
2. $\beta = 38° 37.4'$, $\gamma = 75° 32.8'$, $c = 129.63$.
3. $\alpha = 48° 29.2'$, $\gamma = 115° 33.8'$, $a = 14.829$.
4. $\alpha = 68° 41.5'$, $\gamma = 110° 16.5'$, $c = 9.4326$.

48. Case II. Given two sides and an angle opposite one. Let the given parts be a, b, α.

The solution is effected by the formulas of Arts. 39 and 41. Solving, we have

$$\sin \beta = \frac{b \sin \alpha}{a},$$

$$\gamma = 180° - (\alpha + \beta),$$

$$c = \frac{a \sin \gamma}{\sin \alpha},$$

with the check formula $$c = \frac{b \sin \gamma}{\sin \beta}.$$

An ambiguity arises in this case, however, since to any value of the sine correspond two supplementary angles, one acute, the other obtuse. Thus we also have

$$\beta' = 180° - \beta,$$

$$\gamma' = 180° - (\alpha + \beta'),$$

$$c' = \frac{a \sin \gamma'}{\sin \alpha},$$

$$c' = \frac{b \sin \gamma'}{\sin \beta'}.$$

The nature of this ambiguity will appear from the construction of the triangles with the given parts. If the given angle α is acute, there will be no solution, one solution, or two solutions, according as the free end of a (see Fig. 36), swinging about

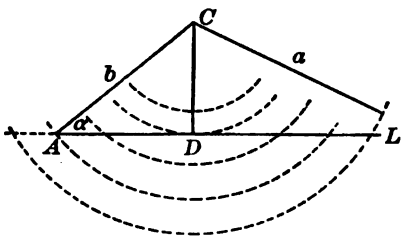

FIG. 36.

C, meets the line AL in no points, one point, or two points; *i.e.* as a is shorter than CD, the perpendicular from C upon AL, longer than AC, or intermediate between CD and AC. For $a = CD$ there is a single right triangle; and for $a = AC$, a single isosceles triangle.

When α is right or obtuse, there is no solution or one solution, according as a is shorter or longer than AC.

These results may be tabulated for reference.

$$\alpha < 90^\circ \begin{cases} a < b \sin \alpha, & \text{no solution,} \\ b \sin \alpha < a < b, & \text{two solutions,} \\ \left.\begin{matrix} a \geqq b, \\ a = b \sin \alpha, \end{matrix}\right\} & \text{one solution.} \end{cases}$$

$$\alpha \gtreqless 90^\circ \begin{cases} a \lesseqgtr b, & \text{no solution,} \\ a > b, & \text{one solution.} \end{cases}$$

If we proceed with the numerical work, without previously testing the number of solutions possible, the case of a single solution will appear from the fact that $\alpha + \beta' > 180^\circ$. (Whence $\alpha + (180^\circ - \beta) > 180^\circ$, or $\alpha - \beta > 0$, or $\beta < \alpha$.) When there is no solution, we shall get $\log \sin \beta > 0$; *i.e.* its augmented characteristic will be 10 or greater. A preliminary free-hand sketch will ordinarily serve to determine the number of possible solutions.

EXAMPLE 1. Given

$$a = 3541,$$
$$b = 4017,$$
$$\alpha = 61^\circ\ 27'.$$

By careful arrangement of the work, we can determine the number of solutions by inspection.

							Check	
$b=4017$	log	3.60390				log	3.60390	
$\alpha=61°\ 27'$	log sin	9.94369 − 10	colog sin	0.05631				
$b \sin \alpha$	log	3.54759						
$a=3541$	log	3.54913	log	3.54913				
$\beta=\ 85°\ 11'$	log sin	9.99846				colog sin	0.00154	
$\alpha+\beta=146°\ 38'$								
$\gamma=33°\ 22'$			log sin	9.74036 − 10		log sin	9.74036 − 10	
$c=2217.16$			log	3.34580				
$c=2217.16$						log	3.34580	

From the logarithms of b, a, and $b \sin \alpha$ it is seen that $b \sin \alpha < a < b$, whence there are two solutions. For the second solution we have:

					Check	
$\alpha=\ 61°\ 27'$	colog sin	0.05631				
$\beta'=\ 94°\ 49'$					colog sin	0.00154
$\alpha+\beta'=156°\ 16'$						
$\gamma'=23°\ 44'$	log sin	9.60474 − 10			log sin	9.60474 − 10
$a=3541$	log	3.54913				
$b=4017$					log	3.60390
$c'=1622.52$	log	3.21018				
$c'=1622.52$					log	3.21018

EXAMPLE 2. How many triangles are determined by the given parts $\alpha = 30°$, $b = 24$, $a = 10$, 12, 20, 24, 30?

Here $b \sin \alpha = 24 \times \frac{1}{2} = 12$. Accordingly, we have, for $a = 10$, no triangle; for $a = 12$, one right triangle; for $a = 20$, two triangles; for $a = 24$, one isosceles triangle; and for $a = 30$, one triangle.

EXAMPLES

1. How many triangles are determined by the given parts $\beta = 43°$, $c = 120$, and $b = 63$, 81.884, 95, 120, 150?

2. How many triangles are determined by the given parts $\gamma = 54°$, $a = 75$, and $c = 51$, 60, 67.5, 70, 75, 100?

Find the remaining parts of all possible triangles, given

3. $a = 62.518$, $b = 72.932$, $\beta = 98°\ 23.5'$.

4. $a = 429.15$, $c = 328.12$, $\alpha = 130°\ 33.7'$.

5. $b = 3912.7$, $c = 3526.5$, $\gamma = 35°\ 25.8'$.

6. $b = 129680$, $c = 152960$, $\beta = 38°\ 28.8'$.

49. Case III. Given two sides and the included angle. Let the given parts be a, b, γ, with $a > b$. The solution is effected by the formulas of Arts. 43, 39, and 41. Solving, we have

$$\tan \tfrac{1}{2}(\alpha - \beta) = \frac{a-b}{a+b} \cot \tfrac{1}{2}\gamma,$$

$$\tfrac{1}{2}(\alpha + \beta) = 90° - \tfrac{1}{2}\gamma,$$

$$c = \frac{a \sin \gamma}{\sin \alpha};$$

with the check formula $$c = \frac{b \sin \gamma}{\sin \beta}.$$

Example. Given

$$a = .745,$$
$$b = .231,$$
$$\gamma = 78° 15'.$$

$\gamma = 78° 15'$			log sin	9.99080 − 10	log sin	9.99080 − 10
$a = .745$			log	9.87216 − 10		
$b = .231$					log	9.36361 − 10
$a - b = .514$	log	9.71096 − 10				
$a + b = .976$	colog	0.01055				
$\frac{\gamma}{2} = 39° 7.5'$	log cot	0.08966				
$\frac{\alpha-\beta}{2} = 32° 55.3'$	log tan	9.81120 − 10				
$\frac{\alpha+\beta}{2} = 50° 52.5'$						
$\alpha = 83° 47.8'$			colog sin	0.00255		
$\beta = 17° 57.2'$					colog sin	0.51109
$c = .73368$			log	19.86551 − 20		
$c = .73367$						19.86550 − 20

Examples

Find the unknown parts, given

1. $b = 284.12$, $c = 361.26$, $\alpha = 125° 32'$.
2. $c = 395.71$, $a = 482.33$, $\beta = 137° 21'$.
3. $a = .06351$, $c = .10329$, $\beta = 83° 29.4'$.
4. $c = .00397$, $b = .00513$, $\alpha = 68° 21.8'$.

50. Case IV. Given the three sides. The given parts are a, b, c.

The solution is effected by the formulas of Art. 44, with the formula for r from Art. 45. We have at once

$$s = \tfrac{1}{2}(a+b+c),$$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}},$$

$$\tan \frac{1}{2}\alpha = \frac{r}{s-a}, \text{ etc.}$$

$\alpha + \beta + \gamma = 180°$, serves as a check formula.

EXAMPLE 1. Given

$a = .05341,$

$b = .06217,$

$c = .03482.$

Then	$2s = .15040$		
	$s = .07520$	colog	1.12378
	$s-a = .02179$	log	8.33826 − 10
	$s-b = .01303$	log	8.11494 − 10
	$s-c = .04038$	log	8.60617 − 10
	r^2	log	16.18315 − 10
	r	log	8.09157 − 10
	$\frac{\alpha}{2} = 29° 32.3'$	log tan	9.75331 − 10
	$\frac{\beta}{2} = 43° 27.6'$	log tan	9.97663 − 10
	$\frac{\gamma}{2} = 17° 0.1'$	log tan	9.48540 − 10
	$\alpha = 59° 4.6'$		
	$\beta = 86° 55.2'$		
	$\gamma = 34° 0.2'$		
sum of angles	$= 180° 0'$		

When the three sides are given and only one angle is required, say β, the two appropriate formulas may be combined into one, as

$$\tan \frac{1}{2}\beta = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}.$$

EXAMPLE 2. Given

$a = 35,$

$b = 64,$

$c = 73.$

Then $2s = 172$

$s =$	86	colog	8.06550 − 10
$s - a =$	51	log	1.70757
$s - b =$	22	colog	8.65758 − 10
$s - c =$	13	log	1.11394
			2)19.54459 − 20
$\frac{1}{2}\beta =$	30° 37.4′	log tan	9.77230 − 10
$\beta =$	61° 14.8′		

EXAMPLES

Find the angles of the following triangles:

1. $a = 6123,$ $b = 7148,$ $c = 6815.$
2. $a = 12{,}545,$ $b = 8612,$ $c = 10{,}353.$
3. $a = .05431,$ $b = .03714,$ $c = .06513.$
4. $a = .006152,$ $b = .008174,$ $c = .007534.$
5. $a = 72{,}584,$ $b = 125{,}217,$ $c = 36{,}925.$
6. $a = 13{,}579,$ $b = 35{,}791,$ $c = 24{,}680$; find $\beta.$
7. $a = 80{,}812,$ $b = 37{,}194,$ $c = 43{,}618.$
8. $a = 36{,}925,$ $b = 25{,}814,$ $c = 14{,}703$; find $\gamma.$

Find the areas in examples 1 and 2.

51. Composition and resolution of forces. Equilibrium. In mechanics the solution of oblique triangles is frequently required in problems relating to the composition and resolution of forces, velocities, and other directed quantities.

In this article will be stated, without proof, some of the laws governing the combination of such quantities, showing the application of trigonometry to certain of the problems involved.

Suppose the line segments AB and AC, Fig. 37, to represent in magnitude and direction two forces acting at a point A, and including between their lines of action the angle ϕ.

Complete the parallelogram $ABDC$. The diagonal AD, drawn from the point A, is the line segment representing the *resultant* of the two given forces, *i.e.* the single force that will produce the same effect as the two given forces. The process of finding the resultant of two or more given forces is called the *composition* of forces.

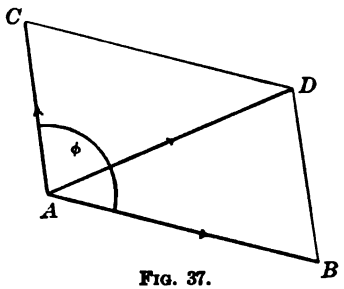

FIG. 37.

Conversely, the two line segments AB and AC may be taken as the components of AD. Thus the two forces AB and AC, acting together at A, produce the same effect as the single force AD. The process of finding two or more forces equivalent to a given force is called the *resolution* of the force into its components.

Since the segment BD is equal and parallel to AC, it follows that the resultant and the two components form a closed triangle ABD, and the relation between the forces may be obtained by solving this triangle. Note that the angle ABD is the supplement of the angle ϕ, so that by Art 37,

$$\cos ABD = -\cos \phi.$$

EXAMPLE 1. Find the resultant of two forces of 320 dynes and 400 dynes, respectively, acting on a common point, at an angle of 54° 28′.

In the triangle ABD, Fig. 37, we have given two sides and the included angle. If only the magnitude of the resultant is desired, it may be obtained by the law of cosines, Art. 42. Thus we obtain

$$AD = \sqrt{\{\overline{AB}^2 + \overline{AC}^2 + 2\,AB \cdot AC \cdot \cos \phi\}}.$$

If the angle formed by the resultant with its components is also required, the logarithmic computation may be effected as in Case III, Art. 49.

EXAMPLE 2. Resolve a force of 40 pounds into components making angles of 32° and 74° 20 with its line of action.

Referring to Fig. 37, we have

$$AD = 40, \angle BAD = 32°, \text{ and } \angle DAC = \angle ADB = 74° 20'.$$

Denoting the sides opposite the angles A, B, D, respectively, by a, b, d, we have from the law of sines,

$$a = b\frac{\sin A}{\sin B}, \qquad d = b\frac{\sin D}{\sin B}.$$

Hence the components may be computed.

Three forces are in equilibrium when the resultant of any two forces is equal and opposite to the third. Thus in Fig. 37, if the direction of the force AD is reversed, it and the forces AB and AC will be in equilibrium. The necessary conditions that three forces shall be in equilibrium are:

1. Their lines of action shall lie in the same plane.
2. Their lines of action shall meet in a point.
3. The line segments representing the three forces when laid off in order shall form a triangle.

In Fig. 38 the forces a, b, and c applied at a common point are in equilibrium. The angles between the lines of action are denoted by A, B, C, as indicated. When the forces are laid off to

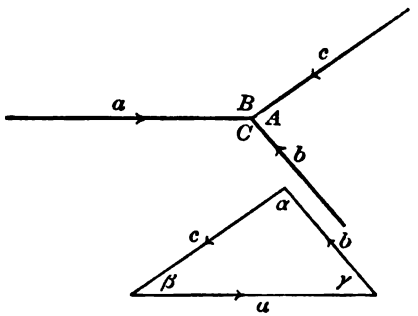

FIG. 38.

form the triangle, the angles of the triangle are seen to be the supplements of the corresponding angles A, B, C.

That is,

$$\alpha = 180° - A, \text{ whence } \sin\alpha = \sin A,$$

$$\beta = 180° - B, \text{ whence } \sin\beta = \sin B.$$

etc. etc.

From the law of sines,

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}.$$

Therefore,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

EXERCISE XIV

Find the unknown parts of the following triangles:

	α	β	γ	a	b	c
1.	62° 35′				82916	59278
2.				75290	92841	69289
3.	25° 36.2′	68° 13.5′		3.9168		
4.		55° 55.4′		.25317		.36291
5.	69° 17.5′			329.12	689.12	
6.		100° 10′			62198	29322
7.				.0000713	.0000987	.0001255
8.	61° 15.2′	49° 16.3′			58.291	
9.			120° 50.2′	2.8315	4.1217	
10.		38° 17.2′		21.992	50.715	
11.	150° 24.2′				.038251	.047319
12.	58° 06.5′			57.15		67.31
13.		75° 19.3′	70° 29.2′		658.42	
14.				100.05	200.07	150.08
15.		126° 26.4′		.0021868		.0032292
16.			10° 32.8′		25.317	37.293
17.				50010	70020	90030
18.		48° 25.3′	56° 34.5′			7219.2
19.			120° 15′	62158		75292
20.			90° 00′	725.63	617.25	

Solve the following triangles, given

21. $a = 2500$, $c = 2125$, $A = 208{,}690$.

22. $b = 103.5$, $c = 90$, $A = 4586.7$.

23. $\alpha = 73° 10′$, $b = 753$, $A = 74{,}803$.

24. $\beta = 57° 25′$, $c = 57.65$, $A = 3055.7$.

25. Find the areas in examples 1, 9, 17.

26. Find the areas in examples 2, 4, 14.

27. Determine the magnitude and direction of the resultant of two forces of magnitudes a and b, if their lines of action include an angle ϕ.

28. Carry out the computation of example 27 in the following cases:

$a = 20$, $b = 36$, $\phi = 45°$; $a = 300$, $b = 540$, $\phi = 64°$;

$a = 75$, $b = 60$, $\phi = 145°$; $a = 250$, $b = 320$, $\phi = 120°$.

29. Find the directions of three forces in equilibrium if $a = 7$, $b = 10$, $c = 15$; also if $a = 24$, $b = 36$, $c = 42$.

30. Referring to Figure 38 solve completely and interpret physically when $a = 695$, $b = 483$, $C = 155°$; $a = 720$, $b = 840$, $B = 100°$.

31. Solve and interpret when $a = 1200$, $B = 135°$, $C = 150°$; $a = 135$, $b = 142$, $c = 95$.

32. Resolve a force of magnitude 84 into two equal components making an angle of 60° with each other.

33. Resolve a force of magnitude 240 into two components of 120 and 180 each and find the directions of the components.

34. Determine the formula for one side of a quadrilateral in terms of the other three sides and their included angles. Compute for $a = 10$, $b = 12$, $c = 15$, $\widehat{ab} = 135°$, $\widehat{bc} = 60°$.

QUERY. How many given parts serve to determine the remaining parts of a quadrilateral?

35. Given the four sides and one angle of a quadrilateral, determine the other angles and the diagonals. Compute for $a = 60$, $b = 72$, $c = 90$, $d = 100$, $\widehat{ab} = 120°$.

36. Given three angles and two sides of a quadrilateral, determine the remaining sides. Compute for $a = 630$, $b = 500$, $\widehat{ab} = 100°$, $\widehat{bc} = 80°$, $\widehat{cd} = 60°$.

37. Find the angles and the lengths of the sides of a regular pentagram, or five-pointed star, inscribed in a circle of radius 8.

38. Compute the volume for each foot in depth of a horizontal cylindrical tank of length 30 feet and radius 6 feet.

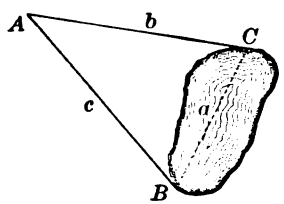

FIG. 40.

39. Having measured the following data, $A = 80° 30'$, $B = 72° 15'$, and $c = 232.5$ feet, compute the inaccessible distance b (Fig. 39).

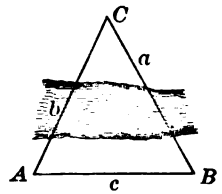

FIG. 39.

40. Compute the distance a across a lake, Fig. 40, having measured A, B, and c, which are respectively 51° 20′, 72° 40′ and 3420.5 feet.

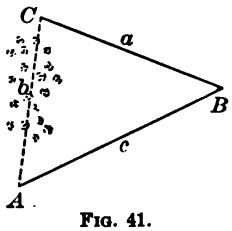

FIG. 41.

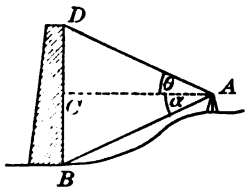

FIG. 42.

41. A being invisible from C, find the distance b through a forest, having measured $a = 1037$ feet, $c = 1208$ feet, $B = 69° 25'$.

42. In Fig. 42, BC, the distance of the foot of a wall below the instrument is 12.3 feet, θ and α, the angles of elevation and depression, are 15° 20′ and 21° 15′, respectively. Find the height of the wall and its distance from the instrument.

43. A pole BC, Fig. 43, is 12 feet long and leans two feet from a vertical toward the instrument at A. If the angles of elevation of the top and bottom are respectively 37° 15′ and 11° 50′, what are the horizontal and vertical distances from the instrument to the foot of the pole?

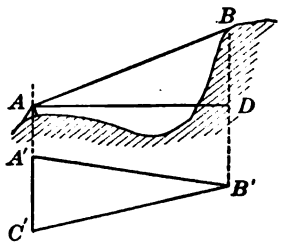

Fig. 44.

44. It is desired to find the horizontal distance and elevation of the inaccessible point B, Fig. 44, with reference to an instrument at A. Having laid out a base line AC, 250 feet long, the angles at A and C are found to be 87° 10′ and 73° 51′, respectively, and from A the angular elevation of B is 11° 32′.

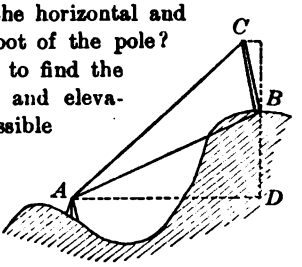

Fig. 43.

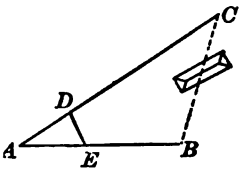

Fig. 45.

45. Given $B = 110°\ 05'$, $AE = AD = 200$ feet, $DE = 125$ feet, and $AB = 632$ feet; find the distance AC to be laid off, and the inaccessible distance BC (Fig. 45).

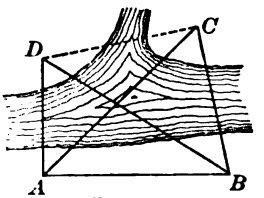

Fig. 46.

46. From measurements we have (Fig. 46) $AB = 600$ feet, $BAC = 70°\ 40'$, $BAD = 92°\ 30'$, $ABD = 65°\ 32'$, $ABC = 89°\ 25'$. Find the inaccessible distances AD and DC, and the angle between DC and AB.

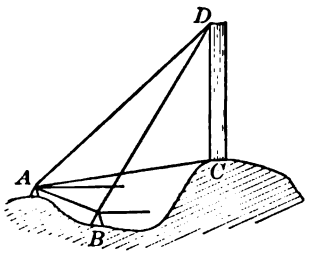

Fig. 47.

47. From the instrument at A (Fig. 47) the angles of elevation to the top and base of the vertical wall are 15° 12′ and 1° 23′, respectively. A base line AB is measured 75 feet toward the wall down a plane inclined 8° 16′, and from B the angle of elevation to the top of the wall is 37° 46′. Compute the height of the wall and its horizontal distance from A.

48. It is required to prolong the line AB (Fig. 48) beyond an obstacle. At B is made an angle 52° 20′ to the right and at C an angle of 110° 00′ to the left, BC being 210 feet. Compute the proper distance CD and angle to the right at D, also the inaccessible distance BD. Note that by making $B = D = 60°$ and $C = 120°$, then $BC = CD = BD$ and all computations are avoided.

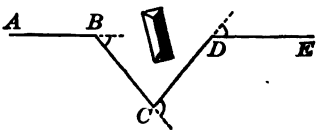

Fig. 48.

49. Having but one point C (Fig. 49) from which both inaccessible points A and B are visible, we are required to find the inaccessible distances AC

and AD and the angle between AB and DC. $ADC = 87° 42'$, $DCA = 60° 32'$, $DCE = 170° 05'$, $BCE = 41° 20'$, $CEB = 111° 35'$, $DC = 365.2$ feet, $CE = 410.7$ feet.

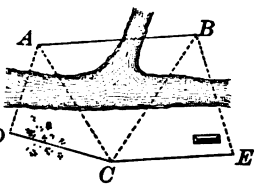

FIG. 49.

50. It is required to ascertain the length and position of an inaccessible line AB (Fig. 50), its extremities not being visible from a common point beyond the obstacles. By chaining we have $CD = 210.7$ feet, $DE = 390.4$ feet, $EF = 173.5$ feet. Then the following angles are measured: $ACD = 83° 41'$, $CDE = 19° 12'$ left ($180°–19° 12'$), $CDA = 79° 49'$, $FEB = 53° 20'$, $DEF = 42° 03'$ left, $EFB = 115° 27'$.

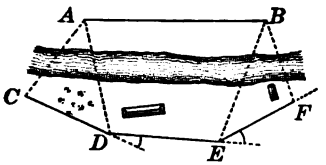

FIG. 50.

In order to locate points suitably upon a map, find lengths AB, AD, and BE.

51. A tower 115 feet high casts a shadow 157 feet long upon a walk which slopes downward from its base at the rate of 1 in 10. What is the elevation of the sun above the horizon?

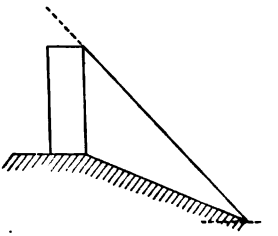
FIG. 51.

CHAPTER VII

THE GENERAL ANGLE

Only those parts of trigonometry that are necessary for the solution of triangles have been developed thus far. In this and the following chapters are considered some of the more important topics of another phase of trigonometry that is no less essential for the further study of pure and applied mathematics.

52. General definition of an angle. If a straight line rotates about one of its points, remaining always in the same plane, it generates an angle. The angle is measured by the amount of rotation by which the line is brought from its original position into its terminal position. For the small rotation leading to acute and obtuse angles this definition agrees with the customary elementary definition, the knowledge of which has been presupposed in the foregoing chapters.

As in Art. 3, counterclockwise rotation generates positive angles; clockwise rotation, negative.

In the sexagesimal system of angle measurement the standard unit is the angle produced by one complete rotation of the generating line. This angle is divided into 360 equal parts called *degrees*, the degree into 60 minutes, and the minute into 60 seconds.

In the circular system the standard unit is the *radian*, the angle produced by such a rotation that each point in the generating line describes an arc equal in length to its radius. Angular magnitudes are stated in radians and decimal fractions thereof.

Instruments are graduated and tables printed in accordance with the sexagesimal system, which is used in practical numerical calculations. Astronomers, however, employ decimal fractions of seconds, while engineers make use of tenths of minutes and decimal divisions of degrees. In theoretical discussions the radian system is commonly employed. Hereafter, in this book, the two systems will be used interchangeably.

Since the circumference of a circle is equal to 2π times its radius, where $\pi = 3.14159\cdots$, we may write the following relations between the two systems:

$$2\pi \text{ radians} = 360°$$

$$1 \text{ radian} = 57.29578°\cdots$$

$$= 57° 17' 44.8''$$

and, in general, the number of degrees in any angle is equal to the number of radians multiplied by $\frac{180}{\pi}$, while the number of radians is equal to the number of degrees multiplied by $\frac{\pi}{180}$. Thus the straight angle is π radians; the right angle, $\frac{\pi}{2}$ radians.

If the radius of the circle is represented by r, the arc by a, and the angle, in radians, by α, we have the important relation

$$a = r\,\alpha.$$

53. Axes, quadrants, etc. Let the two axes of coördinates be assumed as in Art. 4; and, as in Art. 6, let the angle be placed upon the axis, its vertex at the origin, and its initial line extending along the X-axis toward the right. The sign and magnitude of the angle will determine the position of the terminal line, causing it to coincide with one of the axes or to fall in one of the quadrants. An angle is said to be of the first, second, third, or fourth quadrant according as its terminal line falls in that quadrant.

While the acute angle is of the first quadrant, the converse is by no means necessarily true. The terminal line of every angle, however large, must coincide with the terminal line of some positive angle less than 360° (see Fig. 52). For the purpose of trigonometry as developed in the present chapter, for every angle, positive or negative, and of any magnitude, may be substituted a positive angle less than 360°.

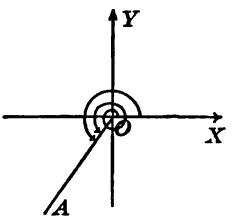

FIG. 52.

54. Definitions of the trigonometric functions. The trigonometric functions of angles of any size are defined identically as in

Art. 6. Thus for all positions of the terminal line, Fig. 53,

$$\frac{y}{v} = \sin \alpha, \qquad \frac{x}{v} = \cos \alpha,$$

$$\frac{y}{x} = \tan \alpha, \qquad \frac{x}{y} = \cot \alpha,$$

$$\frac{v}{x} = \sec \alpha, \qquad \frac{v}{y} = \csc \alpha.$$

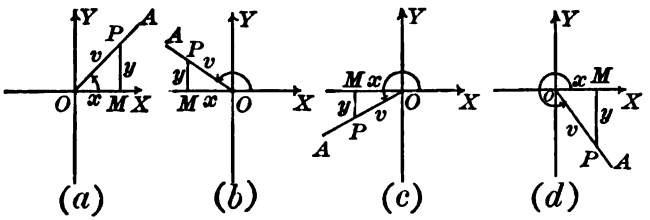

FIG. 53.

55. Signs and limitations in value. The abscissas are positive for all points in the first and fourth quadrants, negative for those in the second and third. Ordinates are positive for all points in the first and second quadrants, negative for those in the third and fourth. The radius vector is, by agreement, considered positive for all points.

In conformity with the sign law of algebra, the functions of angles of the different quadrants will have signs as displayed in the following table:

QUAD.	SINE	COSINE	TANGENT	COTANGENT	SECANT	COSECANT
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−

It will be noticed that for angles of the first quadrant all six functions are positive. In each of the other quadrants one pair of mutually reciprocal functions are positive, the other two pairs are negative. These positive pairs run as follows: second quadrant, sine and cosecant: third quadrant, tangent and cotangent: fourth quadrant, cosine and secant.

The student should establish these statements regarding the signs of the functions and memorize them.

Since the lengths of the abscissa and ordinate can never exceed that of the radius vector, it follows that the sine and cosine can never be numerically greater than unity, and the secant and cosecant can never be numerically less than unity. The tangent and cotangent can have numerical values either greater or less than unity.

EXERCISE XV

1. Express in degrees, minutes, and seconds the angles $\frac{\pi^R}{4}$, $\frac{\pi^R}{3}$, $\frac{5\pi^R}{6}$, $\frac{5\pi^R}{8}$, 3^R, $\frac{3}{4}^R$.

2. Express in radians the angles 30°, 15°, 45°, 120°, 240°, 300°, 450°.

3. In a circle of radius 60 cm., what is the length of the arc which subtends at the center the angle 30°, 60°, $\frac{2\pi}{3}$, $\frac{\pi}{4}$?

4. In a circle of radius 10 inches, what is the circular measure of the angle subtended by an arc whose length is 10, 5, 20, 5π inches?

5. A friction gear consists of two tangent wheels, whose radii are 8 and 12 inches, respectively. The smaller wheel makes 4 revolutions per second. Find the number of revolutions per second made by the larger, the angular velocity of each, and the linear velocity of a point on the circumference of each. If the larger wheel is attached to the rear axle of an automobile whose rear wheel has a diameter of 30 inches, find the speed of progress of the machine.

6. The diameters of the front and rear sprocket wheels of a bicycle are 10 inches and 4 inches, respectively, and the diameter of the rear wheel is 28 inches. Find the rate of pedaling when the bicycle is traveling 12 miles per hour, the corresponding angular velocities of the two sprocket wheels, and the linear velocity of the chain.

7. Determine the quadrant to which each of the following angles belongs: 210°, 465°, 745°, $-830°$, $\frac{10\pi}{3}$, $\frac{15\pi}{4}$, $-\frac{8\pi}{3}$.

8. Determine the signs of the functions of the following angles: 240°, 330°, 400°, $\frac{5\pi}{3}$, $-\frac{7\pi}{4}$, 6^R.

9. Show that the quadrant to which an angle belongs is determined if the signs of any two non-reciprocal functions are given.

10. To what quadrant does an angle belong if its sine and tangent are negative; its secant and cotangent positive; sine and secant negative; tangent and cosine positive?

11. Determine the quadrants of the following angles:

$$\sin^{-1}\tfrac{3}{5};\ \arccos -\tfrac{5}{13};\ \arctan\tfrac{3}{4};\ \cot^{-1}-\tfrac{7}{12}.$$

12. Determine the quadrants of the following angles:

$$\sin^{-1}\tfrac{3}{5}\equiv\cot^{-1}-\tfrac{4}{3};\ \arccos -\tfrac{5}{13}\equiv\operatorname{arccsc}\tfrac{13}{12}.$$

13. For what values of α is $\sin\alpha-\cos\alpha$ positive?

14. For what values of α is $\tan\alpha-\cot\alpha$ negative?

15–20. Find the missing values in the following table:

$\angle$	sin	cos	tan	cot	sec	csc	Quad.
α	$\frac{20}{29}$						II
β		$-\frac{24}{25}$					III
γ			$-\frac{9}{40}$				III
δ				$-\frac{15}{8}$			IV
θ					$\frac{25}{7}$		IV
ϕ						$-\frac{17}{8}$	III

56. Variation of the trigonometric functions. A change in the angle will produce a corresponding change in the values of the coördinates and in their ratios. If, for convenience, the chosen point in the terminal line of the angle is maintained at a constant distance from the vertex, the radius vector will retain the constant value $+v$.

As the angle θ increases continuously from 0° to 360°, the abscissa and ordinate vary continuously between the limits $-v$ and $+v$. As θ increases from 0° to 90°, x is positive and decreases from v to 0; as θ increases from 90° to 180°, x is negative and decreases (algebraically) from 0 to $-v$; as θ increases from 180° to 270°, x is negative and increases from $-v$ to 0; and as θ increases from 270° to 360°, x is positive and increases from 0 to v. As θ increases from 0° to 90°, y is positive and increases from 0 to v; as θ increases from 90° to 180°, y is positive and decreases from v to 0; as θ increases from 180° to 270°, y is negative and decreases from 0 to $-v$; as θ increases from 270° to 360°, y is negative and increases from $-v$ to 0. Upon introducing these varying values into the ratio definitions, we are enabled to trace the variation of the trigonometric functions.

We see, for example, that as θ increases from 0° to 360°, $\tan\theta$ continually increases algebraically, changing sign from negative to positive through the value 0 as θ passes through 0°,

180°, and 360°, and from positive to negative by becoming infinite as θ passes through 90° and 270°. There is an infinite discontinuity in tan θ, for $\theta = 90°$ and $\theta = 270°$.

QUERY. Which of the trigonometric functions other than the tangent become infinite and therefore discontinuous?

The student should trace the variation of each function in detail, stating the narrative verbally.

57. Graphs of the trigonometric functions. The whole behavior of each function can be conveniently represented by means of the graphical method already introduced in Art. 4. Assume a pair

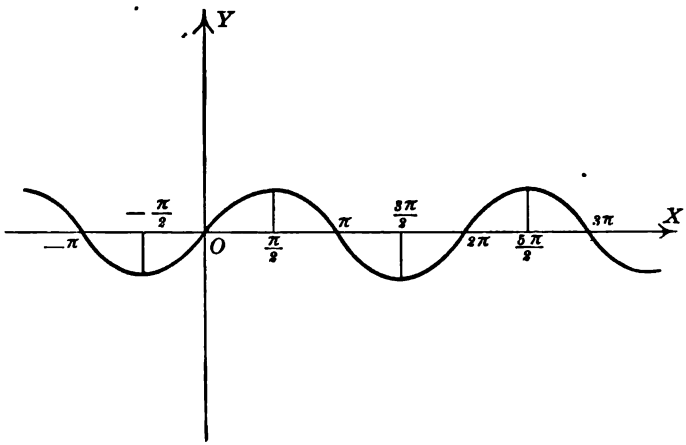

FIG. 54.

of axes of coördinates, as in Art. 4, and along the X-axis to the right lay off equal spaces corresponding to the number of degrees in the angle θ. At each point in the X-axis erect a perpendicular whose length is proportional to the value of the sine of that angle. Each point thus determined has the property that its abscissa represents the angle θ and its ordinate the corresponding value of sin θ. Now having located a sufficient number of points, draw through them a smooth curve. It will be seen that the value, sign, and variation of the sign at each instant is fully exhibited by the ordinate, position, and inclination of the curve or graph. The same may be done for each of the functions.

The graphs of the different functions are here presented. The student should trace carefully the intimate and exact correspondence of the graphical and the verbal narratives.

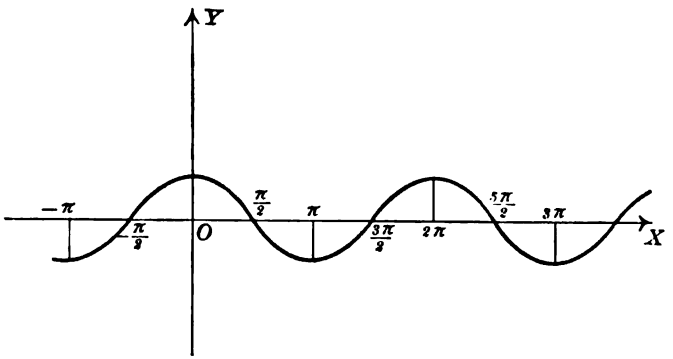
Y
X
O
$-\pi$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$\frac{5\pi}{2}$
3π
$-\frac{\pi}{2}$

FIG. 55.

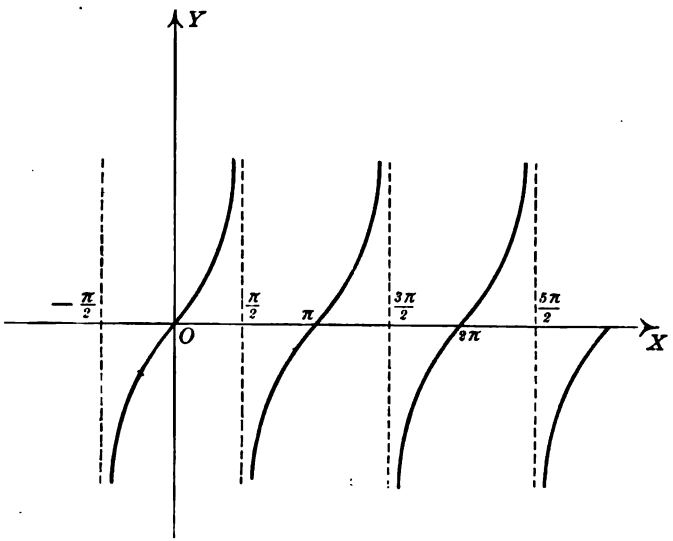
Y
X
O
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$\frac{5\pi}{2}$

FIG. 56.

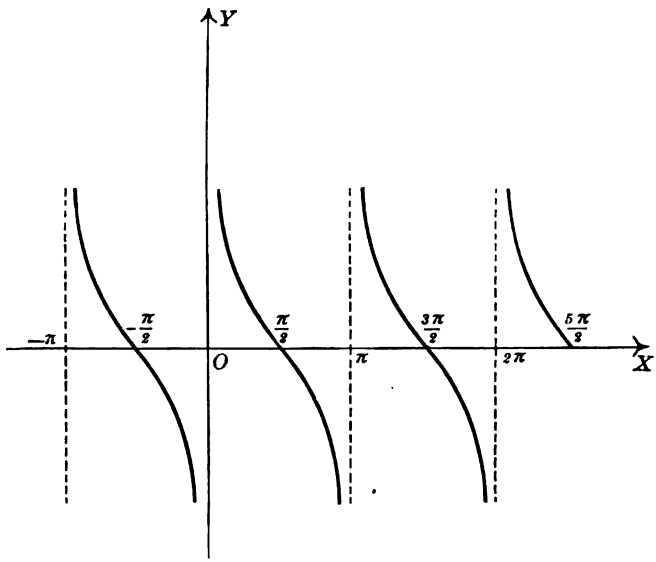
Y
X
O
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$\frac{5\pi}{2}$

FIG. 57.

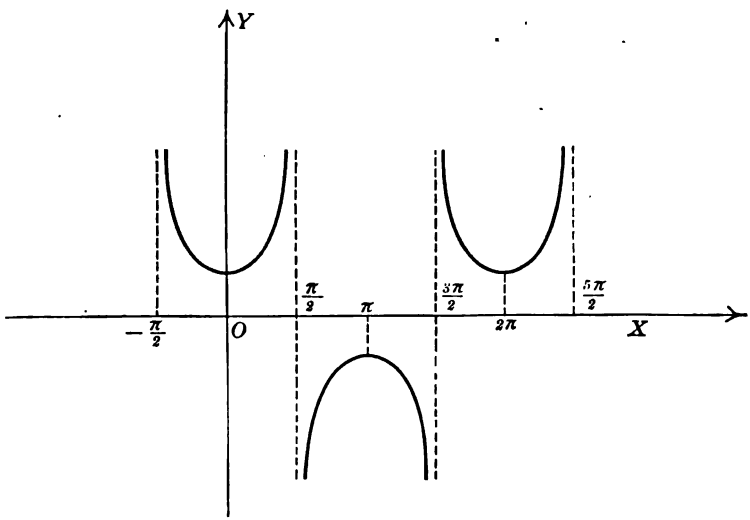
Y
X
O
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$\frac{5\pi}{2}$

FIG. 58.

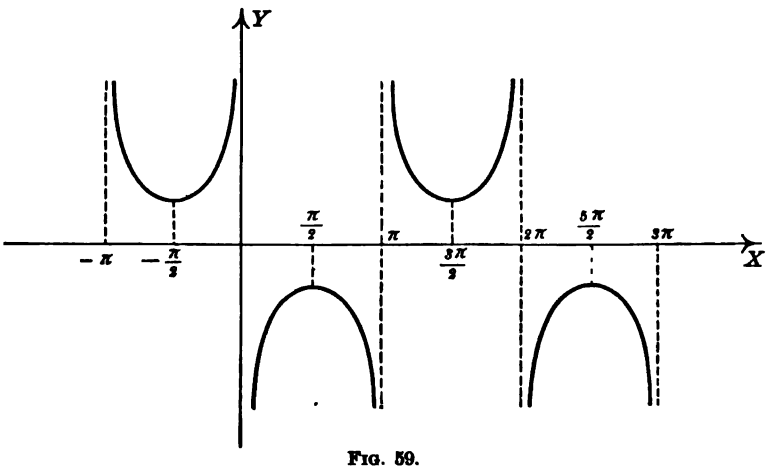

FIG. 59.

58. Functions of 270° and 360°. By the method of limits employed in Art. 12, we get the following sets of values:

$$\sin 270° = -1, \qquad \cos 270° = 0,$$
$$\tan 270° = \infty, \qquad \cot 270° = 0,$$
$$\sec 270° = \infty. \qquad \csc 270° = -1.$$
$$\sin 360° = 0, \qquad \cos 360° = 1,$$
$$\tan 360° = 0, \qquad \cot 360° = \infty,$$
$$\sec 360° = 1, \qquad \csc 360° = \infty.$$

Here ∞ is used as before to denote the value of a fraction whose numerator remains finite while its denominator approaches zero. The sign + or − is prefixed to the symbol ∞ according as the variable becomes ∞ through a positive or a negative sequence of values. In the light of this discussion the values of the functions of $k \times \frac{\pi}{2}$ (k any integer) may be tabulated, the upper of the pair of double signs arising when the angle approaches the critical value from below.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0	∓ 0	$+1$	∓ 0	$\mp\infty$	$+1$	$\mp\infty$
$\frac{\pi}{2}$	$+1$	± 0	$\pm\infty$	± 0	$\pm\infty$	$+1$
π	± 0	-1	∓ 0	$\mp\infty$	-1	$\pm\infty$
$\frac{3}{2}\pi$	-1	∓ 0	$\pm\infty$	± 0	$\mp\infty$	-1
2π	± 0	$+1$	∓ 0	$\mp\infty$	$+1$	$\mp\infty$

EXERCISE XVI

1. Trace the variation, as θ varies, (*a*) of $\sin 2\theta$; (*b*) of $\cot \frac{\theta}{2}$.

2. Trace the variation, as θ varies, (*a*) of $\tan 2\theta$; (*b*) of $\cos \frac{\theta}{2}$.

3. Draw the graph of $\cos 2\theta$.

4. Draw the graph of $\sin 3\theta$.

5. In what points will a horizontal line $\frac{1}{2}$ unit above the X-axis intersect the graph of $\sin\theta$? Explain the significance of the result.

6. In what points will a horizontal line 1 unit above the X-axis intersect the graph of $\tan\theta$? Explain.

7. If the graphs of $\tan\theta$ and $\cot\theta$ are drawn on the same axes to the same scale, where will they intersect? What is the significance?

8. If the graphs of $\sin\theta$ and $\cos\theta$ are drawn on the same axes to the same scale, where will they intersect? What is the significance?

9. Construct the graph of $\log_{10} x$, taking values of the number x as abscissas and the corresponding logarithms as ordinates.

59. Fundamental relations. Just as in Art. 9 we find, by inspection,

$$\csc a = \frac{1}{\sin a}, \tag{1}$$

$$\sec a = \frac{1}{\cos a}, \tag{2}$$

$$\cot a = \frac{1}{\tan a}; \tag{3}$$

by division,

$$\tan a = \frac{\sin a}{\cos a}, \tag{4}$$

$$\cot a = \frac{\cos a}{\sin a}; \tag{5}$$

by virtue of the Pythagorean proposition,

$$\sin^2 a + \cos^2 a = 1, \tag{6}$$

$$\tan^2 a + 1 = \sec^2 a, \tag{7}$$

$$\cot^2 a + 1 = \csc^2 a. \tag{8}$$

The student should prove that all these formulas conform, for angles in all quadrants, to the algebraic law of signs.

60. Line representations of the trigonometric functions. As the names tangent and secant indicate, the trigonometric functions were originally defined as certain lines measured in terms of a standard unit line. The adoption of the abstract ratios, as in this book, is of comparatively recent date. It is both interesting and advantageous to know the line representations and show that they lead to the same science of trigonometry as do the ratio definitions.

The line representations most frequently used involve the use of a unit circle, *i.e.* a circle of radius unity. It is evident that we may replace each of the defining ratios of Art. 54 by an equal ratio so chosen that its denominator is positive unity. The value of the ratio will be equal to that of the numerator. In other words, if a positive unit radius is taken as the denominator, the length and sign of the numerator will represent the function in magnitude and sign. We have, then, simply to select six lines whose ratios to the radius agree with the definitions of Art. 54. The ratio of the subtended arc to the radius is, by Art. 52, the circular measure of the angle.

Suppose, then, a circle of unit radius drawn with its center at the origin of coördinates.

The angle is placed upon the axes just as in Art. 6, and from the point P of intersection of the terminal line with the circle, perpendiculars MP and NP are drawn to the two axes. From the two points A and B where the positive axes cut the circle, tangents AT and BS are drawn meeting the terminal line (produced if necessary) in the points T and S.

Since P, T, S, Figs. 60–63, lie in the terminal line, we have, at once, in accordance with Art. 54 (or Art. 6):

$$\sin\alpha = \frac{MP}{OP}, \qquad \cos\alpha = \frac{NP}{OP},$$

$$\tan\alpha = \frac{AT}{OA}, \qquad \cot\alpha = \frac{BS}{OB},$$

$$\sec\alpha = \frac{OT}{OA}, \qquad \csc\alpha = \frac{OS}{OB}.$$

But by construction,

$$OP = OA = OB = 1.$$

These denominators may then be suppressed and the functions represented graphically as indicated below:

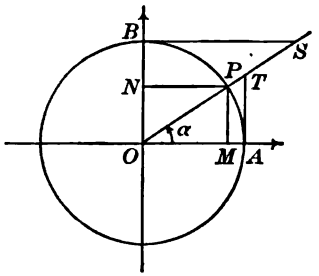

FIG. 60.

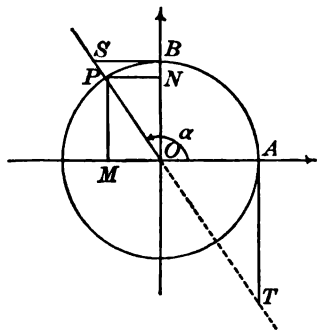

FIG. 61.

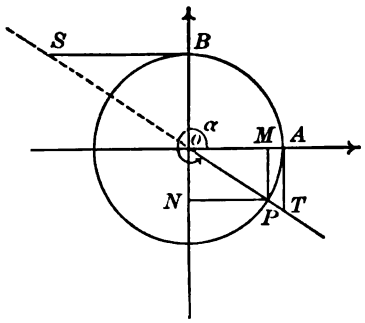

FIG. 62.

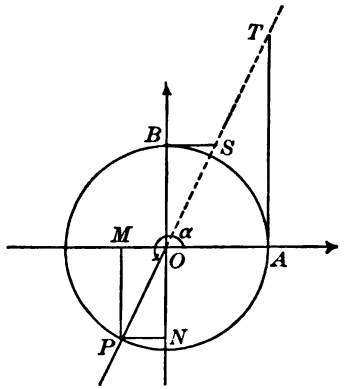

FIG. 63.

$$\sin\alpha = MP, \quad \cos\alpha = NP,$$
$$\tan\alpha = AT, \quad \cot\alpha = BS,$$
$$\sec\alpha = OT, \quad \csc\alpha = OS.$$

Moreover, the angle, in radians, is represented as follows:

$$\alpha = \frac{\text{arc } AP}{OP} = \text{arc } AP.$$

According to the modern view, the line is not the function, but by its length and direction represents the function in magnitude and sign.

Note that the line representing the tangent is always drawn from the point A and that representing the cotangent from B. All the lines are read *from* the axes *to* the terminal line. Horizontal lines are positive toward the right, negative toward the left. Vertical lines are positive upward, negative downward.

By means of the Pythagorean proposition, and the theorems concerning similar triangles, the fundamental relations given in the preceding article, as also the limitations of value stated in Art. 55, are readily established. So, also, the subsequent theorems of trigonometry may be interpreted by means of the line representation of the trigonometric functions. This graphic interpretation frequently presents special advantages. This is the case, for example, in the investigation of the variation of the functions considered in Art. 56. So, too, the construction of the graphs of the functions as treated in Art. 57 is facilitated, since the lengths of the defining lines may be transferred by the use of dividers.

EXERCISE XVII

Find the values of the following expressions:

1. $\cos^4\alpha - \sin^4\alpha$, when $\alpha = \arctan(-\frac{1}{2})$, in the 2d quadrant.

2. $\dfrac{\cos\alpha}{1-\tan\alpha} + \dfrac{\sin\alpha}{1-\cot\alpha}$, when $\alpha = \sec^{-1}(-3)$, in the 3d quadrant.

3. $\dfrac{\tan\alpha + \sec\alpha - 1}{\tan\alpha - \sec\alpha + 1}$, when $\alpha = \arcsin(-\frac{11}{61})$, in the 4th quadrant.

4. $\dfrac{1}{\csc\alpha - \cot\alpha} + \dfrac{1}{\csc\alpha + \cot\alpha}$, when $\alpha = \cos^{-1}\frac{28}{125}$, in the 4th quadrant.

Solve the following equations, finding all the angles less than 2π that satisfy each equation:

5. $\cos\beta = \frac{1}{2}$.

6. $\tan\beta = -\sqrt{3}$.

7. $\sin 2\alpha = -\frac{1}{2}\sqrt{3}$.

8. $\cot 3\alpha = 1$.

9. $4\sin^2\alpha - 4\cos\alpha - 1 = 0$.

10. $3\tan^2\beta - 1 = 0$.

11. $2\sin\beta\cos\beta - \sin\beta = 0$.

12. $2\sin\alpha + \sqrt{3}\tan\alpha = 0$.

In exercises 13–24, verify the given identities by transforming the first member into the second.

13. $(\sin\alpha + \cos\alpha)(\cot\alpha + \tan\alpha) = \sec\alpha + \csc\alpha$.

14. $(\sec\alpha - \cos\alpha)(\csc\alpha - \sin\alpha) = \sin\alpha\cos\alpha$.

15. $\dfrac{\tan\alpha + \cot\beta}{\cot\alpha + \tan\beta} = \tan\alpha\cot\beta$.

16. $(r\cos\theta)^2 + (r\sin\theta\cos\phi)^2 + (r\sin\theta\sin\phi)^2 = r^2$.

17. $\dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} \cdot \dfrac{\cot\alpha\cot\beta + 1}{\cot\beta - \cot\alpha} = 1$.

18. $\csc\alpha(\sec\alpha - 1) - \cot\alpha(1 - \cos\alpha) = \tan\alpha - \sin\alpha$.

19. $(\sin\alpha\cos\beta - \cos\alpha\sin\beta)^2 + (\cos\alpha\cos\beta + \sin\alpha\sin\beta)^2 = 1$.

20. $\sec\alpha\csc\alpha(1 - 2\cos^2\alpha) + \cot\alpha = \tan\alpha$.

21. $(\sin\alpha\cos\beta + \cos\alpha\sin\beta)^2 + (\cos\alpha\cos\beta - \sin\alpha\sin\beta)^2 = 1$.

22. $\sec^2\alpha\csc^2\alpha - \dfrac{(1-\tan^2\alpha)^2}{\tan^2\alpha} = 4$.

23. $(\cos\alpha + \sqrt{-1}\sin\alpha)(\cos\alpha - \sqrt{-1}\sin\alpha) = 1$.

24. $(\cos\alpha + \sqrt{-1}\sin\alpha)^2 + (\cos\alpha - \sqrt{-1}\sin\alpha)^2 = 4\cos^2\alpha - 2$.

25. By means of Fig. 60 show that, when θ is acute and measured in radians, $\sec\theta > \tan\theta > \theta > \sin\theta$.

26. By means of Fig. 60 show that, when θ is acute and measured in radians, $\csc\theta > \cot\theta > \left(\dfrac{\pi}{2} - \theta\right) > \cos\theta$.

61. Periodicity of the trigonometric functions. It was pointed out, in Art. 23, that if two angles differing by an integral multiple of 360° are placed on the axes, their terminal lines coincide. As an immediate consequence, it follows that corresponding functions of the two angles are identical. Thus we may write

$$\sin(2k\pi + \alpha) = \sin\alpha,$$

and, in general,

$$F(2k\pi + \alpha) = F(\alpha),$$

where F denotes the same function in both members of the equation, and k is an integer.

62. Functions of $\left[k\cdot\dfrac{\pi}{2} \pm \alpha\right]$. Precisely as in Art. 10, 37, and 38, we may express the functions of the angles $\pm\alpha$, $90° \pm \alpha$, $180° \pm \alpha$, $270° \pm \alpha$, $360° \pm \alpha$, and other similarly compounded angles in terms of the functions of α, no matter what the quadrant of the angle α. Because of the periodicity brought out in the preceding article, it is not necessary to carry the investigation beyond the five multiples of the right angle mentioned; indeed, the fifth reduces to the first. On account of the double signs and the possibility of α belonging to any one of the four quadrants, there exist thirty-two distinct cases. The demonstration is the same

for all cases, involving the same proportionality of sides of similar triangles and the same question of agreement or opposition of signs. The working out of the proof in three characteristic instances should be sufficient to enable the student to do the same for any and all cases. The theorem is, however, somewhat elusive, and the student can completely master it and render it an infallible instrument only by actual careful construction and proof of most of the cases. Upon first study it may be well to limit consideration to the cases in which α is of the first quadrant.

Let it be required first to express the functions of $(180° + \alpha)$ in terms of functions of α, when α is an angle of the first quadrant. If, in Fig. 64, $\angle XOA = \alpha$, then $\angle XOB = \beta = 180° + \alpha$. The two triangles OMP and ONQ are similar, the pairs of corresponding sides being v and v', x and x', and y and y'. Notice also that x' and y' are negative, all the other sides being positive. Giving due attention to signs, we may write:

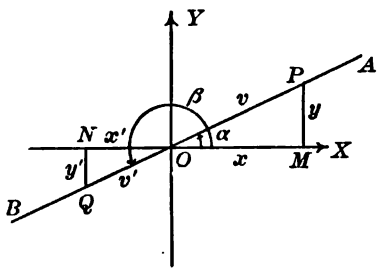

FIG 64.

$$\sin(180° + \alpha) \equiv \sin\beta = \frac{y'}{v'} = -\frac{y}{v} = -\sin\alpha,$$

$$\cos(180° + \alpha) \equiv \cos\beta = \frac{x'}{v'} = -\frac{x}{v} = -\cos\alpha,$$

$$\tan(180° + \alpha) \equiv \tan\beta = \frac{y'}{x'} = \frac{y}{x} = \tan\alpha,$$

$$\cot(180° + \alpha) \equiv \cot\beta = \frac{x'}{y'} = \frac{x}{y} = \cot\alpha,$$

$$\sec(180° + \alpha) \equiv \sec\beta = \frac{v'}{x'} = -\frac{v}{x} = -\sec\alpha,$$

$$\csc(180° + \alpha) \equiv \csc\beta = \frac{v'}{y'} = -\frac{v}{y} = -\csc\alpha.$$

Again, let it be required to express the functions of $(270° - \alpha)$ in terms of functions of α, when α is of the first quadrant. In Fig. 65, $\angle XOA = \alpha$, $\angle XOB = \beta = 270° - \alpha$. The two triangles OMP and ONQ are similar, the pairs of corresponding sides now

being v and v', x and y', and y and x'. The sides x' and y' are negative, all the others positive. We may then write:

$$\sin(270° - \alpha) \equiv \sin\beta = \frac{y'}{v'} = -\frac{x}{v} = -\cos\alpha.$$

$$\cos(270° - \alpha) \equiv \cos\beta = \frac{x'}{v'} = -\frac{y}{v} = -\sin\alpha.$$

$$\tan(270° - \alpha) \equiv \tan\beta = \frac{y'}{x'} = \frac{x}{y} = \cot\alpha.$$

$$\cot(270° - \alpha) \equiv \cot\beta = \frac{x'}{y'} = \frac{y}{x} = \tan\alpha.$$

$$\sec(270° - \alpha) \equiv \sec\beta = \frac{v'}{x'} = -\frac{v}{y} = -\csc\alpha.$$

$$\csc(270° - \alpha) \equiv \csc\beta = \frac{v'}{y'} = -\frac{v}{x} = -\sec\alpha.$$

As a third and especially important instance, let us find the functions of $-\alpha$, when α is of the second quadrant. In Fig. 66, $XOA = \alpha$, $XOB = \beta = -\alpha$. The two triangles OMP and ONQ are similar, the pairs of corresponding sides being v and v', x and x', y and y', while x, x', and y' are negative.

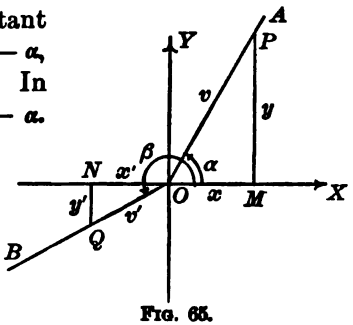

FIG. 65.

We then have, as before:

$$\sin(-\alpha) \equiv \sin\beta = \frac{y'}{v'} = -\frac{y}{v} = -\sin\alpha,$$

$$\cos(-\alpha) \equiv \cos\beta = \frac{x'}{v'} = \frac{x}{v} = \cos\alpha,$$

$$\tan(-\alpha) \equiv \tan\beta = \frac{y'}{x'} = \frac{y}{x} = -\tan\alpha.$$

$$\cot(-\alpha) \equiv \cot\beta = \frac{x'}{y'} = -\frac{x}{y} = -\cot\alpha,$$

$$\sec(-\alpha) \equiv \sec\beta = \frac{v'}{x'} = \frac{v}{x} = \sec\alpha,$$

$$\csc(-\alpha) \equiv \csc\beta = \frac{v'}{y'} = -\frac{v}{x} = -\csc\alpha.$$

It will be noticed that whenever the number of right angles involved is even the pairs of corresponding sides are v and v', x and x', y and y'; while whenever the number of right angles is odd the pairs of corresponding sides are v and v', x *and* y', y *and* x'. Thus we have the theorem: *Any function of an even number of right angles plus or minus* α *is numerically equal to the same function of* α*; any function of an odd number of right angles plus or minus* α *is numerically equal to the corresponding co-function of* α*; the agreement or opposition of signs is to be determined from the quadrants of* α *and of the compound angle.* It may easily be verified that in all cases this agreement or opposition of signs is the same as when α is of first quadrant.

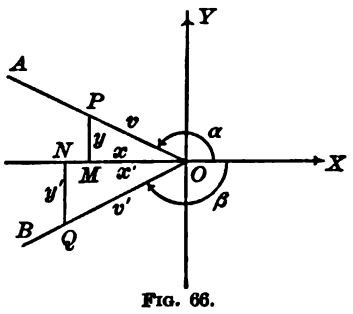

FIG. 66.

The general theorem may also be stated as follows: *If the sum or difference of two angles is an even number of right angles, the functions of the one are numerically equal to the same functions of the other. If the sum or difference of two angles is an odd number of right angles, the functions of the one are numerically equal to the corresponding co-functions of the other. The agreement or opposition of signs is to be determined from the quadrants of the two angles.*

The significance of the theorem is made clear by application to an example: Required to find the value of $\cos(810° + \alpha)$. Here $810° = 9 \times 90°$, an odd number of right angles. When α is considered as of the first quadrant (and its functions consequently positive), the compound angle $(810° + \alpha)$ is of the second quadrant and hence its cosine is negative. The required relation is, therefore,

$$\cos(810° + \alpha) = -\sin\alpha,$$

which holds for all values of α.

Again, to find the value of tan 1230°. We have

$1230° = 14 \times 90° - 30°$, and is of second quadrant.

Then $$\tan 1230° = -\tan 30° = -\frac{1}{\sqrt{3}}.$$

The student may, if he prefers, construct the figure and proceed as in the demonstration just given.

As a consequence of these relations, it follows that to every inverse function correspond two angles, lying between 0 and 2π.

Thus
$$\arcsin a = \alpha \text{ and } \pi - \alpha,$$
$$\arccos b = \alpha \text{ and } 2\pi - \alpha,$$
$$\arctan c = \alpha \text{ and } \pi + \alpha,$$
$$\operatorname{arc\,cot} d = \alpha \text{ and } \pi + \alpha,$$
$$\operatorname{arc\,sec} e = \alpha \text{ and } 2\pi - \alpha,$$
$$\operatorname{arc\,csc} f = \alpha \text{ and } \pi - \alpha.$$

These statements should be verified by the student.

EXERCISE XVIII

Express in terms of a positive angle less than 45°:

1. $\sin 700°$.
2. $\cos 260°$.
3. $\tan 436°$.
4. $\cot -35°$.
5. $\csc 930°$.
6. $\sec 1400°$.

Find the value of $\cos\alpha + \sin\alpha$ and of $\tan\alpha - \cot\alpha$ when α has the value

7. $\frac{\pi}{6}$.
8. $-\frac{2\pi}{3}$.
9. $\frac{19\pi}{4}$.
10. $\frac{7\pi}{6}$.
11. $\frac{11\pi}{3}$.
12. $-\frac{9\pi}{4}$.

Find all the values between 0° and 360° of

13. $\arctan \sqrt{3}$.
14. $\csc^{-1}(-\sqrt{2})$.
15. $\arccos(-.5)$.
16. $\operatorname{arcsec} 2$.
17. $\operatorname{arccot}(-1)$.
18. $\sin^{-1}(-\frac{1}{2}\sqrt{3})$.

Find the value of

19. $\sin 480° \sin 690° + \cos(-420°)\cos 600°$.
20. $\tan 840° \cot 420° + \tan(-300°)\cot(-120°)$.
21. $\tan\frac{17\pi}{6}\tan\frac{14\pi}{3} + \cot\left(-\frac{11\pi}{6}\right)\cot\left(-\frac{4\pi}{3}\right)$.
22. $\sin\frac{19\pi}{6}\cos\left(-\frac{11\pi}{6}\right) - \sin\frac{7\pi}{3}\cos\left(-\frac{4\pi}{3}\right)$.

23. If $\sin 200° 30' = .35$, find $\cos 830° 30'$.

24. If $\tan 558° 26' = \frac{1}{3}$, find $\cot 468° 26'$.

25. If $\cot 520° = -a$, find $\sin 160°$.

26. If $\cos 590° = -m$, find $\tan 850°$.

27. Express $\cos(\alpha - 90°)$ as a function of α.

28. Express $\sin(\alpha - 180°)$ as a function of α.

29. Express $\tan(\alpha - 360°)$ as a function of α.

30. Express $\cot(\alpha - 270)°$ as a function of α.

CHAPTER VIII

FUNCTIONS OF TWO ANGLES

63. Formulas for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$. Suppose α and β to be acute angles. In Fig. 67 $(\alpha+\beta)$ is acute; in Fig. 68 $(\alpha+\beta)$ is obtuse. The following demonstration applies to both figures.

Let $\angle XOA=\alpha$, $\angle AOB=\beta$; then $\angle XOB=\alpha+\beta$. From P, a point in OB, draw PM perpendicular to OX, PQ perpendic-

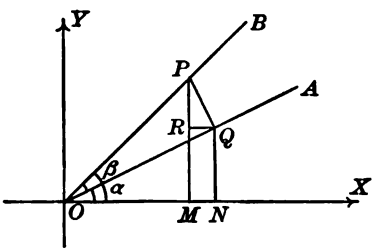

FIG. 67.

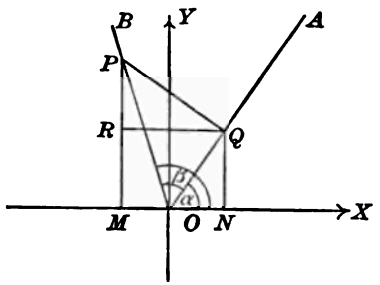

FIG. 68.

ular to OA, and from Q draw QN perpendicular to OX, and QR perpendicular to MP. The angle $RPQ=\alpha$ and $RP=QP\cos\alpha$, $RQ=QP\sin\alpha$, by Art. 16. By the same article,

$$MP=OP\sin(\alpha+\beta).$$

Also
$$\begin{aligned} MP&=MR+RP=NQ+RP\\ &=OQ\sin\alpha+QP\cos\alpha\\ &=OP\sin\alpha\cos\beta+OP\cos\alpha\sin\beta. \end{aligned}$$

Equating the two values of MP and dividing through by the common factor OP, we have the theorem

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta. \qquad (1)$$

In like manner

$$OM = OP \cos(\alpha + \beta),$$

and also
$$\begin{aligned} OM &= ON - MN = ON - RQ \\ &= OQ \cos\alpha - OP \sin\alpha \\ &= OP \cos\alpha \cos\beta - OP \sin\alpha \sin\beta. \end{aligned}$$

Hence the companion theorem

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta. \qquad (2)$$

These are called the *addition formulas* and are fundamental in trigonometry.

64. Extension of addition formulas. The two formulas of the last article were proved only for angles both of the first quadrant. It remains to be shown that they hold when α and β denote any angles.

First, let α be an angle of the second quadrant. Then $\phi\ (= \alpha - 90°)$ is an angle of the first quadrant. Now $\alpha = 90° + \phi$, so that, by Art. 38,

$$\sin\alpha = \cos\phi, \quad \cos\alpha = -\sin\phi.$$

Since ϕ is of the first quadrant, the formulas of Art. 63 apply and we have

$$\begin{aligned} \sin(\alpha + \beta) &= \sin(90° + \overline{\phi + \beta}) \\ &= \cos(\phi + \beta) \\ &= \cos\phi \cos\beta - \sin\phi \sin\beta, \\ &= \sin\alpha \cos\beta + \cos\alpha \sin\beta. \end{aligned}$$

Likewise

$$\begin{aligned} \cos(\alpha + \beta) &= \cos(90° + \overline{\phi + \beta}) \\ &= -\sin(\phi + \beta) \\ &= -\sin\phi \cos\beta - \cos\phi \sin\beta, \\ &= \cos\alpha \cos\beta - \sin\alpha \sin\beta. \end{aligned}$$

The formulas are therefore true when one angle is of the first and the other of the second quadrant. By adding 90° successively to each of the angles, the formulas are established for two positive angles of all quadrants. If one of the angles is negative, it can be augmented by such an integral multiple of 360° as to produce a positive angle possessing the same functions.

The addition formulas are, therefore, true for angles of any size.

EXERCISE XIX

Evaluate the addition formulas for

1. $\alpha = 60°$, $\beta = 30°$.
2. $\alpha = 45°$, $\beta = 90°$.
3. $\alpha = 240°$, $\beta = 150°$.
4. $\alpha = 300°$, $\beta = 150°$.
5. $\alpha = \arctan \frac{3}{4}$, $\beta = \arccos (-\frac{8}{17})$, α first quadrant, β second.
6. $\alpha = \sin^{-1} (-\frac{5}{13})$, $\beta = \cot^{-1} \frac{7}{24}$, α fourth quadrant, β third.

Find the value of

7. $\cos\left(\frac{\pi}{4} + \alpha\right) \cos\left(\frac{\pi}{4} + \beta\right) - \sin\left(\frac{\pi}{4} + \alpha\right) \sin\left(\frac{\pi}{4} + \beta\right)$.
8. $\sin\left(\frac{\pi}{3} + \alpha\right) \cos\left(\frac{\pi}{6} + \beta\right) + \cos\left(\frac{\pi}{3} + \alpha\right) \sin\left(\frac{\pi}{6} + \beta\right)$.
9. $\sin (1 + n)\,\alpha \cos (1 - n)\,\alpha + \cos (1 + n)\,\alpha \sin (1 - n)\,\alpha$.
10. $\cos (1 + n)\,\alpha \cos (1 - n)\,\alpha - \sin (1 + n)\,\alpha \sin (1 - n)\,\alpha$.
11. $\sin (\theta + \phi) \cos (\theta - \phi) + \cos (\theta + \phi) \sin (\theta - \phi)$.
12. $\cos (\theta - \phi) \cos \phi - \sin (\theta - \phi) \sin \phi$.
13. Evaluate the addition formulas for $\alpha = 60°$, $\beta = 45°$, and thus find sin 105°, cos 105°, sin 15°, cos 15°.
14. Evaluate the addition formulas for $\alpha = 45°$, $\beta = 30°$, and thus find sin 75°, cos 75°, sin 15°, cos 15°.

65. Subtraction formulas. In the addition formulas replace β by $-\beta$. We have

$$\sin (\alpha - \beta) = \sin \alpha \cos (-\beta) + \cos \alpha \sin (-\beta).$$

But by Art. 62,

$$\sin (-\beta) = -\sin \beta, \cos (-\beta) = \cos \beta.$$

Making this substitution, we have

$$\sin (\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta. \tag{1}$$

In like manner

$$\cos (\alpha - \beta) = \cos \alpha \cos (-\beta) - \sin \alpha \sin (-\beta),$$

or, by the same substitution,

$$\cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta. \tag{2}$$

66. Formulas for $\tan(\alpha \pm \beta)$, $\cot(\alpha \pm \beta)$. From Arts. 59 and 63 we have

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$= \frac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

or, finally,

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}. \tag{1}$$

In like manner we may derive

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}. \tag{2}$$

Again,

$$\cot(\alpha+\beta) = \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)},$$

$$= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta},$$

$$= \frac{\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} - \dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\dfrac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} + \dfrac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}},$$

or

$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\beta + \cot\alpha}. \tag{3}$$

Likewise

$$\cot(\alpha-\beta) = \frac{\cot\alpha\cot\beta + 1}{\cot\beta - \cot\alpha}. \tag{4}$$

EXERCISE XX

1. Demonstrate geometrically the formula for $\sin(\alpha-\beta)$, when $\alpha > \beta$, both acute.

2. Demonstrate geometrically the formula for $\cos(\alpha-\beta)$, when $\alpha > \beta$, both acute.

Evaluate the formulas of Art. 65 for

3. $\alpha = 60°, \beta = 120°$.
4. $\alpha = 240°, \beta = 150°$.
5. $\alpha = \arcsin \frac{5}{17}, \beta = \arctan -\frac{4}{3}$.
6. $\alpha = \cot^{-1} \frac{11}{60}, \beta = \cos^{-1} \frac{4}{5}$.

Evaluate the formulas of Art. 66 for

7. $\alpha = 330°, \beta = 150°$.
8. $\alpha = 210°, \beta = 300°$.
9. $\alpha = \cos^{-1} \frac{5}{13}, \beta = \tan^{-1} (-\frac{4}{3})$.
10. $\alpha = \operatorname{arccot} \frac{44}{117}, \beta = \arctan \frac{44}{117}$.
11. Find the functions of 15° by putting $\alpha = 45°, \beta = 30°$.
12. Find the functions of 15° by putting $\alpha = 60°, \beta = 45°$.

Show that

13. $\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$.
14. $\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$.

Expand by successive applications of the formulas:

15. $\cos(\alpha+\beta+\gamma)$.
16. $\sin(\alpha+\beta+\gamma)$.
17. $\tan(\alpha+\beta+\gamma)$.
18. $\cot(\alpha+\beta+\gamma)$.

Show that

19. $\sin\left(\frac{\pi}{3}+\alpha\right) - \sin\left(\frac{\pi}{3}-\alpha\right) = \sin\alpha$.
20. $\cos\left(\frac{\pi}{6}+\alpha\right) + \cos\left(\frac{\pi}{6}-\alpha\right) = \cos\alpha$.

67. Functions of twice an angle. In the addition formulas of Arts. 63 and 66, place $\beta = \alpha$. We then obtain

$$\sin 2\alpha = 2\sin\alpha\cos\alpha, \tag{1}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha, \tag{2}$$

$$= 1 - 2\sin^2\alpha, \tag{2a}$$

$$= 2\cos^2\alpha - 1. \tag{2b}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}. \tag{3}$$

$$\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}. \tag{4}$$

68. Functions of half an angle. From Art. 67 we may write

$$\cos 2\beta = 1 - 2\sin^2\beta,$$

and solving for $\sin\beta$,

$$\sin\beta = \sqrt{\tfrac{1}{2}(1-\cos 2\beta)}.$$

Now placing $2\beta = \alpha$, so that $\beta = \frac{\alpha}{2}$, we obtain

$$\sin\tfrac{1}{2}\alpha = \sqrt{\tfrac{1}{2}(1-\cos\alpha)}. \tag{1}$$

Similarly

$$\cos 2\beta = 2\cos^2\beta - 1;$$

so that

$$\cos\beta = \sqrt{\tfrac{1}{2}(1+\cos 2\beta)},$$

and, with the same substitution,

$$\cos\tfrac{1}{2}\alpha = \sqrt{\tfrac{1}{2}(1+\cos\alpha)}. \tag{2}$$

Dividing the first formula by the second, we get

$$\tan\frac{1}{2}\alpha = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}, \tag{3}$$

and inverting,

$$\cot\frac{1}{2}\alpha = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}. \tag{4}$$

Rationalizing the numerators of the last two formulas, we get other useful forms,

$$\tan\frac{1}{2}\alpha = \frac{1-\cos\alpha}{\sin\alpha}, \tag{5}$$

$$\cot\frac{1}{2}\alpha = \frac{1+\cos\alpha}{\sin\alpha}. \tag{6}$$

EXERCISE XXI

Find the values of

1. The functions of 60° from those of 30°.
2. The functions of 120° from those of 60°.
3. The functions of 75° from those of 150°.
4. The functions of 15° from those of 30°.

Find the values of the functions of

5. $2\arctan\frac{5}{12}$.
6. $2\cos^{-1}(-\frac{5}{13})$.
7. $\frac{1}{2}\sin^{-1}(-\frac{24}{25})$.
8. $\frac{1}{2}\arctan\frac{119}{120}$.
9. $\arcsin\frac{5}{13} + 2\operatorname{arccot}\frac{3}{4}$.
10. $\arctan\frac{5}{12} - 2\arccos\frac{3}{5}$.

Transform the first member into the second:

11. $\dfrac{1+\sin\theta-\cos 2\theta}{\cos\theta+\sin 2\theta}=\tan\theta.$

12. $\dfrac{1+\cos\theta+\cos 2\theta}{\sin\theta+\sin 2\theta}=\cot\theta.$

13. $(\sqrt{1+\sin\alpha}+\sqrt{1-\sin\alpha})^2=4\sin^2\tfrac{1}{2}\alpha.$

14. $(\sqrt{1+\sin\alpha}-\sqrt{1-\sin\alpha})^2=4\cos^2\tfrac{1}{2}\alpha.$

15. $\tan\left(\frac{\pi}{4}+\alpha\right)-\tan\left(\frac{\pi}{4}-\alpha\right)=2\tan 2\alpha.$

16. $\cot\left(\frac{\pi}{4}+\alpha\right)-\cot\left(\frac{\pi}{4}-\alpha\right)=-2\tan 2\alpha.$

Find the values of α which satisfy the following equations:

17. $(2+\sqrt{3})(1-\sin 2\alpha)-2\cos^2 2\alpha=0.$

18. $\sin 2\alpha+2\cos 2\alpha=1.$

19. $4\sec^2 2\alpha+\tan 2\alpha=7.$

20. $\csc 2\alpha+\cot 2\alpha=2.$

Show that

21. $\tan^{-1}\dfrac{x-2}{3}=\cos^{-1}\dfrac{3}{\sqrt{x^2-4x+13}}.$

22. $\arctan\dfrac{2}{\sqrt{x^2+2x-3}}=\operatorname{arccsc}\dfrac{x+1}{2}.$

23. Find $\sin\left(\frac{\pi}{2}-2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right).$

24. Find $\sin\left(\sin^{-1}m+\tan^{-1}\dfrac{\sqrt{1-m^2}}{m}\right).$

25. Find $\sin\left(\arccos(1-a)-2\arctan\sqrt{\dfrac{a}{2-a}}\right).$

26. Find $\cos(\arccos(1-2a)-2\arcsin\sqrt{a}).$

69. Conversion formulas for products. Adding the two first formulas of Arts. 63 and 65, we have

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta,$$

or, reversing and dividing by 2,

$$\sin\alpha\cos\beta=\tfrac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]. \qquad (1)$$

If we subtract, instead of adding, we get

$$\sin(\alpha+\beta)-\sin(\alpha-\beta)=2\cos\alpha\sin\beta,$$

or

$$\cos\alpha\sin\beta=\tfrac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]. \qquad (2)$$

Treating the two second formulas in like manner, we obtain

$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)], \tag{3}$$

and

$$\sin\alpha\sin\beta = -\tfrac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)]. \tag{4}$$

By means of these formulas, products of sines and cosines are expressed as sums or differences. By successive applications higher powers and products are reducible to expressions linear in sines and cosines. The same transformations may often be effected by application of the formulas of Art. 67, written in the form

$$\sin\alpha\cos\alpha = \tfrac{1}{2}\sin 2\alpha, \tag{5}$$

$$\sin^2\alpha = \tfrac{1}{2}(1-\cos 2\alpha), \tag{6}$$

$$\cos^2\alpha = \tfrac{1}{2}(1+\cos 2\alpha). \tag{7}$$

EXERCISE XXII

Reduce the following products to linear expressions:

1. $\sin 5\alpha\cos 3\alpha$.
2. $\cos 6\alpha\sin 4\alpha$.
3. $\sin 7\alpha\sin 3\alpha$.
4. $\cos 2\alpha\cos 5\alpha$.
5. $\sin^2\alpha\cos\alpha$.
6. $\sin\alpha\cos^3\alpha$.
7. $\cos^4\alpha$.
8. $\sin^4\alpha$.
9. $\cos^3\alpha\sin^3\alpha$.
10. $\sin^2\alpha\cos^2\alpha$.

Show that

11. $\cos\alpha\sin(\beta-\gamma)+\cos\beta\sin(\gamma-\alpha)+\cos\gamma\sin(\alpha-\beta)=0$.
12. $\sin(\beta-\gamma)\sin(\alpha-\delta)+\sin(\gamma-\alpha)\sin(\beta-\delta)+\sin(\alpha-\beta)\sin(\gamma-\delta)=0$.
13. $\sin\frac{3\pi}{5}\cos\frac{\pi}{5}+\sin\frac{2\pi}{5}\cos\frac{4\pi}{5}=0$.
14. $2\cos\frac{5\pi}{8}\cos\frac{\pi}{4}+\sin\frac{\pi}{8}+\cos\frac{5\pi}{8}=0$.

Solve for α, making use of Art. 10.

15. $\cos(50^\circ+\alpha)\sin(50^\circ-\alpha)-\cos(40^\circ-\alpha)\sin(40^\circ+\alpha)=0$.
16. $\sin(70^\circ+\alpha)\sin(70^\circ-\alpha)+\sin(20^\circ+\alpha)\sin(20^\circ-\alpha)=0$.

Solve for α, making use of Art. 69.

17. $\cos 8\alpha+\cos 9\alpha=0$.
18. $\sin 5\alpha-\sin 10\alpha=0$.

19. $\cos(\alpha+\theta)\cos(\alpha-\theta)+\cos(3\alpha+\theta)\cos(3\alpha-\theta)=\cos 2\theta.$

20. $\sin(\alpha+\theta)\cos(\alpha-\theta)+\sin(3\alpha+\theta)\cos(3\alpha-\theta)=\sin 2\theta.$

70. Conversion formulas for sums and differences. In the process of deriving the formulas of the last article, before reversing and dividing by 2, substitute $\alpha+\beta=\phi$, $\alpha-\beta=\theta$, so that

$$\alpha=\frac{\phi+\theta}{2},\quad \beta=\frac{\phi-\theta}{2}.$$

We then obtain the following formulas:

$$\sin\phi+\sin\theta=2\sin\frac{\phi+\theta}{2}\cos\frac{\phi-\theta}{2}, \tag{1}$$

$$\sin\phi-\sin\theta=2\cos\frac{\phi+\theta}{2}\sin\frac{\phi-\theta}{2}, \tag{2}$$

$$\cos\phi+\cos\theta=2\cos\frac{\phi+\theta}{2}\cos\frac{\phi-\theta}{2}, \tag{3}$$

$$\cos\phi-\cos\theta=-2\sin\frac{\phi+\theta}{2}\sin\frac{\phi-\theta}{2}. \tag{4}$$

These formulas serve to effect transformations converse to those mentioned in Art. 69.

71. Multiple angles. In the formula for $\sin(\alpha+\beta)$ put $\beta=2\alpha$. Then

$$\begin{aligned}\sin 3\alpha&=\sin\alpha\cos 2\alpha+\cos\alpha\sin 2\alpha\\&=\sin\alpha-2\sin^3\alpha+2\sin\alpha\cos^2\alpha\\&=3\sin\alpha-4\sin^3\alpha.\end{aligned}$$

Again,

$$\begin{aligned}\cos 3\alpha&=\cos\alpha\cos 2\alpha-\sin\alpha\sin 2\alpha\\&=2\cos^3\alpha-\cos\alpha-2\sin^2\alpha\cos\alpha\\&=4\cos^3\alpha-3\cos\alpha.\end{aligned}$$

In like manner the other functions of 3α and, by repeating the process, the functions of any integral multiple of α may be expressed in terms of functions of α.

EXERCISE XXIII

Show that

1. $\dfrac{\sin 6\alpha+\sin 4\alpha}{\cos 6\alpha+\cos 4\alpha}=\tan 5\alpha.$

2. $\dfrac{\cos 3\alpha-\cos 5\alpha}{\sin 3\alpha+\sin 5\alpha}=\tan\alpha.$

3. $\dfrac{\sin 7\alpha - \sin 5\alpha}{\cos 7\alpha + \cos 5\alpha} = \tan \alpha.$

4. $\dfrac{\cos 4\alpha - \cos 2\alpha}{\sin 4\alpha - \sin 2\alpha} = -\tan 3\alpha.$

5. $\dfrac{\sin \alpha - \sin \beta}{\sin \alpha + \sin \beta} = \cot \dfrac{\alpha + \beta}{2} \tan \dfrac{\alpha - \beta}{2}.$

6. $\dfrac{\cos \alpha + \cos \beta}{\cos \alpha - \cos \beta} = -\cot \dfrac{\alpha + \beta}{2} \cot \dfrac{\alpha - \beta}{2}$

7. $\dfrac{\cos 3\theta + 2\cos 5\theta + \cos 7\theta}{\sin 3\theta + 2\sin 5\theta + \sin 7\theta} = \cot 5\theta.$

8. $\dfrac{\sin \theta - 2\sin 4\theta + \sin 7\theta}{\cos \theta - 2\cos 4\theta + \cos 7\theta} = \tan 4\theta.$

Solve the following equations:

9. $\cos \theta + \cos 5\theta = \cos 3\theta.$

10. $\sin \theta + \sin 5\theta = \sin 3\theta.$

11. $\sin 2\theta + 2\sin 4\theta + \sin 6\theta = 0.$

12. $\cos 3\theta + 2\cos 4\theta + \cos 5\theta = 0.$

Derive the formulas for:

13. $\cot 3\alpha.$ (In terms of $\cot \alpha$.)

14. $\tan 3\alpha.$ (In terms of $\tan \alpha$.)

15. $\sin 4\alpha.$

16. $\cos 4\alpha.$

Solve the equations:

17. $\sin 3\alpha = \sqrt{2} \sin 2\alpha.$

18. $\sqrt{3} \cos 3\alpha + 2\sin 2\alpha = 0.$

19. $\cos 3\alpha = \cos \alpha \cos 2\alpha.$

20. $\sin 3\alpha = \sin \alpha \cos 2\alpha.$

CHAPTER IX

ANALYTIC TRIGONOMETRY

The foregoing chapters constitute an introduction to the elementary principles of trigonometry. The student ought now to be prepared for a more advanced study of the theory of the trigonometric functions, which may be entitled analytic trigonometry. It is beyond the scope of this book to consider more than a few of the most important topics which might be discussed under this head. For a more extended treatment the student is referred to the treatises by Henrici and Treutlein, Hobson, Lock, Loney, Todhunter, and others, and, of course, to articles in the various mathematical journals.

72. Limits of $\theta/\sin\theta$ and $\theta/\tan\theta$ as θ approaches zero. Let θ be an acute angle measured in radians. Construct, as in Fig. 69, the angle $XOP = \theta$, repeated symmetrically as XOQ. Draw through P the arc PAQ with center O, the chord PMQ, and the broken or double tangent PTQ. Then

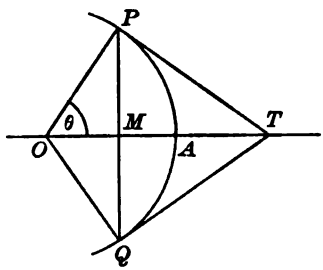

FIG. 69.

$$\frac{AP}{OP} = \theta, \qquad \frac{MP}{OP} = \sin\theta, \qquad \frac{TP}{OP} = \tan\theta.$$

By elementary geometry,

$$PMQ < PAQ < PTQ.$$

Whence, dividing by 2 and by OP,

$$\sin\theta < \theta < \tan\theta. \tag{1}$$

Dividing equation (1) through by $\sin\theta$, we have

$$1 < \frac{\theta}{\sin\theta} < \sec\theta.$$

Now in Art. 12 it was proved that as θ approaches the limit 0, $\cos\theta$ and its reciprocal $\sec\theta$ approach the limit 1. Thus, the value of $\theta/\sin\theta$ is always intermediate between 1 and a number that approaches the limit 1, as θ approaches 0. The ratio $\theta/\sin\theta$

must, therefore, approach the limit 1 at the same time. This is expressed symbolically by writing

$$\lim_{\theta \doteq 0} \left(\frac{\theta}{\sin \theta} \right) = 1.$$

Again, dividing equation (1) through by $\tan \theta$, we get

$$\cos \theta < \frac{\theta}{\tan \theta} < 1.$$

Now as θ approaches 0, $\cos \theta$ approaches 1, and hence, as before, $\theta / \tan \theta$ approaches the limit 1 at the same time. Symbolically,

$$\lim_{\theta \doteq 0} \left(\frac{\theta}{\tan \theta} \right) = 1.$$

NOTE. — Since $\sin \theta$ and $\tan \theta$ both approach 0 along with θ, it might seem that they therefore approach equality, and then the theorems would follow. The fallacy of assuming that the limiting form $\frac{0}{0}$ has the value 1 will appear on considering the following instances. The circumference and area of a circle approach zero simultaneously with the radius. We have, however, the general relations

$$\frac{\text{Circumference}}{\text{Radius}} = \frac{2\pi r}{r} = 2\pi = 6.28318 \cdots,$$

$$\frac{\text{Area}}{\text{Radius}} = \frac{\pi r^2}{r} = \pi r = 3.14159 \cdots r.$$

Now when r approaches the limit 0, the limit of the first ratio is the constant 2π, and the limit of the second ratio is 0.

The limiting form $\frac{0}{0}$ will be discussed at length in calculus. (See Townsend and Goodenough's "First Course in Calculus," Art. 18.)

EXAMPLE. If θ is increased by an angle δ, let it be required to determine the limit of the ratio of the consequent increase in $\sin \theta$ to the increment δ of θ, as that increment δ approaches zero. By Art. 70, we have

$$\frac{\sin(\theta + \delta) - \sin \theta}{\delta} = \frac{2 \cos\left(\theta + \frac{\delta}{2}\right) \sin \frac{\delta}{2}}{\delta},$$

$$= \cos\left(\theta + \frac{\delta}{2}\right) \cdot \frac{\sin \frac{\delta}{2}}{\frac{\delta}{2}}.$$

Now when δ approaches 0, $\cos\left(\theta+\frac{\delta}{2}\right)$ approaches $\cos\theta$ and

$$\frac{\sin\frac{\delta}{2}}{\frac{\delta}{2}} \text{ approaches } 1.$$

Hence

$$\lim_{\delta=0}\frac{\sin(\theta+\delta)-\sin\delta}{\delta}=\cos\theta.$$

It will be noticed that the numerator and denominator approach 0 simultaneously, but that the limit of the value of their ratio is a number somewhere between -1 and $+1$, and depending upon the value of θ.

EXAMPLES

In like manner find the limits as $\delta=0$, of

1. $\frac{\cos(\theta+\delta)-\cos\theta}{\delta}$.
2. $\frac{\sec(\theta+\delta)-\sec\theta}{\delta}$. (SUGGESTION. Express in terms of cosine.)
3. $\frac{\csc(\theta+\delta)-\csc\theta}{\delta}$.
4. $\frac{\tan(\theta+\delta)-\tan\theta}{\delta}$. (SUGGESTION. Express in terms of sine and cosine.)
5. $\frac{\cot(\theta+\delta)-\cot\theta}{\delta}$.

73. De Moivre's theorem. If we adopt the customary notation $i=\sqrt{-1}$, so that $i^2=-1$, we have, on performing the multiplication,

$$\begin{aligned}(\cos\alpha+i\sin\alpha)(\cos\beta+i\sin\beta)&=\cos\alpha\cos\beta-\sin\alpha\sin\beta\\&\quad+i(\sin\alpha\cos\beta+\cos\alpha\sin\beta)\\&=\cos(\alpha+\beta)+i\sin(\alpha+\beta),\end{aligned}\tag{1}$$

a relation which holds for all values of α and β, whether positive or negative.

Putting $\beta=\alpha$, we get

$$(\cos\alpha+i\sin\alpha)^2=\cos 2\alpha+i\sin 2\alpha.$$

Again, putting $\beta=2\alpha$ in equation (1) and making use of the relation just established, we get

$$\begin{aligned}(\cos\alpha+i\sin\alpha)^3&=(\cos\alpha+i\sin\alpha)(\cos 2\alpha+i\sin 2\alpha)\\&=\cos 3\alpha+i\sin 3\alpha.\end{aligned}$$

Repetition of this process proves the relation

$$(\cos\alpha + i\sin\alpha)^n = \cos n\alpha + i\sin n\alpha \qquad (2)$$

for all positive, integral values of n.

It is evident, upon multiplying, that

$$(\cos\alpha + i\sin\alpha)(\cos\alpha - i\sin\alpha) = 1,$$

whence

$$(\cos\alpha + i\sin\alpha)^{-1} = \cos\alpha - i\sin\alpha.$$

Suppose n to be a negative integer. Let $n = -m$, where m is a positive integer. Now

$$(\cos\beta - i\sin\beta)^{-m} = (\cos\beta + i\sin\beta)^m$$
$$= \cos m\beta + i\sin m\beta.$$

Substituting $m = -n$ and $\beta = -\alpha$, we get

$$(\cos\alpha + i\sin\alpha)^n = \cos n\alpha + i\sin n\alpha,$$

true also for negative integral values of n.

Suppose n to be a fraction, either positive or negative. Let $n = \frac{r}{s}$, where r and s are integers. Now

$$(\cos\beta + i\sin\beta)^{\frac{r}{s}} = (\cos r\beta + i\sin r\beta)^{\frac{1}{s}}.$$

Raising both members to the sth power,

$$(\cos s\beta + i\sin s\beta)^{\frac{r}{s}} = \cos r\beta + i\sin r\beta.$$

Introducing $\frac{r}{s} = n$, and putting $s\beta = \alpha$, so that $r\beta = \frac{r}{s} \cdot s\beta = n\alpha$,

we get $$(\cos\alpha + i\sin\alpha)^n = \cos n\alpha + i\sin n\alpha.$$

This relation, therefore, holds for all rational values of n. By an argument involving the method of limits it can be proved also for all irrational values of n. This is De Moivre's theorem, an instrument of great importance in some branches of mathematics.

EXAMPLE. An illustration of its use is afforded by applying it to the derivation of the formulas for the sines and cosines of multiple angles. Thus

$$\cos 3\alpha + i\sin 3\alpha = (\cos\alpha + i\sin\alpha)^3$$
$$= \cos^3\alpha + 3i\cos^2\alpha\sin\alpha - 3\cos\alpha\sin^2\alpha - i\sin^3\alpha.$$

On equating the real terms on each side, and also the imaginary terms, separately, we have at once

$$\cos 3\alpha = \cos^3\alpha - 3\cos\alpha\sin^2\alpha$$
$$= 4\cos^3\alpha - 3\cos\alpha.$$
$$\sin 3\alpha = 3\cos^2\alpha\sin\alpha - \sin^3\alpha$$
$$= 3\sin\alpha - 4\sin^3\alpha.$$

The functions of 4α and of higher multiples of α are as readily found. The simplicity and beauty of the method appears on comparison with that of Art. 71.

Examples

1. Show that $\dfrac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta} = \cos(\alpha - \beta) + i\sin(\alpha - \beta)$.

2. Show that $\left(\cos\dfrac{2\pi + \alpha}{n} + i\sin\dfrac{2\pi + \alpha}{n}\right)^n = \cos\alpha + i\sin\alpha$.

3. Show that $\left(\cos\dfrac{2k\pi + \alpha}{n} + i\sin\dfrac{2k\pi + \alpha}{n}\right)^n = \cos\alpha + i\sin\alpha$, where k is any integer.

4. Show that the angle $\dfrac{2k\pi + \alpha}{n}$ has n different values as k takes the successive values, $0, 1, 2, \cdots n-1$ (n being a positive integer). Show also that for all integral values of k outside these limits, the terminal sides of the angles coincide with those of the n angles already found.

5. Since $\cos 0 + i\sin 0 = 1$, find the n different nth roots of 1, of which all but one are imaginary. Making use of the tables of natural sines and cosines compute for $n = 2, 3, 4, 6$.

6. Since $\cos\pi + i\sin\pi = -1$, find the n different nth roots of -1, of which all but one are imaginary when n is odd, and all imaginary when n is even. Compute for $n = 2, 3, 4, 6$.

74. Graphical representation of complex numbers. An interesting application of De Moivre's theorem is found in the graphical representation of complex numbers, devised by Wessel, a Danish mathematician, and published by Argand in 1608. The treatment of this topic belongs rather to the courses in algebra and function theory. (See Rietz and Crathorne's "Algebra.") Only so much of the rudiments of the method will be developed here as possess a trigonometric interest.

A pure imaginary is an indicated square root of a negative number. A complex number is an indicated sum of a real number

and a pure imaginary. All pure imaginaries can be expressed in the form yi, and all complex numbers in the form $x+yi$. Here $i=\sqrt{-1}$, so that $i^2=-1$; while x and y are real numbers, either rational or irrational.

Argand's method makes use of a pair of mutually perpendicular axes. The Argand diagram must not, however, be confused with the Cartesian scheme of coördinates.

All real numbers, rational or irrational, are represented by distances from the origin to points in the horizontal axis, called now the axes of reals, positive to the right, negative to the left. To every real number corresponds a point in this axis, and conversely, to every point in this axis corresponds a real number. Thus there is said to be a one-to-one correspondence between the totality of real numbers and the totality of points in the line.

All pure imaginaries are represented by distances from the origin to points in the vertical axis, now called the axis of imaginaries, points above and below the origin giving, respectively, positive and negative coefficients for the imaginary unit factor $i\equiv\sqrt{-1}$. Here again there exists a one-to-one correspondence between the totality of pure imaginaries and the totality of points in the vertical axis.

Notice that the origin alone, of all points in the plane, is on both axes. The number zero belongs to both systems. With this single exception, no pure imaginary can equal a real number, since the directions of the two axes are essentially different.

In order to represent the complex number $x+yi$ recourse must be had to the method of adding coplanar but non-collinear directed line segments employed in the graphical composition and resolution of forces in physics. Since directed line segments may undergo translation, the segment yi may be placed with its initial point upon the terminus of the segment x. The complex number is therefore represented by the right line segment (radius vector) v from the origin to the resulting terminus of the segment yi. For $y=0$ we have real numbers, for $x=0$ we have pure imaginaries.

As the lengths of the horizontal segment x and the vertical segment y measure respectively the magnitudes of the reals and the pure imaginaries, so the length of the radius vector v may be said to measure the absolute magnitude of the complex number $v=x+yi$. This is called the absolute or numerical value of v, and is denoted by the letter r. Evidently all points on the unit circle about the origin possess the absolute value 1.

The directed line segment, or radius vector, v makes in general an oblique angle with the axis of reals, and its direction is determined by the angle it forms with the positive axis of reals. This angle is denoted by θ, and is called the *amplitude* of the complex number. All points lying on the same radius have a common amplitude, while radii vectores extending from the origin in opposite directions have amplitudes differing by π. All positive real numbers have the amplitude 0; negative reals, π; pure imaginaries, $\frac{\pi}{2}$ or $\frac{3\pi}{2}$.

The right triangle formed by x, y, and v yields the relations

$$r = \sqrt{x^2+y^2}, \qquad \theta = \arctan\frac{v}{x},$$

$$x = r\cos\theta, \qquad y = r\sin\theta.$$

We may write interchangeably,

$$v, \text{ or } x+yi, \text{ or } r(\cos\theta + i\sin\theta).$$

The expression $\cos\theta + i\sin\theta$ consequently denotes a unit segment (complex unit) with the amplitude θ, while r is a purely arithmetical factor.

Conjugate complex numbers, $x+yi$ and $x-yi$, evidently have the same absolute value and amplitudes which are negatives of each other.

Addition is effected graphically by placing the initial point of the second segment upon the terminus of the first and connecting the initial point of the first to the terminus of the second. Thus in Fig. 70,

$$\begin{aligned} v &= v_1 + v_2 \\ &= x_1 + iy_1 + x_2 + iy_2 \\ &= (x_1+x_2) + i(y_1+y_2). \end{aligned}$$

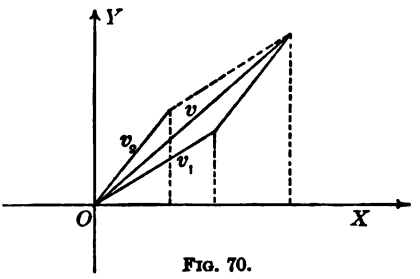

FIG. 70.

The values of r and θ in terms of r_1, r_2, θ_1 and θ_2 are readily determined, but exhibit little of present interest. Suffice it to point out that

$$r < r_1 + r_2,$$
$$\theta \neq \theta_1 + \theta_2.$$

Subtraction reduces at once to addition on reversing the subtrahend segment.

On attacking the problem of multiplication, we must define the product of a directed rectilinear segment by the imaginary unit i as a segment of equal length turned through a positive right angle. Thus $v = x + iy = r(\cos\theta + i\sin\theta)$ multiplied by i gives

$$v' = -y + ix = r\left[\cos\left(\frac{\pi}{2}+\theta\right) + i\sin\left(\frac{\pi}{2}+\theta\right)\right].$$

The absolute value is unchanged, while the amplitude is increased by $\frac{\pi}{2}$. This is consistent with the original scheme of representation, since reals multiplied by i give pure imaginaries, and these multiplied by i give -1 times the original, *i.e.* the original radius vector reversed.

Multiplying a directed segment by a positive real number simply stretches it, multiplying its length and leaving its direction unchanged. Multiplying

$$v = x + iy = r(\cos\theta + i\sin\theta) \text{ by } k, \text{ we get}$$

$$v' = kv = kx + iky = kr(\cos\theta + i\sin\theta).$$

The absolute value is multiplied by the factor k, while the amplitude is unchanged.

Multiplication of one complex number by another is effected by combining the two processes just described, applying the associative and distributive laws. Thus

$$\begin{aligned} v = v_1 \cdot v_2 &= (x_1 + iy_1)\cdot(x_2 + iy_2) \\ &= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2) \\ &= (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1). \end{aligned}$$

Using the other notation and applying De Moivre's theorem,

$$\begin{aligned} v = v_1 \cdot v_2 &= r_1(\cos\theta_1 + i\sin\theta_1)\cdot r_2(\cos\theta_2 + i\sin\theta_2) \\ &= r_1r_2\cdot[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]. \end{aligned}$$

Figure 71 illustrates the multiplication of $5-2i$ by $2+3i$. The product is shown to be $16+11i$. We have then the law that the absolute value of the product of two complex numbers equals the product of their absolute values, while the amplitude of the product equals the sum of their amplitudes.

The inverse process of division is readily performed, with the result

$$v = \frac{v_1}{v_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2},$$

or

$$v = \frac{v_1}{v_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)},$$

$$v = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)].$$

The absolute value of the quotient is equal to the quotient of the absolute values, while the amplitude of the quotient is equal to the difference of the amplitudes.

We have further,

$$v = v_1^n = r_1^n(\cos n\theta_1 + i\sin n\theta_1).$$

The absolute value of the power is equal to the power of the absolute value, while the amplitude of the power is equal to the amplitude of the number multiplied by the index of the power. Here "power" is used to denote the result of affecting the number by the exponent n, whatever the value of n. This includes both involution and evolution. In particular let n be the reciprocal of a positive integer m. Then

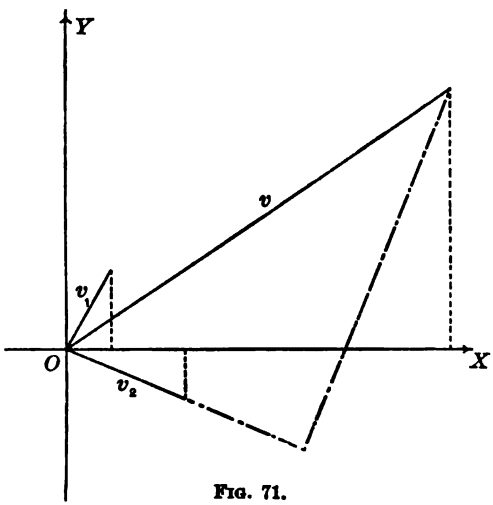

FIG. 71.

$$v = \sqrt[m]{v_1} = v_1^{\frac{1}{m}} = \sqrt[m]{r_1} \cdot \left(\cos\frac{\theta_1}{m} + i\sin\frac{\theta_1}{m}\right).$$

But v_1 is just as well and exactly represented by

$$r[\cos(2k\pi + \theta) + i\sin(2k\pi + \theta)],$$

where k is any integer. Thus the mth root just found is only one of an infinite number, all given by the form

$$v_1^{\frac{1}{m}} = \sqrt[m]{r_1}\left[\cos\frac{2k\pi + \theta_1}{m} + i\sin\frac{2k\pi + \theta_1}{m}\right],$$

in which k assumes all integral values. This form gives m different values for the root, corresponding to $k = 0, 1, 2, \cdots m-1$. All the others are repetitions of these m roots, since the terminal sides of all the other amplitude angles will coincide with the terminal sides of the m amplitudes specified.

Hence every complex number has m different mth roots, whose common absolute value is the arithmetical mth root of the absolute value of the number, while their amplitudes have the m different values,

$$\frac{\theta_1}{m}, \frac{2\pi+\theta_1}{m}, \frac{4\pi+\theta_1}{m}, \ldots, \frac{2(m-1)\pi+\theta_1}{m},$$

all less than 2π.

In the special case of any positive real number x_1, whose amplitude is therefore zero, we obtain m different mth roots with the common absolute value $\sqrt[m]{r_1}$, which is called the principal value of $\sqrt[m]{x_1}$, and the m different amplitudes,

$$0, \frac{2\pi}{m}, \frac{4\pi}{m}, \frac{6\pi}{m}, \ldots \frac{2(m-1)\pi}{m}.$$

Only one of these is real, the first, and it is called the principal mth root of the positive real number.

The student should construct figures to illustrate the foregoing theorems. Still another analytic notation for complex numbers will be brought out in Art. 75.

Examples

1. Represent by Argand's diagrams the numbers $2, -3, 3i, -4i, 3+5i, 4-3i, -2+i, -5-3i, 4+\sqrt{-3}, \sqrt{5}-\sqrt{-2}$.

2. Write the numbers the termini of whose radii vectores have the Cartesian coördinates $(3, 4)$, $(-3, 2)$, $(7, -3)$, $(-5, -2)$, $(6, 0)$, $(0, 5)$, $(-2, 0)$, $(0, -6)$, $(0, 0)$, $(\sqrt{3}, \sqrt{5})$.

3. Find the absolute values and the amplitudes (expressed in degrees and minutes) of the numbers in examples 1 and 2.

4. Describe the situation of the number points which have: (1) the common absolute value 3; (2) the common amplitude 30°; (3) the amplitudes 45° and 225°.

5. Perform graphically: $(3+4i)+(7-2i)$; $(-3+2i)+(6-3i)$; $(7-3i)-(4+2i)$; $(3-2i)-(-6-3i)$; $(5+2i)+(3-4i)-(6-3i)$.

6. Perform graphically, taking the first factor in each case as the multiplier: $3\cdot(5+2i)$; $i\cdot(3+5i)$; $2i\cdot(6-3i)$; $-4\cdot(2+5i)$; $-6i\cdot(3+2i)$; $(4+2i)\cdot(3+4i)$; $(3+4i)\cdot(4+2i)$.

7. Construct the quotient of $\frac{21+i}{3+2i}$; $\frac{6-17i}{4-3i}$.

8. Construct: $(3+2i)^2$; $\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^3$; $(1-i\sqrt{3})^3$; i^4.

9. Find by construction: $\sqrt{7-24i}$; $\sqrt[4]{-119+120i}$; $(-5+12i)^{\frac{1}{3}}$; $\sqrt{-4}$; $\sqrt[3]{8}$; $\sqrt[4]{16}$.

10. Write the general solution of the binomial equation: $x^n - a^n = 0$.

11. Find *all* the roots of the equations $x^2-1=0$; $x^2+1=0$; $x^3-1=0$; $x^3-8=0$.

75. Exponential values of the trigonometric functions. The first form of De Moivre's theorem, Art. 73, Eq. (1), may be written symbolically,

$$F(\alpha)\cdot F(\beta) = F(\alpha+\beta),$$

which is read, function of α times (the same) function of β equals (the same) function of $(\alpha+\beta)$; or, the product of the (same) functions of two numbers equals the (same) function of the sum of the two numbers. Now this is identically the characteristic relation or law governing the exponential function, that is, a function of the form a^x; thus,

$$a^\alpha \cdot a^\beta = a^{\alpha+\beta}.$$

For reasons discussed in Art. 77, it is found that instead of the more general function a^α, we must place

$$\cos\alpha + i\sin\alpha = e^{i\alpha}, \tag{1}$$

where $e = 2.71828183\cdots$ is the base of the Naperian, or natural, system of logarithms given in Art. 23.

Note that the law of exponents, derived for positive integral exponents, and assumed to hold also for negative, fractional, and irrational exponents, is still further assumed for exponents which are pure imaginaries and complex numbers. As in the former cases, the significance must be determined in conformity to the action of the assumed law. Indeed, the *law* defines the *function.*

Since $\cos\alpha - i\sin\alpha = \dfrac{1}{\cos\alpha + i\sin\alpha}$, we have also

$$\cos\alpha - i\sin\alpha = e^{-i\alpha}. \tag{2}$$

Adding and dividing by 2, we obtain

$$\cos\alpha = \frac{e^{i\alpha}+e^{-i\alpha}}{2}; \tag{3}$$

again, subtracting and dividing by $2i$,

$$\sin \alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}. \tag{4}$$

These values were first given by Euler in 1743. Starting from these two exponential values as fundamental definitions, and defining further

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}, \quad \cot \alpha = \frac{1}{\tan \alpha}, \quad \sec \alpha = \frac{1}{\cos \alpha}, \quad \csc \alpha = \frac{1}{\sin \alpha},$$

it is possible to develop all the laws and formulas of trigonometry as contained in Arts. 59 and 63–71, quite apart from any geometric meaning attached to the functions or their argument α. The analogous derivation of those trignometric theorems dependent on the periodicity of the trigonometric functions involves the periodicity of the logarithm, and is therefore postponed until the later mathematical study of the student.

A third notation for complex numbers now becomes manifest; for

$$v = x + iy = r(\cos \theta + i \sin \theta) = re^{i\theta}.$$

The consequent theorems regarding the absolute values and amplitudes of products, quotients, powers, and roots follow readily, and should be worked out by the student.

Examples

1. Find the exponential values of $\tan \alpha$, $\cot \alpha$, $\sec \alpha$, $\csc \alpha$.

2. Derive from the exponential values the laws $\sin^2 \alpha + \cos^2 \alpha = 1$, etc., of Art. 59.

3. Derive from the exponential values the formulas of Arts. 63–71.

4. Derive from the exponential notation the laws for the absolute values and amplitudes of products, quotients, powers, and roots of complex numbers.

76. Hyperbolic functions. Closely allied to Euler's forms of the last article are the two interesting and important forms,

$$\frac{e^{\alpha} + e^{-\alpha}}{2} \text{ and } \frac{e^{\alpha} - e^{-\alpha}}{2}.$$

They are called, by analogy, the hyperbolic cosine and hyperbolic sine. Thus, employing the customary notation,

$$\cosh \alpha = \frac{e^{\alpha} + e^{-\alpha}}{2}, \quad \sinh \alpha = \frac{e^{\alpha} - e^{-\alpha}}{2}.$$

The remaining hyperbolic functions are defined from these, just as in Art. 75 :

$$\tanh \alpha = \frac{\sinh \alpha}{\cosh \alpha}, \quad \coth \alpha = \frac{1}{\tanh \alpha}, \quad \operatorname{sech} \alpha = \frac{1}{\cosh \alpha}, \quad \operatorname{csch} \alpha = \frac{1}{\sinh \alpha}.$$

A very simple relation exists between the hyperbolic and the circular (*i.e.* ordinary trigonometric) functions. Evidently

$$\cosh \alpha = \cos i\alpha,$$
$$\sinh \alpha = -i \sin i\alpha,$$
$$\tanh \alpha = -i \tan i\alpha;$$

and conversely,

$$\cos \alpha = \cosh i\alpha,$$
$$\sin \alpha = -i \sinh i\alpha,$$
$$\tan \alpha = -i \tanh i\alpha.$$

To each formula of Chapter VIII corresponds a formula for the hyperbolic functions, which may be deduced either directly from the exponential definitions, or by substituting the values just given in the formulas for the circular functions. The student should derive these formulas by both methods.

The analogue to De Moivre's theorem is

$$(\cosh \alpha + \sinh \alpha)^n = \cosh n\alpha + \sinh n\alpha.$$

Cosh α and sinh α possess an imaginary period $2\pi i$, since $e^u = e^{u+2k\pi i}$, k being any integer. (See treatises on the theory of functions.)

77. Exponential and trigonometric series. In the present article values in the form of infinite series will be derived for certain exponential, logarithmic, and trigonometric functions. In the proof, however, the use of the binomial formula and the manipulation of the series introduce a lack of rigor requiring extended consideration in the subsequent courses in algebra, the calculus, and the theory of functions.

(1) *Exponential series.*

Expanding by the binomial formula,*

$$\left(1+\frac{x}{n}\right)^n = 1 + \frac{n}{1!}\cdot\frac{x}{n} + \frac{n(n-1)}{2!}\cdot\frac{x^2}{n^2} + \frac{n(n-1)(n-2)}{3!}\cdot\frac{x^3}{n^3} + \cdots$$

* The symbol $\lfloor\underline{k}$, or $k!$, is used to denote the product $1 \cdot 2 \cdot 3 \cdots k$, where k is any positive integer, and is read "factorial k".

$$= 1 + 1 \cdot \frac{x}{1!} + 1 \cdot \left(1 - \frac{1}{n}\right) \cdot \frac{x^2}{2!} + 1 \cdot \left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right) \cdot \frac{x^3}{3!} + \cdots.$$

Now as n becomes infinite, the binomial factors $\left(1 - \frac{1}{n}\right)$, $\left(1 - \frac{2}{n}\right)$, etc., all approach the common limit 1, and we shall have, in the limit,

$$e^x = \lim_{n \to \infty}\left[\left(1 + \frac{x}{n}\right)^n\right] = 1 + \frac{x}{1!} + \frac{x}{2!} + \frac{x}{3!} + \cdots. \qquad (1)$$

This series is convergent for all finite values of x. (See Rietz and Crathorne's "Algebra.")

For $x = 1$ we get

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots \qquad (2)$$

$$= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \cdots.$$

The terms diminish rapidly in value and, when expressed decimally, the value of e is found to be 2.71828183 ….

The series for e^x is valid also for negative and imaginary values of x; thus, substituting successively $-x$, ix, and $-ix$ for x, we have

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots,$$

$$e^{ix} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + i\left[\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right],$$

$$e^{-ix} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots - i\left[\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right].$$

(2) *Logarithmic series.*

From the expansion just obtained for e^x can be derived a series for $\log_e(1 + y)$.

Since* $u^x = e^{x \log_e u}$,

we have $$u^x = 1 + \frac{x}{1!}(\log_e u) + \frac{x^2}{2!}(\log_e u)^2 + \frac{x^3}{3!}(\log_e u)^3 + \cdots.$$

Placing $u = 1 + y$,

$$(1 + y)^x = 1 + \frac{x}{1!}\log_e(1 + y) + \frac{x^2}{2!}[\log_e(1 + y)]^2 + \frac{x^3}{3!}[\log_e(1 + y)]^3 + \cdots.$$

*Let $w = u^x$. Taking logarithms to base e, we have $\log_e w = x \log_e u$. Now taking exponentials to base e, $w = u^x = e^{x \log_e u}$.

Expanding the first member by the binomial formula,

$$(1+y)^x = 1 + \frac{x}{1!}y + \frac{x(x-1)}{2!}y^2 + \frac{x(x-1)(x-2)}{3!}y^3 + \cdots.$$

Picking out and equating the coefficients of x in the two expressions, the required expansion is obtained,

$$\log_e(1+y) = \frac{y}{1} - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \cdots. \tag{3}$$

This series is convergent for $-1 < y \leqq +1$.

If $w = \log_a u$, we have $a^w = u$; whence, taking logarithms to base e, $w \log_e a = \log_e u$. Therefore $w = \log_a u = \frac{1}{\log_e a} \cdot \log_e u$.

Substituting from (3) we see that

$$\log_a(1+y) = \frac{1}{\log_e a}\left(\frac{y}{1} - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \cdots\right). \tag{4}$$

(3) *Trigonometric series.*

From De Moivre's theorem

$$\cos m\theta + i \sin m\theta = (\cos\theta + i\sin\theta)^m.$$

Expanding by the binomial formula and separating the real terms from the imaginary,

$$\cos m\theta + i\sin m\theta = \cos^m\theta - \frac{m(m-1)}{2!}\cos^{m-2}\theta\sin^2\theta + \frac{m(m-1)(m-2)(m-3)}{4!}\cos^{m-4}\theta\sin^4\theta \cdots + i\left(\frac{m}{1!}\cos^{m-1}\theta\sin\theta - \frac{m(m-1)(m-2)}{3!}\cos^{m-3}\theta\sin^3\theta + \cdots\right).$$

Equating separately the real and the imaginary parts,

$$\cos m\theta = \cos^m\theta - \frac{m(m-1)}{2!}\cos^{m-2}\theta\sin^2\theta + \cdots,$$

$$\sin m\theta = \frac{m}{1!}\cos^{m-1}\theta\sin\theta - \frac{m(m-1)(m-2)}{3!}\cos^{m-3}\theta\sin^3\theta + \cdots.$$

Place now

$$m\theta = \alpha, \text{ so that } \theta = \frac{\alpha}{m};$$

$$\cos\alpha = \cos^m\left(\frac{\alpha}{m}\right) - \frac{m(m-1)}{2!}\cos^{m-2}\left(\frac{\alpha}{m}\right)\sin^2\left(\frac{\alpha}{m}\right) + \cdots,$$

$$\sin\alpha = \frac{m}{1!}\cos^{m-1}\left(\frac{\alpha}{m}\right)\sin\left(\frac{\alpha}{m}\right) - \frac{m(m-1)(m-2)}{3!}\cos^{m-3}\left(\frac{\alpha}{m}\right)\sin^3\left(\frac{\alpha}{m}\right) + \cdots.$$

Now let θ approach the limit zero and m become infinite, while still obeying the condition that $m\theta = \alpha$, where α remains finite. By Art. 12, $\cos\frac{\alpha}{m}$, $\cos^2\frac{\alpha}{m}$, etc., approach the limit 1 as m becomes infinite, and in the calculus the same is shown for $\cos^m\left(\frac{\alpha}{m}\right)$, $\cos^{m-1}\left(\frac{\alpha}{m}\right)$, etc.

Again, $$m\sin\left(\frac{\alpha}{m}\right) = \alpha \cdot \frac{\sin\theta}{\theta},$$

$$m(m-1)\sin^2\left(\frac{\alpha}{m}\right) = \alpha(\alpha-\theta)\cdot\left(\frac{\sin\theta}{\theta}\right)^2,$$

$$m(m-1)(m-2)\sin^3\left(\frac{\alpha}{m}\right) = \alpha(\alpha-\theta)(\alpha-2\theta)\cdot\left(\frac{\sin\theta}{\theta}\right)^3, \text{ etc.}$$

Since $\lim_{\theta=0}\frac{\sin\theta}{\theta} = 1$, the limits approached by these expressions, as $\theta \doteq 0$, are α, α^2, α^3, etc.

Making these substitutions, we obtain, in the limit,

$$\cos\alpha = 1 - \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} - \frac{\alpha^6}{6!} + \cdots, \tag{5}$$

$$\sin\alpha = \frac{\alpha}{1!} - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots. \tag{6}$$

These series are convergent for all values of α.

Tan α may also be expanded into the series

$$\tan\alpha = \frac{\alpha}{1} + \frac{\alpha^3}{3} + \frac{\alpha^5}{15} + \frac{17\,\alpha^7}{315} + \cdots. \tag{7}$$

It will be noticed that the series for $\cos\alpha$ contains only even powers of α, while those for $\sin\alpha$ and $\tan\alpha$ contain only the odd powers of α. (See the third example worked out in Art. 62.)

The assumption of Art. 75 may now be justified. For, on substituting for e^{ix}, e^{-ix}, $\sin x$, and $\cos x$ their expansions in series, we obtain

$$\cos x + i\sin x = e^{ix},$$

$$\cos x - i\sin x = e^{-ix},$$

and

$$\cos x = \frac{e^{ix} + e^{-ix}}{2},$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}.$$

(4) *Hyperbolic series.*

From the analogous relations the expansions for the hyperbolic functions are readily obtained.

Thus $$\cosh \alpha = \frac{e^{\alpha} + e^{-\alpha}}{2} = 1 + \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \cdots, \tag{8}$$

$$\sinh \alpha = \frac{e^{\alpha} - e^{-\alpha}}{2} = \frac{\alpha}{1!} + \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \frac{\alpha^7}{7!} + \cdots. \tag{9}$$

EXAMPLES

1. By substituting $-\alpha$ for α, find the series for $\sin(-\alpha)$, $\cos(-\alpha)$, $\tan(-\alpha)$, and by comparison, verify the corresponding relations of Art. 62.

2. By substituting $i\alpha$ for α, verify the relations of Art. 76, $\cosh \alpha = \cos i\alpha$, etc.

3. Using two terms of the expansions for $\sin \alpha$ and $\cos \alpha$, and retaining only powers of α below the fifth, obtain an approximate verification of the following formulas:

$\sin^2 \alpha + \cos^2 \alpha = 1$, $\sin(\alpha + \beta) = \cdots$, $\cos(\alpha + \beta) = \cdots$, $\sin 2\alpha = \cdots$, $\cos 2\alpha = \cdots$.

4. Do as required in example 3 for the hyperbolic functions.

5. Repeat examples 3 and 4, using three terms of the series and retaining powers of α below the seventh, thus arriving at a closer approximation.

78. Computation of trigonometric tables. The numerical values of the sine, cosine, and other trigonometric functions of angles from 0° to 90°, as tabulated in Table III, may be calculated by means of various trigonometric formulas, or better, by the use of the series derived in Art. 77.

Euler gave the following series, carrying the computation to 28 decimal places and a corresponding number of terms: Place $\alpha = m \cdot \frac{\pi}{2}$, in the series for $\sin \alpha$ and $\cos \alpha$; whence

$$\begin{aligned}
\sin\left(m \cdot \frac{\pi}{2}\right) &= 1.570\,796\, m \; - 0.645\,964\, m^3 \\
&\quad + 0.079\,693\, m^5 - 0.004\,682\, m^7 \\
&\quad + 0.000\,160\, m^9 - 0.000\,004\, m^{11} \\
&\quad + \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad ., \\
\cos\left(m \cdot \frac{\pi}{2}\right) &= 1.000\,000 \quad - 1.233\,700\, m^2 \\
&\quad + 0.253\,699\, m^4 - 0.020\,863\, m^6 \\
&\quad + 0.000\,919\, m^8 - 0.000\,025\, m^{10} \\
&\quad + \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad ..
\end{aligned}$$

We need to calculate the sines and cosines of angles up to 45° only, so that m is a fraction and always less than $\frac{1}{2}$. The terms of the series converge rapidly and a few terms suffice to give values correct to a small number of decimal places.

More extended discussion of this topic may be found in Hobson's "Trigonometry," Chap. IX; Todhunter's "Plane Trigonometry," Chap. X; and other advanced treatises on trigonometry.

The series for $\log(1+y)$ converges too slowly for convenient calculation, but a modified form is easily obtained. Manifestly

$$\log(1-y) = -y - \frac{y^2}{2} - \frac{y^3}{3} - \frac{y^4}{4} - \cdots$$

and

$$\log\frac{1+y}{1-y} = \log(1+y) - \log(1-y)$$
$$= 2\left[\frac{y}{1} + \frac{y^3}{3} + \frac{y^5}{5} + \cdots\right].$$

Place $y = \dfrac{1}{2v+1}$, whence $\dfrac{1+y}{1-y} = \dfrac{v+1}{v}$; then

$$\log\frac{v+1}{v} = 2\left(\frac{1}{2v+1} + \frac{1}{3(2v+1)^3} + \frac{1}{5(2v+1)^5} + \cdots\right),$$

or

$$\log(v+1) = \log v + 2\left[\frac{1}{2v+1} + \frac{1}{3(2v+1)^3} + \frac{1}{5(2v+1)^5} + \cdots\right].$$

This series converges rapidly and by it $\log 2$ can be computed from $\log 1 = 0$, $\log 3$ from $\log 2$, etc. Logarithms of composite numbers can be checked by adding the logarithms of their factors.

79. Proportional parts. In using the logarithmic and trigonometric tables it was assumed, as stated in Art. 26, that for small differences in the number, the differences in the logarithm are proportional to the differences in the number, and that likewise, for small differences in the angle, the differences in the sine (or other trigonometric function, or logarithmic function) are proportional to the differences in the angle.

We have

$$\log(x+\delta) - \log x = \log\frac{x+\delta}{x} = \log\left(1+\frac{\delta}{x}\right)$$
$$= \frac{\delta}{x} - \frac{\delta^2}{2x^2} + \frac{\delta^3}{3x^3} - \frac{\delta^4}{4x^4} + \cdots$$
$$= \frac{\delta}{x}. \qquad \text{(Approximately for small values of } \delta.\text{)}$$

Therefore, we have approximately

$$\frac{\log(x+\delta_1)-\log x}{\log(x+\delta_2)-\log x}=\frac{\delta_1/x}{\delta_2/x}=\frac{\delta_1}{\delta_2}, \tag{1}$$

for small differences.

Again,

$$\sin(\delta+\theta)-\sin\theta=2\cos\left(\theta+\frac{\delta}{2}\right)\sin\frac{\delta}{2}$$

$$=\cos\theta\cdot\delta. \quad \text{(Approximately for small values of } \delta.\text{)}$$

Hence, approximately

$$\frac{\sin(\theta+\delta_1)-\sin\theta}{\sin(\theta+\delta_2)-\sin\theta}=\frac{\delta_1\cos\theta}{\delta_2\cos\theta}=\frac{\delta_1}{\delta_2}. \tag{2}$$

For the other functions, the proof follows exactly similar lines, and can easily be supplied by the student.

Full discussion along this line may be found in Loney's "Plane Trigonometry," Chap. XXX; Lock's "Higher Trigonometry," Chap. VIII; Hobson's "Trigonometry," Chap. IX; etc.

80. General inverse functions. In Art. 14 only acute angles were under consideration, so that the relations

$$m=\sin\alpha, \qquad \alpha=\arcsin m,$$

expressed a one-to-one correspondence. In other words, under the condition that

$$0°\leqq\alpha\leqq 90°, \qquad 0\leqq m\leqq 1,$$

to each value of α there corresponds one and only one value of m and conversely.

On considering the general angle, it became evident that to any one angle there corresponds one and only one value of the sine (or other function), but that to one value of the sine (or other function) correspond many angles. We *define* arcsin m, arccos m, arctan m, etc., as the numerically smallest angle having the given sine, cosine, tangent, etc. It follows that arcsin m, arctan m, arccot m, arccsc m always lie between $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$, while arccos m and arcsec m always lie between 0 and $+\pi$. These are called the *principal* values of the general inverse func-

tions Arcsin m, Arccos m, etc. As results of Arts. 61, 62, we may write, if k is any integer,

$$\begin{aligned}
\operatorname{Arcsin} m &= 2k\pi + \arcsin m,\\
\operatorname{Arccos} m &= 2k\pi + \arccos m,\\
\operatorname{Arctan} m &= k\pi + \arctan m,\\
\operatorname{Arccot} m &= k\pi + \operatorname{arccot} m,\\
\operatorname{Arcsec} m &= 2k\pi + \operatorname{arcsec} m,\\
\operatorname{Arccsc} m &= 2k\pi + \operatorname{arccsc} m.
\end{aligned}$$

Similar relations exist for the inverse hyperbolic functions, the periods being imaginary, $2k\pi i$ and $k\pi i$.

From the relations of Art. 76 may be derived the following:

$$\begin{aligned}
\arccos m &= (\pm)\, i \operatorname{inv}\cosh m,\\
\arcsin m &= -\, i \operatorname{inv}\sinh im,\\
\arctan m &= -\, i \operatorname{inv}\tanh im;
\end{aligned}$$

and

$$\begin{aligned}
\operatorname{inv}\cosh m &= (\pm)\, i \arccos m,\\
\operatorname{inv}\sinh m &= -\, i \arcsin im,\\
\operatorname{inv}\tanh m &= -\, i \arctan im.
\end{aligned}$$

81. Logarithmic values of inverse functions. Since the circular and hyperbolic functions are expressible as exponential functions, it would seem that the inverse functions should be expressible as logarithmic functions. Such is, indeed, the case, and the desired values may be found by solving for α the forms given in Arts. 75 and 76.

(1) *Circular functions.*

If, for example,

$$z = \sin\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2i},$$

we have the quadratic equation in $e^{i\alpha}$,

$$e^{2i\alpha} - 2ize^{i\alpha} - 1 = 0,$$

whose roots are

$$e^{i\alpha} = iz \pm \sqrt{1 - z^2}.$$

Choosing the upper sign, and taking logarithms to base e, we get

$$i\alpha = \log(iz + \sqrt{1 - z^2}),$$

whence

$$\alpha = \arcsin z = -\, i \log(iz + \sqrt{1 - z^2})$$

gives the principal value of the arcsine.

(2) *Hyperbolic functions.*

As may be expected, the values of the inverse hyperbolic functions are real in form. Thus from

$$z = \sinh \alpha = \frac{e^{\alpha} - e^{-\alpha}}{2},$$

we get, on solving,

$$\alpha = \text{inv}\sinh z = \log (z + \sqrt{z^2+1}).$$

EXAMPLES

Obtain

1. $\arccos z = -i \log (z + \sqrt{z^2-1})$.

2. $\text{arc}\tan z = \frac{1}{2} i \log \frac{1+iz}{1-iz}$.

3. $\text{inv}\cosh z = \log z (z + \sqrt{z^2-1})$.

4. $\text{inv}\tanh z = \frac{1}{2} \log \frac{1+z}{1-z}$.

REVIEW EXERCISES

1. To what quadrant do the following angles belong: 560°, 653°, 1030°, $-425°$, $-1260°$?

2. To what quadrant do the following angles belong: $\frac{2\pi^R}{5}$, $\frac{5\pi^R}{3}$, 8^R, $\frac{13\pi_R}{3}$, 25^R, $-\frac{7\pi^R}{4}$, $-\frac{10\pi^R}{3}$?

3. Reduce to radians: 75°, $-300°$, $-250°$, 2000°, 465° 20′.

4. Reduce to the degree system: 4^R, -6^R, $\frac{4\pi^R}{3}$, $\frac{8\pi^R}{5}$, $-\frac{7\pi^R}{2}$.

5. Find the lengths of the arcs subtended by the following angles at the center of a circle of radius 6: 45°, 120°, 270°, $\frac{3\pi^R}{4}$, $\frac{5\pi^R}{8}$, $\frac{5\pi^R}{3}$.

6. A polygon of n sides is inscribed in a circle of radius r. Find the length of the arc subtended by one side. Compute the numerical values if $r = 10$ and $n = 3, 4, 5, 6, 8$.

7. Taking the radius of the earth to be 4000 miles, find the difference in latitude of two points on the same meridian 300 miles apart.

8. Find the difference in longitude of two points on the equator 1200 miles apart.

9. Find the distance in degrees between two points, one of which is 800 miles due north of the other.

10. A city is surrounded by a circular belt line 5 miles in radius. How long will a train require to go at a speed of 20 miles an hour from a station due east of the center to one due northwest, if the motion is clockwise; if counter-clockwise?

11. Find with the protractor the angles formed successively by the radii vectores of the points (3, 0), (2, 4), $(-3, 5)$, (0, 6), $(-4, 2)$, $(-2, 1)$, $(5, -3)$, (8, 0).

12. Find with the protractor the angles of the triangles formed by the abscissa, ordinate, and radius vector of each of the following points: (4, 4), (1, 3), $(3, -3)$, $(-2, 2)$, $(-4, -8)$.

13. Find by measurement the coördinates of the point whose radius vector is 4 and makes an angle of 30° with the positive x-axis; 5 and 120°; 8 and 225°.

14. Find by measurement the length and inclination angle of the radii vectores of the points whose coördinates are (2, 5), $(-5, 12)$, $(-8, -15)$.

15. If the earth were assumed to be a plane, and one degree of latitude or longitude were 60 miles, what would be the distance and direction from a point in 20° N. lat., 60° E. long., to one in 50° N. lat., 30° E. long.; from a point in 30° S. lat., 15° E. long., to one in 45° N. lat., 40° W. long.?

NOTE. This assumption is made by navigators as a basis for what is known as Plane Sailing. In Great Circle Sailing the earth is considered a sphere. Let the student devise a system of coördinates for the latter.

16. Find by measurement the six trigonometric functions of 36°, 155°, 285°, − 130°.

17. Find by measurement the following angles: arccot $\frac{3}{4}$ and of 1st quadrant; arcsin $\frac{1}{3}$ and of 2d quadrant; arccos (− 0.3) and of 3d quadrant; arcsec 0.6 and of 4th quadrant.

18. Find the lacking functions in the following table:

ANGLE	SINE	COSINE	TANGENT	COTANGENT	SECANT	COSECANT	QUADRANT
α	$-\frac{8}{17}$						III
β		$\frac{3}{5}$					IV
γ			$-\frac{1}{2}$				II
δ				$\frac{24}{7}$			III
θ					-2		III
ϕ						3	I

19. Find the value of $\dfrac{\cos\alpha}{1-\tan\alpha}+\dfrac{\sin\alpha}{1-\cot\alpha}$ if $\alpha=\arcsin\dfrac{3}{5}$ and of 2d quadrant.

20. Find the value of $\dfrac{\tan\beta+\sec\beta-1}{\tan\beta-\sec\beta+1}$ if $\beta=\operatorname{arccsc}\left(-\dfrac{13}{5}\right)$ and of 3d quadrant.

21. Find the value of $\dfrac{\tan^2\alpha-\sin^2\alpha}{\cos^2\alpha}$ if $\alpha=\operatorname{arccot}\left(-\dfrac{1}{2}\right)$ and of 4th quadrant.

22. Find the value of $\dfrac{1+\cos\gamma-2\sec\gamma}{3+\cos\gamma+2\sec\gamma}$ if $\gamma=\arctan\dfrac{9}{40}$ and of 1st quadrant.

23. Express $\dfrac{\sin^2\alpha+\cos^2\alpha}{\sec^2\alpha+\csc^2\alpha}$ in terms of $\tan\alpha$.

24. Express $\dfrac{4-\sin\beta-3\csc\beta}{\sin\beta-1-6\csc\beta}$ in terms of $\cos\beta$.

25. Express $\dfrac{(1-\cos\alpha)(1+\sec\alpha)}{(1-\sin\alpha)(1+\csc\alpha)}$ in terms of $\tan\alpha$.

26. In the following identity transform the first member into the second,

$$\frac{(1+\tan\gamma)(\cos\gamma-\cot\gamma)}{(1+\cot\gamma)(\sin\gamma-\tan\gamma)}=-\cot\gamma.$$

27. Show that $\dfrac{(1-\tan\alpha)(1+\cot\alpha)}{(1+\tan\alpha)(1-\cot\alpha)} = 1.$

28. Show that $\dfrac{\sin^2\beta+\cos^2\beta}{\sin^2\beta+\cos^2\beta} + \dfrac{\sin^2\beta-\cos^2\beta}{\sin^2\beta-\cos^2\beta} = -2\sin^2\beta\cos^2\beta.$

Solve the following equations and find the angle in degrees:

29. $4\sin^2\gamma - \tan^2\gamma = 0.$

30. $2\tan^2\alpha - \sec\alpha = 4.$

31. $4\csc\beta + \cot^2\beta = 5.$

32. $\tan^2\gamma + 3\cot^2\gamma = 4.$

33. $\sin^2\alpha + \sin^2\beta = \frac{1}{2}$, $\cos^2\alpha + \cos^2\beta = 0.$

34. $2\cos^2\alpha + \sin^2\beta = 2$, $\sin\alpha + \cos^2\beta = 0.$

35. For what range of values of α between 0 and 2π is $\sin\alpha + \cos\alpha$ positive; negative?

36. For what range of values of β between 0 and 2π is $\tan\beta - \cot\beta$ positive; negative?

37. Show that $\tan\gamma + \cot\gamma$ must always be numerically greater than unity.

38. Trace the variation of $\sin^2\theta$ as θ varies from 0 to 2π.

39. Trace the variation of $\cos^2\theta$ as θ varies from 0 to 2π.

40. Trace the variation of $1 - \sin\theta$ as θ varies from 0 to 2π.

41. Trace the variation of $1 - \cos\theta$ as θ varies from 0 to 2π.

42. Find by inspection $\log_8 .625$, $\log_{81} 27$, $\log_{25} .008$.

43. What numbers correspond to the following logarithms to base 9: -3, -2, -1.5, -1, 0, .5, 1, 2, 3?

44. In the formula $W = \dfrac{144\,n}{n-1}\,p_1v_1\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$, which gives the work of an air compressor, find W when $n = 1.3$, $p_1 = 14.7$, $p_2 = 72$, $v_1 = 6$.

45. Work the following with the slide rule:

(*a*) $\dfrac{.72 \times 137 \times 14}{372 \times 778} = ?$ (*b*) $\left(\dfrac{120}{42}\right)^{1.3} = ?$ (*c*) $\dfrac{42\sin 27°}{13.24} = ?$

46. Solve for x: $5^{3x} = 6$; $8^{x-1} = 7$.

QUERY. Does the result depend on the base of the system of logarithms used?

47. Solve for x: $3^{2x} - 4\cdot 3^x + 3 = 0.$

48. Find the amount of \$2000 in 5 years at 4% compound interest.

49. At what rate, compound interest, will \$45,000 amount in 8 years to \$60,000?

50. In how many years will a city become three times its original size if it increases $\frac{1}{4}$ each year?

51. Derive the formulas of Art. 42 from those of Art. 40. [SUGGESTION. Multiply respectively by a, b, and $-c$ and add.]

52. Derive the formulas of Art. 40 from those of Art. 42. [SUGGESTION. Solve for $a\cos\beta$ and $b\cos\alpha$ and add.]

53. From the law of sines, Art. 41, show that $\dfrac{b-c}{b+c}=\dfrac{\sin\beta-\sin\gamma}{\sin\beta+\sin\gamma}$.

54. By applying the formulas of Art. 70 to the result obtained in example 53, derive the law of tangents of Art. 43.

55. From the formulas of Arts. 42 and 68 derive the results

$$\sin\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}};\quad \cos\frac{\alpha}{2}=\sqrt{\frac{s(s-a)}{bc}};\quad \tan\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

56. Draw the graph of $\sin\theta+\cos\theta$ and thus trace its variation. What values of θ cause the given expression to assume maximum values; minimum?

57. Draw the graph of $\tan\theta+\cot\theta$ and thus trace its variation. What values of θ make the given expression a maximum; a minimum?

58. Draw the graph of $\arcsin u$ and trace its variation. [SUGGESTION. Lay off the values of u as abcissas, of $\arcsin u$ as ordinates.]

59. Draw the graph of $\arccos u$ and trace its variation.

60. Draw the graph of $\arctan u$ and trace its variation.

61. Draw the graph of $\operatorname{arccot} u$ and trace its variation. What discontinuities are exhibited by the functions of examples 58–61?

62. Find from the table the values of $\cos 625°\,12'$; of $\sin 238°\,25'$; of $\tan 324°\,6'$; of $\cot 921°\,32'$.

63. Find without reference to the table the value of

$$\cos 285°\cos 345°+\sin 195°\sin 465°.$$

64. Find without reference to the table the value of

$$\tan 205°\cot 335°+\tan 295°\cot 115°.$$

65. Find all the values between 0 and 2π of

$$\arcsin\left(-\frac{1}{2}\right);\quad \arccos\left(-\frac{1}{\sqrt{2}}\right);\quad \arctan\frac{1}{3}.$$

66. Find all the values of α between 0 and 2π if

$$\sin 3\alpha=\frac{3}{\sqrt{2}};\ \text{if}\ \tan 2\alpha=-\sqrt{3};\ \text{if}\ \cos\frac{\alpha}{2}=\frac{1}{\sqrt{2}};\ \text{if}\ \cot\frac{\alpha}{3}=-1.$$

67. Find the value of $\sin(2\alpha+\beta)$ if $\alpha=\arcsin\frac{4}{5}$ and of 2d quadrant, $\beta=\arctan\frac{1}{3}$ and of 1st quadrant.

68. Find the value of $\tan(3\alpha+2\beta)$ if $\alpha=\arcsin\frac{1}{3}$ and of 2d quadrant, $\beta=\arccos\frac{4}{5}$ and of 4th quadrant.

69. Derive the formula for $\sin(\alpha+\beta+\gamma)$ in terms of sine and cosine.

70. Derive the formula for $\cos(\alpha+\beta+\gamma)$ in terms of sine and cosine.

71. Derive the formula for $\tan(\alpha + \beta + \gamma)$ in terms of tangent.

Discuss the results of examples 69–71 in case $\alpha + \beta + \gamma = \pi$.

72. In the results of examples 69–71 put $\beta = \gamma = \alpha$, and thus obtain the formulas for $\sin 3\alpha$, $\cos 3\alpha$, and $\tan 3\alpha$.

73. Find the value of $\cos(3\alpha - 2\beta)$ if $\alpha = \arccos(-\frac{8}{17})$ and of 3d quadrant and $\beta = \arctan\frac{3}{4}$ and of 1st quadrant.

74. Find the value of $\cot(4\alpha - \beta)$ if $\alpha = \arcsin\frac{3}{5}$ and of 1st quadrant and $\beta = \arccos(-\frac{12}{13})$ and of 2d quadrant.

75. Find the value of $\sec(\alpha + \beta)$ if $\alpha = \arctan\frac{5}{12}$ and $\beta = \arcsin\frac{7}{25}$, both of 1st quadrant.

76. Show that $\sin 2 = \dfrac{2\tan\alpha}{1 + \tan^2\alpha}$.

77. Show that $\cos 2\alpha = \dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$.

78. Show that $\tan\left(45° - \dfrac{\alpha}{2}\right) = \csc\alpha + \cot\alpha$.

79. Show that $\cot\left(45° - \dfrac{\alpha}{2}\right) = \csc\alpha - \cot\alpha$.

Transform into products or quotients the following expressions (80–84):

80. $\cot\alpha + \tan\alpha$.

81. $\cot\alpha - \tan\alpha$.

82. $1 + \tan\alpha\tan\beta$.

83. $\cot\alpha - \tan\beta$.

84. $\dfrac{\cot\alpha + \cot\beta}{\tan\alpha + \tan\beta}$.

If $\alpha + \beta + \gamma = \pi$, show that (85–87)

85. $\sin\alpha + \sin\beta + \sin\gamma = 4\cos\dfrac{\alpha}{2}\cos\dfrac{\beta}{2}\cos\dfrac{\gamma}{2}$. [SUGGESTION. Apply Art. 70 to the first two terms and Art. 67 to the third term.]

86. $\cos\alpha + \cos\beta + \cos\gamma = 1 + 4\sin\dfrac{\alpha}{2}\sin\dfrac{\beta}{2}\sin\dfrac{\gamma}{2}$.

87. $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$. (See example 71.)

88. How does it appear from example 87 that either all three angles of a triangle are acute or else two are acute and one obtuse? (Consider the signs.)

89. How does it appear from example 87 that if one angle of a triangle is obtuse, it is numerically nearer 90° than either of the acute angles?

In the following equations find the angle:

90. $\tan 2\alpha\tan\alpha = 1$.

91. $\sin(60° - \beta) - \sin(60° + \beta) = \frac{1}{2}$.

92. $\cos 6\gamma - \cos 2\gamma = 0$.

93. $r \sin\theta = 8, r\cos\theta = 15$.

94. $r\sin\theta\cos\phi = 3, r\sin\theta\sin\phi = 4, r\cos\theta = 12$.

95. Show that $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5} = \frac{5}{16}$.

96. Show that $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15} = \frac{1}{16}$.

97. Show that $2\sin\frac{\alpha}{2} = \pm\sqrt{1+\sin\alpha} \pm \sqrt{1-\sin\alpha}$.

98. Show that $2\cos\frac{\alpha}{2} = \pm\sqrt{1+\sin\alpha} \mp \sqrt{1-\sin\alpha}$.

99. The formula for the horizontal range of a projectile fired at an elevation α with a muzzle speed u, is $\frac{u^2}{g}\sin 2\alpha$. Show that the maximum range is attained for an elevation of 45°.

100. A triangle is formed by two given sides of constant length b and c, including a variable angle α. For what value of α is the third side a maximum ; the area a maximum?

SECONDARY TRIGONOMETRIC FUNCTIONS

In addition to the trigonometric functions defined in Art. 6, 32, and 54, there are certain other expressions which are also functions of the angle. While of less importance than the six primary functions, an investigation of their properties will be valuable, not only for the results obtained, but as a review of the fundamental principles of trigonometry.

We may define, then,

$$\text{versed sine } \alpha = \text{vers } \alpha = 1 - \cos\alpha,$$
$$\text{coversed sine } \alpha = \text{covers } \alpha = 1 - \sin\alpha,$$
$$\text{exsecant } \alpha = \text{exsec } \alpha = \sec\alpha - 1,$$
$$\text{excosecant } \alpha = \text{excsc } \alpha = \csc\alpha - 1.$$

101. By reference to Fig. 53, show that

$$\text{vers } \alpha = 1 - \frac{x}{v}, \qquad \text{covers } \alpha = 1 - \frac{y}{v},$$
$$\text{exsec } \alpha = \frac{v}{x} - 1, \qquad \text{excsc } \alpha = \frac{y}{v} - 1.$$

102. By reference to Figs. 60–63, show that, in line representations,

$$\text{vers } \alpha = MA, \qquad \text{covers } \alpha = NB,$$
$$\text{exsec } \alpha = PT, \qquad \text{excsc } \alpha = PS.$$

103. Determine the signs and limitations in value of each of the four secondary functions in the different quadrants.

104. Show that if the quadrant of the angle and the value of any one of its ten functions are given, the values of the other nine can be found.

105. Find the values of the four secondary functions, given:

$\alpha = \arcsin(-\frac{10}{101})$ and of 3d quadrant; $\beta = \arccos\frac{19}{37}$ and of 4th quadrant;

$\gamma = \arctan(-\frac{11}{144})$ and of 2d quadrant; $\delta = \operatorname{arccot}\frac{44}{117}$ and of 1st quadrant.

106. Find all ten functions, given:

$\alpha = \operatorname{arcvers}\frac{29}{41}$ and of 4th quadrant; $\beta = \operatorname{arccovers}\frac{44}{125}$ and of 2d quadrant;

$\gamma = \operatorname{arcexsec}\frac{5}{51}$ and of 1st quadrant; $\delta = \operatorname{arcexcsc}\frac{52}{7}$ and of 3d quadrant.

107. Trace the variation of each of the secondary functions as the angle varies from 0 to 2π.

108. Draw the graph of each of the secondary functions. What discontinuities, if any, are present.

109. Find the secondary functions of $k \cdot \frac{\pi}{4}$, for $k = 1, 2, \cdots 8$.

110. Find the secondary functions of $k \cdot \frac{\pi}{6}$, for $k = 1, 2, \cdots 12$.

111. Verify the relations of Art. 61 for the secondary functions.

112. Determine the relations analogous to those of Art. 62 affecting the secondary functions. [SUGGESTION. Use Art. 60.]

113. By means of the cosine series, Art. 77, Eq. (5), show that $\lim_{\theta=0} \frac{\operatorname{vers}\theta}{\theta} = 0$.

114. From the preceding example, show that $\lim_{\theta=0} \frac{\operatorname{exsec}\theta}{\theta} = 0$.

115. Show that

$$\frac{\operatorname{vers}\alpha + \operatorname{covers}\alpha}{\operatorname{vers}\alpha - \operatorname{covers}\alpha} - \frac{\operatorname{exsec}\alpha + \operatorname{excsc}\alpha}{\operatorname{exsec}\alpha - \operatorname{excsc}\alpha} = \frac{2\operatorname{vers}\alpha\operatorname{covers}\alpha}{\operatorname{vers}\alpha - \operatorname{covers}\alpha}.$$

116. Show that $\operatorname{exsec}^2\beta + 2\operatorname{exsec}\beta = \tan^2\beta$.

117. Solve and find γ in degrees: $2\operatorname{vers}\gamma(2 - \operatorname{vers}\gamma) = 1$.

118. Solve and find δ in degrees: $\tan^2\delta + \operatorname{exsec}\delta = 4$.

119. Show that, if α is the angle at the center of a circle of radius r, the ordinate at the middle of the chord is given by the formula $m = r\operatorname{vers}\frac{\alpha}{2}$. Find m for $r = 1433$, $\alpha = 11° 32'$.

120. If τ is the intersection angle of two tangents to a circle of radius r, the shortest distance of their point of intersection from the arc is given by the formula $d = r\operatorname{exsec}\frac{\tau}{2}$. Find d for $r = 5730$, $\tau = 5° 32'$.

121. Reduce the first member to the second in the identity

$$(\operatorname{exsec}\alpha + \operatorname{vers}\alpha)(\operatorname{excsc}\alpha + \operatorname{covers}\alpha) = \sin\alpha\cos\alpha.$$

122. Show that $\operatorname{vers} 2\alpha = 2\sin^2\alpha$.

123. Show that $\operatorname{exsec} 2\alpha = \frac{2\sin^2\alpha}{1 - 2\sin^2\alpha}$.

124. Show that $\operatorname{excsc} 2\alpha \cos 2\alpha = \tan\alpha$.

125. At points in a straight line ordinates are erected such that for each point (x, y), $x = \text{vers}(\arcsin y)$. Show that the graph thus determined is a circle tangent to the Y-axis at the origin.

126. At each point in a circular arc the radius is extended an amount equal to the exsecant (in line values) of the arc measured from a fixed point in it. What is the graph thus determined?

127. Two equal circles have their centers in the same horizontal line. Show that the horizontal distance between two points in the neighboring arcs is equal to twice the versed sine (in line values) of the arc, measured from the point of tangency.

128. Two tangents to a circle intersect at an angle τ. Show that the distance of the point of intersection from the midpoint of the chord of contact equals $\text{exsec}\,\tau + \text{vers}\,\tau$ (in line values).

129. Show how the versed sine is of practical use in staking out a circular railroad track passing through three given points.

130. Show how the exsecant is of practical use in staking out a circular railroad spur of given radius branching tangentially from a straight track.

Compute the missing parts of the following triangles, distinguishing right from oblique:

	α	β	γ	a	b	c
131.	37° 42.8′		90°	6244.8		
132.	72° 25.6′		90°			64.863
133.			90°	375.84	296.57	
134.		54° 36.9′		24.465	42.850	
135.		136° 36.8′		36902		37490
136.	68° 51.5′		90°		7532.8	
137.			90°	396.45		531.53
138.			90°	.005428	.006395	
139.	148° 24′			7.4536	5.3648	
140.				.038456	.028638	.051524
141.	125° 34.6′	35° 25.3′				2584.6
142.			24° 36.8′	2.4657	3.6542	
143.		80° 04.5′	90°			30.007
144.			94° 46.8′		34.086	52.475
145.				.93274	.40586	.63208
146.		76° 46.3′	85° 38.7′			8.4637
147.			90°		29846	53857
148.	29° 57.4′	43° 52.6′		64.475		
149.	17° 46.8′				.39475	.29478
150.				36875	28467	48542

151. In a given triangle $a = 280$, $c = 420$, $\gamma = 38°$; find the radius of the circumscribed circle.

152. In a given circle $a = 63$, $b = 81$, $\gamma = 54°$; find the lengths of the bisectors of the interior angles.

153. The sides of a given triangle are 220, 350, 440; find the lengths of the three medians.

154. In a given triangle $b = 340$, $\alpha = 48°$, $\gamma = 63°$; find the lengths of the radii of the inscribed and of the three escribed circles.

155. A boat drifts in a stream whose current runs 4 miles an hour due east under a breeze of 10 miles an hour from the southwest. Determine the motion during 35 minutes, if the resistance reduces the effect of the wind 30%.

156. Three forces of 1800, 2200, and 2700 dynes are in equilibrium; find the angles they make with one another.

157. A helical spring is fastened to the door 16 inches from the axis of the hinges, and to the jamb 4 inches from the same line in the same horizontal plane. Find the length of the spring when the door is closed, open at 30°, 45°, 70°, 90°, 120°. Neglect the thickness of the door.

158. A cable 30 feet long is suspended from the tops of two vertical poles 20 feet apart and 15 and 18 feet high, and bears a load of 200 pounds hanging from it by a trolley. Find the position of the trolley when at rest, and the lengths, inclinations to the horizon, and (common) tensions of the segments of the cable. Neglect the weight of the cable.

159. Let the data be as in the preceding example, save that the load hangs from a ring knotted at the center of the cable; find the inclinations to the horizon and the (unequal) tensions of the segments. Solve when the ring is knotted at a point 12 feet from the lower end of the cable.

160. The eye is 40 inches in front of a mirror and an object appears to be 35 inches back of it, while the line of sight makes an angle of 48° with the mirror. Find the distance and direction of the object from the eye. (NOTE. The angles of incidence and reflection are equal.)

161. The line from the eye to the object recedes from the mirror at an angle of 32°, while the object is 36 inches from the eye and 12 inches from the mirror. Find the angles of incidence and reflection, and the point of reflection.

162. Two railway tracks intersect at an angle of 75°, and are connected by a circular "Y" of 800 feet radius lying in the obtuse angle and tangent to the two tracks. Find the distances of the points of tangency from the crossing and the length of the "Y".

163. Two railway tracks, intersecting at an angle of 62°, are joined by a circular "Y" in the acute angle and tangent to the two tracks at points 900 feet from the crossing. Find the radius of the "Y" and its length.

164. In setting a door frame 6 feet wide and 8 feet high, the vertical side is found to be 2 inches (horizontally) out of plumb. Find the angles of the parallelogram and the lengths of the diagonals. Is the diagonal of the true rectangle the average (arithmetic mean) of these two?

165. The staking out of a certain building requires the setting of stakes at the vertices of a rectangle 32 x 48 feet. A test of the trial setting shows the figure to be a parallelogram whose sides are as given above, but whose diagonals differ by 9 inches. Find the angle through which the longer sides must be swung to correct distortion, and the chord of the arc through which the back corners must be moved.

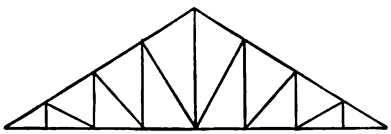

FIG. 72.

166. A triangular roof truss is 80 feet long and divided into 8 equal segments, by vertical members, as shown in Fig. 72. The height being 25 feet, what are the lengths and inclinations to the horizon of the various members?

167. The triangular roof truss shown in Fig. 73 is 60 feet long and 20 feet high. The bottom chord and rafters are divided into equal segments. Find the lengths and inclinations of the members.

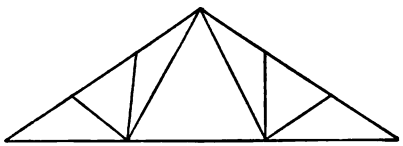

FIG. 73.

168. In order to determine the exact location of the point G (Fig. 74), a base line AB is laid off due north and south, measuring precisely 130 rods. Convenient intermediate stations are chosen, and angles measured as follows: $ABC = 36° 35'$, $BAC = 61° 10'$, $BAD = 43° 54'$, $CAD = 17° 16'$, $DCE = 66° 36'$, $CDE = 47° 41'$, $EDF = 55° 48'$, $DEF = 73° 12'$, $FEG = 58° 32'$, $EFG = 70° 28'$. Compute the distances composing the sides of the triangles in the figure.

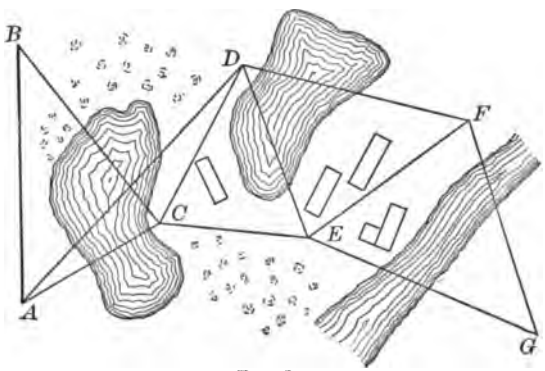

FIG. 74.

169. By projecting the distances AC, CE, EG (Fig. 74), perpendicular and parallel to AB, compute the easterly and southerly distances of G from A; find thence the direct distance and direction of G from A.

FORMULAS

GENERAL TRIGONOMETRY

$$\csc \alpha = \frac{1}{\sin \alpha}.$$

$$\sec \alpha = \frac{1}{\cos \alpha}.$$

$$\cot \alpha = \frac{1}{\tan \alpha}.$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}.$$

$$\cot \alpha = \frac{\cos \alpha}{\sin \alpha}.$$

$$\sin^2 \alpha + \cos^2 \alpha = 1.$$

$$\tan^2 \alpha + 1 = \sec^2 \alpha.$$

$$\cot^2 + 1 = \csc^2 \alpha.$$

$$2\,\pi^R = 360°.$$

$$F(2k\pi + \alpha) = F(\alpha),\ k \text{ an integer.}$$

$$F\left(k \cdot \frac{\pi}{2} \pm \alpha\right) = \pm F(\alpha),\ k \text{ an even integer.}$$

$$F\left(k \cdot \frac{\pi}{2} \pm \alpha\right) = \pm \text{ co-}F(\alpha),\ k \text{ an odd integer.}$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta.$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta.$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}.$$

$$\cot(\alpha \pm \beta) = \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha}.$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha.$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha.$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}.$$

$$\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}.$$

$$\sin\tfrac{1}{2}\alpha = \sqrt{\tfrac{1}{2}(1-\cos\alpha)}.$$

$$\cos\tfrac{1}{2}\alpha = \sqrt{\tfrac{1}{2}(1+\cos\alpha)}.$$

$$\tan\frac{1}{2}\alpha = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha}.$$

$$\cot\frac{1}{2}\alpha = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \frac{1+\cos\alpha}{\sin\alpha}.$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)].$$

$$\cos\alpha\sin\beta = \tfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)].$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-b)].$$

$$\sin\alpha\sin\beta = -\tfrac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)].$$

$$\sin\alpha\cos\alpha = \tfrac{1}{2}\sin 2\alpha.$$

$$\cos^2\alpha = \tfrac{1}{2}(1+\cos 2\alpha).$$

$$\sin^2\alpha = \tfrac{1}{2}(1-\cos 2\alpha).$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}.$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}.$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}.$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}.$$

RIGHT TRIANGLES

$$a^2 + b^2 = c^2.$$

$$\alpha + \beta = 90°.$$

$$\frac{a}{c} = \sin\alpha = \cos\beta.$$

$$\frac{b}{c} = \cos \alpha = \sin \beta.$$

$$\frac{a}{b} = \tan \alpha = \cot \beta.$$

$$A = \tfrac{1}{2} ab = \tfrac{1}{2} bc \sin \alpha = \tfrac{1}{2} c^2 \sin \alpha \cos \alpha = \tfrac{1}{4} c^2 \sin 2\alpha.$$

OBLIQUE TRIANGLES

$$\alpha + \beta + \gamma = 180°.$$

$$c = b \cos \alpha + a \cos \beta, \text{ etc.}$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}.$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma, \text{ etc.}$$

$$\tan \frac{\beta - \gamma}{2} = \frac{b - c}{b + c} \cot \frac{\alpha}{2}, \text{ etc.}$$

$$\tan \frac{\alpha}{2} = \frac{r}{s - a}, \text{ etc.}$$

$$s = \tfrac{1}{2}(a + b + c).$$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

$$A = \tfrac{1}{2} bc \sin \alpha = rs = \sqrt{s(s-a)(s-b)(s-c)}.$$

ANALYTIC TRIGONOMETRY

$$\lim_{\theta \doteq 0} \left[\frac{\theta}{\sin \theta}\right] = 1.$$

$$\lim_{\theta \doteq 0} \left[\frac{\theta}{\tan \theta}\right] = 1.$$

$$(\cos \alpha + i \sin \alpha)(\cos \beta + i \sin \beta) = \cos (\alpha + \beta) + i \sin (\alpha + \beta).$$

$$(\cos \alpha + i \sin \alpha)^n = \cos n\alpha + i \sin n\alpha.$$

$$\cos \alpha + i \sin \alpha = e^{i\alpha}.$$

$$\cos \alpha - i \sin \alpha = e^{-i\alpha}.$$

$$\cos \alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}.$$

$$\sin \alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}.$$

$$\cosh \alpha + \sinh \alpha = e^{\alpha}.$$

$$\cosh \alpha - \sinh \alpha = e^{-\alpha}.$$

$$\cosh \alpha = \frac{e^{\alpha} + e^{-\alpha}}{2}.$$

$$\sinh \alpha = \frac{e^{\alpha} - e^{-\alpha}}{2}.$$

$$e^x = \lim_{n=\infty}\left[\left(1+\frac{x}{n}\right)^n\right] = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots.$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = 2.71\cdots.$$

$$\log(1+y) = \frac{y}{1} - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \cdots.$$

$$\cos \alpha = 1 - \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} - \frac{\alpha^6}{6!} + \cdots.$$

$$\sin \alpha = \frac{\alpha}{1!} - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots.$$

$$\tan \alpha = \frac{\alpha}{1} + \frac{\alpha^3}{3} + \frac{\alpha^5}{15} + \frac{17\,\alpha^7}{315} + \cdots.$$

$$\cosh \alpha = 1 + \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \cdots.$$

$$\sinh \alpha = \frac{\alpha}{1!} + \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \frac{\alpha^7}{7!} + \cdots.$$

CONSTANTS

$$i = \sqrt{-1}.$$

$$\pi = 3.14159265\cdots.$$

$$\pi^R = 180°.$$

$$1^R = 57.2957795° \cdots = 57°\,17'\,44.8''\cdots.$$

$$e = 2.7182818285\cdots.$$

$$\text{Mod.}_{10} = \frac{1}{\log_e 10} = .4342944819\cdots.$$

1 inch = 2.54001 ··· centimeters.

1 foot = .3048 ··· meters.

1 mile = 1.60935 ··· kilometers.

1 centimeter = .3937 ··· inches.

1 meter = 3.28083 feet = 1.09361 yards.

1 kilometer = .62137 miles.

$g = 32.086528 + .171293 \sin^2 \phi$ feet per second per second.

$= 9.779886 + .05221 \sin^2 \phi$ meters per second per second at sea level for latitude ϕ.

ANSWERS TO EXERCISES

(Answers are omitted in case their knowledge would detract from the value of the exercise.)

Exercise II

6. $(0, 0)$, $(a, 0)$, (a, a), $(0, a)$; $\left(\frac{a}{2}\sqrt{2}, 0\right)$, $\left(0, \frac{a}{2}\sqrt{2}\right)$, $\left(-\frac{a}{2}\sqrt{2}, 0\right)$, $\left(0, -\frac{a}{2}\sqrt{2}\right)$.

7. $(5, 0)$, $(0, -5)$, $(-4.33, -2.5)$, $(-3.54, 3.54)$.

9. 5.6569; 7.6158.

11. Cross country distances, in miles: 5.099; 2.828; 2.236; 2.236; 6.325.

12. Distances saved, in yards: 773.2; 644.4; 1288.7; 128.9.

Exercise IV

9. $\frac{1681}{1519}$.

10. $\frac{1}{2}$.

11. $\frac{13}{10}$.

12. $\frac{13}{12}$.

13. $\cos \alpha$.

14. $2 \csc \beta$.

15. $\sqrt{\frac{1-\sin \gamma}{1+\sin \gamma}}$.

16. $2(1+\tan^2 \gamma)$.

Exercise V

9. $\frac{1}{2}$.

10. $\frac{1}{2}\sqrt{3}$.

11. $\frac{5}{6}$.

12. $\frac{1}{4}(3\sqrt{3}-2)$.

13. 60°.

14. 0° and 60°.

15. 60° and 90°.

16. 45°.

17. (*a*) 60°; (*b*) 19.05 ft.

18. 8.08; 16.17.

19. 452.39.

20. 60°.

Exercise VI

11. 0° and 60°. **12.** 30°. **13.** 0°, 30°, and 45°. **14.** 0°, 30°, and 45°.

Exercise VII

1. (*a*) 6.7, 6.7; (*b*) 8.23, 4.75.

2. (*a*) 8; (*b*) 13.86.

3. (*a*) 20, 34.64; (*b*) 28.28, 28.28.

4. 15.6; 9.

5. (*a*) 2598.16; (*b*) 1500.

6. 24 miles per hour, 30° east of north.

Article 18

1. $a = 40.32$, $b = 11.76$.

2. $a = 20.25$, $c = 33.75$.

3. $b = 151.5$, $c = 381.6$.

4. $a = 118.98$, $b = 133.24$.

Exercise VIII

1. $\beta = 64° 50'$, $a = 14.46$, $b = 30.77$.
2. $\beta = 37° 40'$, $a = 57.79$, $b = 44.61$.
3. $\alpha = 28° 45'$, $a = 116.88$, $b = 213.04$.
4. $\alpha = 11° 25'$, $a = 123.11$, $b = 510.68$.
5. $\beta = 68° 35'$, $b = 599.13$, $c = 643.66$.
6. $\beta = 17° 15'$, $b = 223.56$, $c = 753.93$.
7. $\alpha = 9° 30'$, $b = 7170.96$, $c = 7272.73$.
8. $\alpha = 72° 30'$, $a = 3257.40$, $c = 4988.36$.
9. $\alpha = 41° 49'$, $\beta = 48° 11'$, $b = 268.33$.
10. $\alpha = 32° 12'$, $\beta = 57° 48'$, $b = 605.03$.
11. $\alpha = 34° 13'$, $\beta = 55° 47'$, $a = 354.25$.
12. $\alpha = 53° 8'$, $\beta = 36° 52'$, $a = 1120$.
13. $\alpha = 36° 52'$, $\beta = 53° 8'$, $c = 1080$.
14. $\alpha = 44° 46'$, $\beta = 45° 14'$, $c = 845.07$.
15. $\alpha = 29° 11'$, $\beta = 60° 49'$, $c = 440.94$.
16. $\alpha = 59° 42'$, $\beta = 30° 18'$, $c = 4184.34$.
17. $\alpha = 29° 29'$, $\beta = 60° 31'$, $b = 168.00$, $c = 192.99$.
18. $\alpha = 12° 5'$, $\beta = 77° 55'$, $a = 57.50$, $c = 274.78$.
19. $\beta = 15° 40'$, $a = 93.47$, $b = 26.21$, $c = 97.08$.
20. $\alpha = 65° 10'$, $a = 24.86$, $b = 53.71$, $c = 59.19$.

21. 200.1 ft.
22. 1501.73 ft.
23. 2° 23′.
24. 4° 46′.
25. 1° 9′.
26. 33° 41′, 26° 34′, 45°.
27. .134 pitch, .2887 pitch, $\frac{1}{4}$ pitch.
28. 19° 28′ inclination.
29. 8° 8′ inclination.
30. 0° 9′, 0° 17′, 1° 26′.
31. 26° 34′.
32. 5759.71 ft.
33. 32° 28′.
34. 21.73 ft. = 21 ft. 8$\frac{3}{4}$ in.
35. 260.4 ft.
36. 0° 20′.

37. 5° 54′ (=.1029 radians). Note that .1029 = sin 5° 52.5′, an approximation. See Arts. 72 and 77 (3).

38. 2.468 miles.
39. 502.2 ft.

40. $A = (0, 0)$, $B = (240.9, 0)$, $C = (385.9, 274.8)$,
$D = (98.7, 814.6)$, $E = (162.8, 1043.0)$, $F = (649.1, 1248.8)$.

41. $\beta = 110°$, $b = 34.41$, $A = 828.81$; $\beta = 36°$, $a = 202.27$, $A = 24{,}049.53$; $\alpha = 14° 54'$, $\beta = 150° 12'$, $A = 31{,}500$; $\alpha = 63°$, $b = 326.88$, $A = 58{,}838.4$; $\alpha = 64° 17'$, $a = 553.12$, $A = 119{,}694.88$; $\alpha = 41° 1'$, $\beta = 97° 58'$, $A = 153{,}432.72$.

42. 51.76, 61.80, 68.40, 76.54, 1, 121.76, 141.42, 173.20.

43. $S = 2\,s\cos\frac{\alpha}{4}$.

44. $s = 2\,R\sin\frac{180°}{n} = 2\,r\tan\frac{180°}{n}$.

45. $C = 2\,\pi R$, $P = 2\,nR\sin\frac{180°}{n}$, $c = 2\,\pi R\cos\frac{180°}{n}$.

$C = 62.83$, $P_4 = 56.57$, $c_4 = 44.43$;
$P_8 = 61.23$, $c_8 = 58.05$;
$P_{16} = 62.43$, $c_{16} = 61.62$;
$P_{32} = 62.72$, $c_{32} = 62.53$.

46. $A_c = \pi R^2$, $A_p = nR^2 \sin\frac{180°}{n}\cos\frac{180°}{n} = \frac{1}{2}nR^2 \sin\frac{360°}{n}$,

$$A_i = \pi R^2 \cos^2\frac{180°}{n} = \frac{1}{2}\pi R^2\left(1 + \cos\frac{360°}{n}\right).$$

$A_c = 314.16$, $A_{p4} = 200.00$, $A_{i4} = 157.08$;
$A_{p8} = 282.84$, $A_{i8} = 268.15$;
$A_{p16} = 306.16$, $A_{i16} = 302.21$;
$A_{p32} = 312.16$, $A_{i32} = 311.14$.

47. $C = 62.83$, $P_6 = 60.00$, $c_6 = 54.41$;
$P_{12} = 62.11$, $c_{12} = 60.69$;
$P_{24} = 62.64$, $c_{24} = 62.29$;
$P_{48} = 62.78$, $c_{48} = 62.69$.

48. $A_c = 314.16$, $A_{p6} = 259.80$, $A_{i6} = 235.62$;
$A_{p12} = 300.00$, $A_{i12} = 293.11$;
$A_{p24} = 310.56$, $A_{i24} = 308.80$;
$A_{p48} = 313.20$, $A_{i48} = 312.81$.

49. $X = -30$, $Y = -17.321$, $R = 34.641$, 30° south of west.

50. $X = 6r$, $Y = 0$, $R = 6r$, due east.

51. Distance from center $= r\cos\theta$. **52.** $x = -15$.

53. Component along plane $= g\sin\alpha$, component perpendicular to plane $= g\cos\alpha$.

54. 16, 27.71; 8.27, 30.91; 5.56, 31.51; 2.79, 31.88.

55. 50 pounds pressure, 141.42 pounds along ladder.

56. $X = 57.28$, $Y = 30.73$. **57.** $R = 18.44$, $\theta = 49° 24'$. **58.** $c = 11.99$.

Exercise IX

1. 4, 1.5, −2.5.
2. 4, $\frac{4}{3}$, $\frac{4}{9}$.
3. 1, 4, 16, 32, 64, $\frac{1}{16}$, $\frac{1}{32}$.
4. 1, 8, 16, $\frac{1}{4}$, $\frac{1}{64}$.
5. (*a*) .4724; (*b*) .01614.
6. (*a*) 28.16; (*b*) .01913; (*c*) 2.465.
7. 17.978. **9.** 268.71. **11.** $x = 1.79$. **13.** $4136.09. **15.** 11.6 years.
8. .07637. **10.** $x = 2.29$. **12.** $4656.20. **14.** 5.2%. **16.** 3.8 years.

Exercise X

1. 4.86025, 2.79187, 9.84198, 5.80872 − 10, 21.47712.
2. 4.96088, 1.15821, 11.50651, 5.89510 − 10, 24.30108.
3. 516.35, 4.0966×10^{12}, .016335.
4. 16361, 5.64325×10^{11}, .00013671.
5. 9,967,800,000. **7.** 88.594 cm. **9.** $13{,}231 \times 10^{10}$.
6. 7.0048×10^{10} cm. **8.** 71.68 cm. **10.** 2,754,200.
11. 9.63459 − 10, 9.52928 − 10, 0.01824. **13.** 78° 1.1′, 81° 43.6′, 76° 17.1′.
12. 9.97455 − 10, 0.21672, 0.04197. **14.** 25° 20.7′, 27° 32.6′, 35° 3.6′.
15. 13.861. **16.** .91136. **17.** 399.32 **18.** .38875.
19. 1.3365 inches. **22.** .074765.
20. .1111 feet (= 1.3332 inches). **23.** 6.711; 8.381.
21. 1.7% less than the true value. **24.** −11.85; −61.38.

Exercise XII

1. $\beta = 27°$, $a = 2302.6$, $b = 1172.1$.
2. $\alpha = 60° 37.6'$, $\beta = 29° 22.4'$, $b = 4238.9$.
3. $\beta = 14° 44.8'$, $b = 254.07$, $c = 998.12$.
4. $\alpha = 15° 39.6'$, $\beta = 74° 20.4'$, $b = 168.36$, $c = 174.85$.
5. $\alpha = 50° 13.1'$, $\beta = 39° 46.9'$, $c = 9.5378$.
6. $\beta = 71° 34.5'$, $a = 10.417$, $b = 31.271$.
7. $\alpha = 83° 38.4'$, $\beta = 6° 21.6'$, $b = 14.81$, $c = 133.89$.
8. $\alpha = 75° 33'$, $\beta = 14° 27'$, $c = 54.953$.
9. $\alpha = 64° 48.5'$, $\beta = 25° 11.5'$, $b = 31{,}037$.
10. $\beta = 60° 9.8'$, $a = 5.854$, $c = 11.766$.
11. $\beta = 64° 42.6'$, $a = 19.467$, $b = 40.264$, $c = 44.531$.
12. $\alpha = 28° 23.6'$, $\beta = 61° 36.4'$, $c = .00042$.
13. $\alpha = 26° 47.3'$, $a = 3.2159$, $b = 6.3696$.
14. $\alpha = 38° 23.3'$, $\beta = 51° 36.7'$, $a = .056677$.
15. $\alpha = 54° 43.6'$, $b = .44535$, $c = .77120$.
16. $\alpha = 54° 43.2'$, $a = 242.79$, $b = 343.16$, $c = 420.87$.
17. $\alpha = 55° 59.3'$, $\beta = 34° 0.7'$, $c = .0074192$.
18. $\alpha = 9° 47.5'$, $a = .89928$, $c = 6.6577$.
19. $\alpha = 63° 20.7'$, $\beta = 26° 39.3'$, $a = .014523$.
20. $\alpha = 64° 41.8'$, $a = 1563.4$, $b = 739.12$.
21. 8.2583 feet.
22. 21 feet 3.1 inches.
23. 141.42 square feet.
24. $60° 1.8'$.
25. 1237.8 feet.
26. $16\frac{1}{4}$ miles, $36° 52.2'$ north of west.
29. $nR^2 \sin \frac{180°}{n} \cos \frac{180°}{n}$.
30. 2177.4.
31. 178.8 miles.
32. $0° 32'$.
33. .78523 feet.
34. 606.73 feet.
35. 425.64 feet.
36. 142.4 feet.
37. 118.1 feet.
38. 554.06; 145.17.

Article 47

1. $\alpha = 33° 19.9'$, $a = 434.04$, $c = 788.24$.
2. $\alpha = 65° 49.8'$, $a = 122.13$, $b = 835.60$.
3. $\beta = 15° 57.0'$, $b = 5.442$, $c = 17.865$.
4. $\beta = 1° 2.0'$, $a = 9.4526$, $b = .1826$.

Article 48

3. $\alpha = 57° 59.9'$, $\gamma = 23° 36.6'$, $c = 29.526$.
4. $\beta = 13° 55.6'$, $\gamma = 35° 30.7'$, $b = 135.96$.
5. $\begin{cases} \alpha = 104° 31.3', \beta = 40° 2.9', a = 5889.9; \\ \alpha = 4° 37.1', \beta' = 139° 57.1', a' = 489.8. \end{cases}$
6. $\begin{cases} \alpha = 94° 17.9', \gamma = 47° 13.3', a = 207{,}810; \\ \alpha' = 47° 4.6', \gamma' = 132° 46.7', a' = 152{,}600. \end{cases}$

Article 49

1. $\beta = 59° 14.8'$, $\gamma = 30° 32.6'$, $a = 192.14$.
2. $\alpha = 23° 31.8'$, $\gamma = 19° 7.2'$, $b = 820.26$.
3. $\alpha = 33° 17.4'$, $\gamma = 63° 13.2'$, $b = .11496$.
4. $\beta = 32° 16.1'$, $\gamma = 11° 0.3'$, $a = .00893$.

Article 50

1. $\alpha = 51° 57.2'$, $\beta = 66° 49.4'$, $\gamma = 61° 13.4'$; $A = 1.9017 \times 10^9$.
2. $\alpha = 82° 16.4'$, $\beta = 42° 51.8'$, $\gamma = 54° 51.8'$; $A = 1.7797 \times 10^9$.
3. $\alpha = 56° 28.8'$, $\beta = 34° 45.4'$, $\gamma = 88° 45.8'$.
4. $\alpha = 45° 53.4'$, $\beta = 72° 33.2'$, $\gamma = 61° 33.4'$.
5. Impossible. Why?
6. $\beta = 136° 46.8'$.
7. Impossible. Why?
8. $\gamma = 8° 58.3'$.

Exercise XIV

1. $\beta = 74° 0.3'$, $\gamma = 43° 24.7'$, $a = 76{,}567$.
2. $\alpha = 52° 56.6'$, $\beta = 79° 47.8'$, $\gamma = 47° 15.6'$.
3. $\gamma = 86° 10.3'$, $b = 9.0192$, $c = 9.6890$.
4. $\alpha = 43° 29.3'$, $\gamma = 80° 35.3'$, $b = .80470$.
5. Impossible. Why?
6. $\alpha = 52° 11.2'$, $\gamma = 27° 38.8'$, $a = 49{,}921$.
7. $\alpha = 34° 32.1'$, $\beta = 51° 41.8'$, $\gamma = 93° 46.1'$.
8. $\gamma = 69° 28.5'$, $a = 68.693$, $c = 73.377$.
9. $\alpha = 23° 34.1'$, $\beta = 35° 35.7'$, $c = 6.0804$.
10. $\alpha = 15° 35.2'$, $\gamma = 126° 7.6'$, $c = 66.113$.
11. $\beta = 13° 11.7'$, $\gamma = 16° 24.1'$, $a = .14571$.
12. $\begin{cases} \beta = 32° 8.5', \gamma = 89° 45', b = 34.993; \\ \beta' = 31° 38.5', \gamma' = 90° 15', b' = 36.210. \end{cases}$
13. $\alpha = 34° 10.5'$, $a = 282.33$, $c = 641.53$.
14. $\alpha = 29° 57.8'$, $\beta = 104° 27.0'$, $\gamma = 46° 35.2'$.
15. $\alpha = 21° 13.9'$, $\gamma = 32° 19.7'$, $b = .0048578$.
16. $\alpha = 68° 48.7'$, $\gamma = 15° 38.5'$, $a = 124.12$.
17. $\alpha = 33° 33.1'$, $\beta = 50° 42.0'$, $\gamma = 95° 44.9'$.
18. $\alpha = 75° 0.2'$, $a = 8355.2$, $b = 6470.6$.
19. $\alpha = 45° 29.5'$, $\beta = 14° 15.5'$, $b = 2146.7$.
20. $\alpha = 49° 36.8'$, $\beta = 40° 23.2'$, $c = 952.67$.
21. $\alpha = 151° 51.6'$, $\beta = 4° 30.4'$, $\gamma = 23° 38.0'$, $b = 416.44$.
22. $\alpha = 80° 0\,0'$, $\beta = 54° 45.2'$, $\gamma = 45° 14.8'$, $a = 124.81$.
23. $\beta = 90° 50'$, $\gamma = 16° 0'$, $a = 720.81$, $c = 207.58$.
24. $\alpha = 95° 26.6'$, $\gamma = 27° 8.4'$, $a = 125.81$, $b = 106.59$.
25. 2.1815×10^9; 5.0117; 1,742,040,000.
26. 6.5878×10^{18}; .037875; 7270.3.
27. Case III.

28. $\begin{cases} \alpha = 15^\circ\,45', & \beta = 29^\circ\,15', & c = 52.1; \\ \alpha = 21^\circ\,52.6', & \beta = 42^\circ\,7.4', & c = 723.6; \\ \alpha = 91^\circ\,54.7', & \beta = 58^\circ\,5.3', & c = 48.042; \\ \alpha = 47^\circ\,59.5', & \beta = 72^\circ\,0.5', & c = 291.48. \end{cases}$

29. $\begin{cases} A = 156^\circ\,55.6', & B = 145^\circ\,57.2', & C = 57^\circ\,7.2'; \\ A = 145^\circ\,13.6', & B = 121^\circ\,11.4', & C = 93^\circ\,35.0'. \end{cases}$

30. $\begin{cases} A = 63^\circ\,25.9', & B = 141^\circ\,34.1', & c = 328.4; \\ A = 122^\circ\,25.4', & C = 137^\circ\,34.6', & c = 575.41. \end{cases}$

31. $\begin{cases} A = 75^\circ, & b = 878.58, & c = 621.14; \\ A = 113^\circ\,58.6', & B = 106^\circ\,2.2', & C = 139^\circ\,59.2'. \end{cases}$

32. $a = b = 48.5$.

33. $A = 151^\circ\,2.7'$, $B = 133^\circ\,25.9'$, $C = 75^\circ\,31.4'$.

34. $d^2 = a^2 + b^2 + c^2 - 2\,ab\cos\widehat{ab} - 2\,bc\cos\widehat{bc} + 2\,ac\cos(\widehat{ab}+\widehat{bc})$; $d = 12.98$.

35. $\widehat{bc} = 82^\circ\ 13.0'$, $\widehat{cd} = 75^\circ\ 57.3'$, $\widehat{da} = 81^\circ\ 50.7'$, $ab - cd = 109.06$, $bc - da = 117.16$.

36. $c = 1001.1$, $d = 568.6$. **37.** $\alpha = 36^\circ$, $s = 15.217$.

38. $V = 6\pi \arccos\dfrac{6-h}{6} - 30\,(6-h)\sqrt{12\,h - h^2}$: $V_1 = 135.0$, $V_2 = 371.7$, $V_3 = 663.3$, $V_4 = 990.0$, $V_5 = 1338.0$, $V_6 = 1696.4$, $V_7 = 2054.8$, $V_8 = 2402.8$, $V_9 = 2729.5$, $V_{10} = 3021.1$, $V_{11} = 3257.8$, $V_{12} = 3392.8$.

39. $b = 483.4$. **40.** $a = 3221.5$. **41.** $b = 1286$.

42. Distance = 31.63, total height = 20.97.

43. Distance = 24.24, height = 5.08.

44. $AD = 738.2$, $DB = 150.6$.

45. $AC = 1075.1$, $BC = 679.5$.

46. $AD = 1460.0$, $DC = 673.8$, angle BXC at left = $17^\circ\ 27'$.

47. Distance = 135.86, height = 33.61.

48. $D = 57^\circ\ 40'$, $CD = 196.73$, $BD = 233.55$.

49. $AD = 603.94$, $AC = 693.12$, $BC = 838.82$, $BE = 595.76$, $AB = 867.48$, angle CXB at left = $4^\circ\ 27.7'$.

50. $AC = 828.24$, $AD = 836.42$, $BE = 805.4$, $BF = 713.8$, $AB = 185.34$.

51. $39^\circ\ 54'$.

Exercise XV

1. 45°, 60°, 150°, $112^\circ\ 30'$, $171^\circ\ 53'\ 14.4''$, $42^\circ\ 58'\ 18.6''$.

2. $\dfrac{\pi^R}{6}$, $\dfrac{\pi^R}{12}$, $\dfrac{\pi^R}{4}$, $\dfrac{2\,\pi^R}{3}$, $\dfrac{4\,\pi^R}{3}$, $\dfrac{5\,\pi^R}{3}$, $\dfrac{5\,\pi^R}{2}$.

3. 31.416 cm., 62.832 cm., 125.664 cm., 47.124 cm.

4. 1^R, $\dfrac{1^R}{2}$, 2^R, $\dfrac{\pi^R}{2}$.

5. Smaller sprocket: 4 revolutions per second, angular velocity = $8\,\pi$ radians per second, linear velocity of circumference = 201.06 inches per second; larger sprocket: $\frac{1}{2}$ revolutions per second, angular velocity = $\dfrac{64\,\pi}{8}$ radians per second, linear velocity of circumference = 201.06 inches per second. Speed of machine = 20.444 feet per second = 14.38 miles per hour.

6. Linear velocity of chain and of circumferences of both sprockets = 75.43 inches per second; angular velocity of larger sprocket = 15.09 radians per second, of smaller sprocket = 37.76 radians per second; smaller sprocket makes 5.1 revolutions per second.

13. $\frac{\pi}{4} < \alpha < \frac{5\pi}{4}$.

14. $0 < \alpha < \frac{\pi}{4}$, $\frac{\pi}{2} < \alpha < \frac{3\pi}{4}$, $\pi < \alpha < \frac{5\pi}{4}$, $\frac{3\pi}{2} < \alpha < \frac{7\pi}{4}$.

Exercise XVI

5. $\left(\frac{\pi}{6}, \frac{1}{2}\right)$, etc.

6. $\left(\frac{\pi}{4}, 1\right)$, etc.

7. $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{3\pi}{4}, -1\right)$, etc.

8. $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{5\pi}{4}, -\frac{\sqrt{2}}{2}\right)$, etc.

Exercise XVII

1. $\frac{1}{3}$.
2. $-\frac{1+2\sqrt{2}}{3}$.
3. $\frac{1}{3}$.
4. $-\frac{11}{61}$.
5. 60°, 300°.
6. 120°, 300°.
7. 120°, 150°, 300°, 330°.
8. 15°, 75°, 135°, 195°, 255°, 315°.
9. 60°, 300°.
10. 30°, 150°, 210°, 330°.
11. 0°, 60°, 180°, 300°, 360°.
12. 0°, 120°, 180°, 240°, 360°.

Exercise XVIII

1. $-\cos 30°$.
2. $-\sin 10°$.
3. $\cot 14°$.
4. $-\cot 35°$.
5. $-\csc 30°$.
6. $\sec 40°$.
7. $\frac{1+\sqrt{3}}{2}$, $-\frac{2\sqrt{3}}{3}$.
8. $-\frac{1+\sqrt{3}}{2}$, $-\frac{2\sqrt{3}}{3}$.
9. 0, 0.
10. $-\frac{1+\sqrt{3}}{2}$, $\frac{2\sqrt{3}}{3}$.
11. $\frac{\sqrt{3}-1}{2}$, $-\frac{2\sqrt{3}}{3}$.
12. 0, 0.
19. $-\frac{1+\sqrt{3}}{4}$.
20. 0.
21. 0.
22. 0.
23. $-.35$.
24. $-\frac{1}{3}$.
25. $+\sqrt{1-a^2}$.
26. $-\frac{\sqrt{1-m^2}}{m}$.
27. $\sin\alpha$.
28. $-\sin\alpha$.
29. $\tan\alpha$.
30. $-\tan\alpha$.

Exercise XIX

5. $\alpha+\beta = \sin^{-1}\frac{44}{125} = \cos^{-1}-\frac{44}{117}$, II quadrant.
6. $\alpha+\beta = \arcsin -\frac{244}{725} = \arccos -\frac{244}{725}$, III quadrant.
7. $-\sin(\alpha+\beta)$.
8. $\cos(\alpha+\beta)$.
9. $\sin 2\alpha$.
10. $\cos 2\alpha$.
11. $\sin 2\theta$.
12. $\cos\theta$.
13. $105° = \arcsin\frac{\sqrt{2}+\sqrt{6}}{4} = \arccos\frac{\sqrt{2}-\sqrt{6}}{4}$;

$$15° = 105° - 90° = \arcsin\frac{\sqrt{6}-\sqrt{2}}{4} = \arccos\frac{\sqrt{6}+\sqrt{2}}{4}.$$

14. $75° = \arcsin\frac{\sqrt{6}+\sqrt{2}}{4} = \arccos\frac{\sqrt{6}-\sqrt{2}}{4}$;

$$15° = 90° - 75° = \arcsin\frac{\sqrt{6}-\sqrt{2}}{4} = \arccos\frac{\sqrt{6}+\sqrt{2}}{4}.$$

Exercise XX

11. $\tan 15° = 2 - \sqrt{3}$, $\cot 15° = 2 + \sqrt{3}$.

15. $\sin(\alpha+\beta+\gamma) = \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\beta\cos\gamma + \cos\alpha\cos\beta\sin\gamma - \sin\alpha\sin\beta\sin\gamma$.

16. $\cos(\alpha+\beta+\gamma) = \cos\alpha\cos\beta\cos\gamma - \sin\alpha\sin\beta\cos\gamma - \sin\alpha\cos\beta\sin\gamma - \cos\alpha\sin\beta\sin\gamma$.

17. $\tan(\alpha+\beta+\gamma) = \dfrac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\beta\tan\gamma-\tan\gamma\tan\alpha-\tan\alpha\tan\beta}$.

18. $\cot(\alpha+\beta+\gamma) = \dfrac{\cot\beta\cot\gamma+\cot\gamma\cot\alpha+\cot\alpha\cot\gamma}{\cot\alpha\cot\beta\cot\gamma-\cot\alpha-\cot\beta-\cot\gamma}$.

Note the symmetry in the last four formulas.

Exercise XXI

5. $2\alpha = \sin^{-1}\frac{240}{1681} = \cos^{-1}\frac{1519}{1681}$.

6. $2\alpha = \arcsin\pm\frac{119}{169} = \arccos-\frac{119}{169}$. Explain the signs.

7. $\sin\frac{1}{2}\alpha = \pm\frac{1}{5}$ and $\pm\frac{4}{5}$, $\cos\frac{1}{2}\alpha = \pm\frac{4}{5}$ and $\pm\frac{3}{5}$.

8. $\sin\frac{1}{2}\alpha = \pm\dfrac{7\sqrt{2}}{26}$ and $\pm\dfrac{17\sqrt{2}}{26}$, $\cos\frac{1}{2}\alpha = \pm\dfrac{17\sqrt{2}}{26}$ and $\pm\dfrac{7\sqrt{2}}{26}$.

9. $\sin(\alpha+2\beta) = \frac{336}{625}$ and $-\frac{336}{625}$, $\cos(\alpha+2\beta) = -\frac{527}{625}$ and $-\frac{527}{625}$.

10. $\sin(\alpha-2\beta) = \pm\frac{336}{625}$ and $\mp\frac{336}{625}$, $\cos(\alpha-2\beta) = \mp\frac{527}{625}$ and $\mp\frac{527}{625}$.

17. $\alpha = 30°, 45°, 60°, 210°, 225°, 240°$. **18.** $\alpha = 0°, 180°$, and $\frac{1}{2}\arccos\frac{1}{3}$.

19. $\alpha = 67°30', 157°30', 247°30', 337°30'$, and $\frac{1}{2}\arctan\frac{1}{2}$.

20. $\alpha = 90°, 270°$, and $\frac{1}{2}\cos^{-1}\frac{1}{3}$. **23.** $2x$. **24.** 1. **25.** 0. **26.** 1.

Exercise XXII

1. $\frac{1}{2}[\sin 8\alpha + \sin 2\alpha]$.

2. $\frac{1}{2}[\sin 10\alpha - \sin 2\alpha]$.

3. $\frac{1}{2}[\cos 4\alpha - \cos 10\alpha]$.

4. $\frac{1}{2}[\cos 7\alpha + \cos 3\alpha]$.

5. $\frac{1}{2}[\cos\alpha - \cos 3\alpha]$.

6. $\frac{1}{2}[2\sin 2\alpha + \sin 4\alpha]$.

7. $\frac{1}{2}[3 + 4\cos 2\alpha + \cos 4\alpha]$.

8. $\frac{1}{2}[3 - 4\cos 2\alpha + \cos 4\alpha]$.

9. $\frac{1}{16}[3\sin 2\alpha - \sin 6\alpha]$.

10. $\frac{1}{16}[1 - \cos 4\alpha]$.

15. $k\cdot\frac{\pi}{4}$; $[k = 0, 1, 2, 3, 4]$.

16. $(2k+1)\frac{\pi}{4}$; $[k = 0, 1, 2, 3]$.

17. $(2k+1)\frac{\pi}{12}$; $[k = 0, 1, 2, \cdots 11]$ and $(3k+1)\frac{\pi}{6}$; $[k = 0, 1, 2, 3]$.

18. $(2k+1)\frac{\pi}{30}$; $[k = 0, 1, 2, \cdots 29]$ and $(2k+1)\frac{\pi}{10}$; $[k = 0, 1, 2, \cdots 9]$.

19. $k\cdot\frac{\pi}{4}$; $[k = 0, 1, 2, \cdots 7]$.

20. $k\cdot\frac{\pi}{4}$; $[k = 0, 1, 2, \cdots 7]$.

Exercise XXIII

9. $(2k+1)\frac{\pi}{6}$; $[k = 0, 1, 2, \cdots]$.
10. $\frac{k\pi}{3}$.
11. $\frac{k\pi}{4}$.
12. π and $(2k+1)\frac{\pi}{8}$.
13. $\frac{\cot^3\alpha - 3\cot\alpha}{3\cot^2\alpha - 1}$.
14. $\frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$.
15. $4\sin\alpha\cos^3\alpha - 4\sin^3\alpha\cos\alpha$.
16. $8\cos^4\alpha - 8\cos^2\alpha - 3$.
17. 0°, 15°, 105°, 180°, 255°, 345°. (See Exercise XIX, examples 13 and 14.)
18. $\alpha = 60°, 90°, 120°, 270°$, and $\arcsin -\frac{1}{2\sqrt{3}}$.
19. $\frac{k\pi}{2}$.
20. $\frac{k\pi}{2}$.

Article 72

1. $-\sin\theta$.
2. $\sec\theta\tan\theta$.
3. $-\csc\theta\cot\theta$.
4. $\sec^2\theta$.
5. $-\csc^2\theta$.

Article 73

5. $\cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$; ± 1; $1, \frac{1 \pm \sqrt{-3}}{2}$; $\pm 1, \pm i$; $\pm 1, \frac{\pm 1 \pm \sqrt{-3}}{2}$.

6. $\cos\frac{(2k+1)\pi}{n} + i\sin\frac{(2k+1)\pi}{n}$; $\pm i$; $-1, \frac{1 \pm \sqrt{-3}}{2}$; $\frac{\pm 1 \pm i}{2}$; $\pm i, \frac{\pm\sqrt{3} \pm i}{2}$.

Article 74

6. Products: $15 + 6i$; $-5 + 3i$; $6 + 12i$; $-8 - 20i$; $12 - 18i$; $4 + 22i$; $4 + 22i$.
7. Quotients: $5 - 3i$; $3 - 2i$.
8. Results: $4 + 12i$; 1; -8; 1.
9. Roots: $4 - 3i$; $3 + 2i$; $-46 + 9i$; $\pm 2i$.
10. $\sqrt[n]{a}\left[\cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}\right]$; $[k = 0, 1, 2, \cdots (n-1)]$.
11. ± 1; $\pm i$; $1, \frac{-1 \pm \sqrt{-3}}{2}$; $2, -1 \pm \sqrt{-3}$.

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TABLES

TABLE I

COMMON LOGARITHMS

OF NUMBERS

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
0	— Infinity.	30	1.47 712	60	1.77 815	90	1.95 424	120	2.07 918
1	0.00 000	31	1.49 136	61	1.78 533	91	1.95 904	121	2.08 279
2	0.30 103	32	1.50 515	62	1.79 239	92	1.96 379	122	2.08 636
3	0.47 712	33	1.51 851	63	1.79 934	93	1.96 848	123	2.08 991
4	0.60 206	34	1.53 148	64	1.80 618	94	1.97 313	124	2.09 342
5	0.69 897	35	1.54 407	65	1.81 291	95	1.97 772	125	2.09 691
6	0.77 815	36	1.55 630	66	1.81 954	96	1.98 227	126	2.10 037
7	0.84 510	37	1.56 820	67	1.82 607	97	1.98 677	127	2.10 380
8	0.90 309	38	1.57 978	68	1.83 251	98	1.99 123	128	2.10 721
9	0.95 424	39	1.59 106	69	1.83 885	99	1.99 564	129	2.11 059
10	1.00 000	40	1.60 206	70	1.84 510	100	2.00 000	130	2.11 394
11	1.04 139	41	1.61 278	71	1.85 126	101	2.00 432	131	2.11 727
12	1.07 918	42	1.62 325	72	1.85 733	102	2.00 860	132	2.12 057
13	1.11 394	43	1.63 347	73	1.86 332	103	2.01 284	133	2.12 385
14	1.14 613	44	1.64 345	74	1.86 923	104	2.01 703	134	2.12 710
15	1.17 609	45	1.65 321	75	1.87 506	105	2.02 119	135	2.13 033
16	1.20 412	46	1.66 276	76	1.88 081	106	2.02 531	136	2.13 354
17	1.23 045	47	1.67 210	77	1.88 649	107	2.02 938	137	2.13 672
18	1.25 527	48	1.68 124	78	1.89 209	108	2.03 342	138	2.13 988
19	1.27 875	49	1.69 020	79	1.89 763	109	2.03 743	139	2.14 301
20	1.30 103	50	1.69 897	80	1.90 309	110	2.04 139	140	2.14 613
21	1.32 222	51	1.70 757	81	1.90 849	111	2.04 532	141	2.14 922
22	1.34 242	52	1.71 600	82	1.91 381	112	2.04 922	142	2.15 229
23	1.36 173	53	1.72 428	83	1.91 908	113	2.05 308	143	2.15 534
24	1.38 021	54	1.73 239	84	1.92 428	114	2.05 690	144	2.15 836
25	1.39 794	55	1.74 036	85	1.92 942	115	2.06 070	145	2.16 137
26	1.41 497	56	1.74 819	86	1.93 450	116	2.06 446	146	2.16 435
27	1.43 136	57	1.75 587	87	1.93 952	117	2.06 819	147	2.16 732
28	1.44 716	58	1 76 343	88	1.94 448	118	2.07 188	148	2.17 026
29	1.46 240	59	1.77 085	89	1.94 939	119	2.07 555	149	2.17 319
30	1.47 712	60	1.77 815	90	1.95 424	120	2.07 918	150	2.17 609

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
01	432	475	518	561	604	647	689	732	77$\bar{3}$	817
02	860	903	945	988	*030	*072	*11$\bar{5}$	*157	*199	*242
03	01 284	326	368	410	452	494	536	578	620	662
04	703	745	787	828	870	912	953	99$\bar{5}$	*036	*078
05	02 119	160	202	243	284	325	366	407	449	490
06	531	572	612	653	694	73$\bar{5}$	776	816	857	898
07	938	979	*019	*060	*100	*141	*181	*222	*262	*302
08	03 342	383	423	463	503	543	583	623	663	703
09	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	25$\bar{8}$	297	336	376	41$\bar{5}$	454	493
11	532	571	610	65$\bar{0}$	689	727	766	805	844	883
12	922	961	999	*038	*077	*115	*154	*192	*231	*269
13	05 308	346	38$\bar{5}$	423	461	$\bar{5}$00	538	576	614	652
14	690	729	767	80$\bar{5}$	843	881	918	956	994	*032
15	06 070	108	145	183	221	258	296	333	371	408
16	446	483	521	55$\bar{8}$	595	633	670	707	744	781
17	819	856	893	930	967	*004	*041	*078	*11$\bar{5}$	*151
18	07 188	22$\bar{5}$	262	298	335	372	408	445	482	518
19	55$\bar{3}$	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*13$\bar{5}$	*171	*207	*243
21	08 279	314	350	386	422	458	493	529	56$\bar{5}$	600
22	636	672	707	743	778	814	849	884	920	955
23	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
24	09 342	377	412	447	482	517	552	587	621	656
25	691	726	760	795	830	864	899	934	968	*003
26	10 037	072	106	140	17$\bar{5}$	209	243	278	312	346
27	380	41$\bar{5}$	449	483	517	551	585	619	653	687
28	721	75$\bar{5}$	789	823	857	890	924	958	992	*025
29	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
31	727	760	793	826	860	893	926	959	992	*024
32	12 057	090	123	156	189	222	254	287	320	352
33	385	418	450	483	516	548	581	613	646	678
34	710	743	775	808	840	872	90$\bar{5}$	937	969	*001
35	13 033	066	098	130	162	194	226	258	290	322
36	354	386	418	45$\bar{0}$	481	513	545	577	609	640
37	672	704	735	767	799	830	862	893	92$\bar{5}$	956
38	988	*019	*051	*082	*114	*14$\bar{5}$	*176	*208	*239	*270
39	14 301	333	364	395	426	457	489	520	551	582
140	613	644	67$\bar{5}$	706	737	768	799	829	860	891
41	922	953	983	*014	*04$\bar{5}$	*076	*106	*137	*168	*198
42	15 229	259	290	320	351	381	412	442	473	503
43	534	564	594	62$\bar{5}$	65$\bar{5}$	685	715	746	776	806
44	836	866	897	927	957	987	*017	*047	*077	*107
45	16 137	167	197	227	256	286	316	346	376	406
46	435	465	49$\bar{5}$	524	554	584	613	643	673	702
47	732	761	791	820	85$\bar{0}$	879	909	938	967	997
48	17 026	056	08$\bar{5}$	114	143	173	202	231	260	289
49	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	72$\bar{5}$	754	782	811	840	869
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.			
	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8
	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1
	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4
	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.5	29.7
	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	723	754	782	811	840	869
51	898	926	955	984	*013	*041	*070	*099	*127	*156
52	18 184	213	241	270	298	327	355	384	412	441
53	469	498	526	554	583	611	639	667	696	724
54	752	780	808	837	865	893	921	949	977	*005
55	19 033	061	089	117	145	173	201	229	257	285
56	312	340	368	396	424	451	479	507	535	562
57	590	618	645	673	700	728	756	783	811	838
58	866	893	921	948	976	*003	*030	*058	*085	*112
59	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
61	683	710	737	763	790	817	844	871	898	925
62	952	978	*005	*032	*059	*085	*112	*139	*165	*192
63	21 219	245	272	299	325	352	378	405	431	458
64	484	511	537	564	590	617	643	669	696	722
65	748	775	801	827	854	880	906	932	958	985
66	22 011	037	063	089	115	141	167	194	220	246
67	272	298	324	350	376	401	427	453	479	505
68	531	557	583	608	634	660	686	712	737	763
69	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
71	300	325	350	376	401	426	452	477	502	528
72	553	578	603	629	654	679	704	729	754	779
73	805	830	855	880	905	930	955	980	*005	*030
74	24 055	080	105	130	155	180	204	229	254	279
75	304	329	353	378	403	428	452	477	502	527
76	551	576	601	625	650	674	699	724	748	773
77	797	822	846	871	895	920	944	969	993	*018
78	25 042	066	091	115	139	164	188	212	237	261
79	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
81	768	792	816	840	864	888	912	935	959	983
82	26 007	031	055	079	102	126	150	174	198	221
83	245	269	293	316	340	364	387	411	435	458
84	482	505	529	553	576	600	623	647	670	694
85	717	741	764	788	811	834	858	881	905	928
86	951	975	998	*021	*045	*068	*091	*114	*138	*161
87	27 184	207	231	254	277	300	323	346	370	393
88	416	439	462	485	508	531	554	577	600	623
89	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
91	28 103	126	149	171	194	217	240	262	285	307
92	330	353	375	398	421	443	466	488	511	533
93	556	578	601	623	646	668	691	713	735	758
94	780	803	825	847	870	892	914	937	959	981
95	29 003	026	048	070	092	115	137	159	181	203
96	226	248	270	292	314	336	358	380	403	425
97	447	469	491	513	535	557	579	601	623	645
98	667	688	710	732	754	776	798	820	842	863
99	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.		
	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
01	320	341	363	384	406	428	449	471	492	514
02	535	557	578	600	621	643	664	685	707	728
03	750	771	792	814	835	856	878	899	920	942
04	963	984	*006	*027	*048	*069	*091	*112	*133	*154
05	31 175	197	218	239	260	281	302	323	345	366
06	387	408	429	450	471	492	513	534	555	576
07	597	618	639	660	681	702	723	744	765	785
08	806	827	848	869	890	911	931	952	973	994
09	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
11	428	449	469	490	510	531	552	572	593	613
12	634	654	675	695	715	736	756	777	797	818
13	838	858	879	899	919	940	960	980	*001	*021
14	33 041	062	082	102	122	143	163	183	203	224
15	244	264	284	304	325	345	365	385	405	425
16	445	465	486	506	526	546	566	586	606	626
17	646	666	686	706	726	746	766	786	806	826
18	846	866	885	905	925	945	965	985	*005	*025
19	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
21	439	459	479	498	518	537	557	577	596	616
22	635	655	674	694	713	733	753	772	792	811
23	830	850	869	889	908	928	947	967	986	*005
24	35 025	044	064	083	102	122	141	160	180	199
25	218	238	257	276	295	315	334	353	372	392
26	411	430	449	468	488	507	526	545	564	583
27	603	622	641	660	679	698	717	736	755	774
28	793	813	832	851	870	889	908	927	946	965
29	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
31	361	380	399	418	436	455	474	493	511	530
32	549	568	586	605	624	642	661	680	698	717
33	736	754	773	791	810	829	847	866	884	903
34	922	940	959	977	996	*014	*033	*051	*070	*088
35	37 107	125	144	162	181	199	218	236	254	273
36	291	310	328	346	365	383	401	420	438	457
37	475	493	511	530	548	566	585	603	621	639
38	658	676	694	712	731	749	767	785	803	822
39	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
41	202	220	238	256	274	292	310	328	346	364
42	382	399	417	435	453	471	489	507	525	543
43	561	578	596	614	632	650	668	686	703	721
44	739	757	775	792	810	828	846	863	881	899
45	917	934	952	970	987	*005	*023	*041	*058	*076
46	39 094	111	129	146	164	182	199	217	235	252
47	270	287	305	322	340	358	375	393	410	428
48	445	463	480	498	515	533	550	568	585	602
49	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.		
	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
250	39 794	811	829	846	863	881	898	915	933	950	
51	967	983	*002	*019	*037	*054	*071	*088	*106	*123	18
52	40 140	157	173	192	209	226	243	261	278	295	1 1.8
53	312	329	346	364	381	398	415	432	449	466	2 3.6
54	483	500	518	535	552	569	586	603	620	637	3 5.4
55	654	671	688	705	722	739	756	773	790	807	4 7.2
56	824	841	858	875	892	909	926	943	960	976	5 9.0
57	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	6 10.8
58	41 162	179	196	212	229	246	263	280	296	313	7 12.6
59	330	347	363	380	397	414	430	447	464	481	8 14.4
260	497	514	531	547	564	581	597	614	631	647	9 16.2
61	664	681	697	714	731	747	764	780	797	814	17
62	830	847	863	880	896	913	929	946	963	979	1 1.7
63	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	2 3.4
64	42 160	177	193	210	226	243	259	275	292	308	3 5.1
65	325	341	357	374	390	406	423	439	455	472	4 6.8
66	488	504	521	537	553	570	586	602	619	635	5 8.5
67	651	667	684	700	716	732	749	765	781	797	6 10.2
68	813	830	846	862	878	894	911	927	943	959	7 11.9
69	975	991	*008	*024	*040	*056	*072	*088	*104	*120	8 13.6
270	43 136	152	169	185	201	217	233	249	265	281	9 15.3
71	297	313	329	345	361	377	393	409	425	441	16
72	457	473	489	505	521	537	553	569	584	600	1 1.6
73	616	632	648	664	680	696	712	727	743	759	2 3.2
74	775	791	807	823	838	854	870	886	902	917	3 4.8
75	933	949	965	981	996	*012	*028	*044	*059	*075	4 6.4
76	44 091	107	122	138	154	170	185	201	217	232	5 8.0
77	248	264	279	295	311	326	342	358	373	389	6 9.6
78	404	420	436	451	467	483	498	514	529	545	7 11.2
79	560	576	592	607	623	638	654	669	685	700	8 12.8
280	716	731	747	762	778	793	809	824	840	855	9 14.4
81	871	886	902	917	932	948	963	979	994	*010	15
82	45 025	040	056	071	086	102	117	133	148	163	1 1.5
83	179	194	209	225	240	255	271	286	301	317	2 3.0
84	332	347	362	378	393	408	423	439	454	469	3 4.5
85	484	500	515	530	545	561	576	591	606	621	4 6.0
86	637	652	667	682	697	712	728	743	758	773	5 7.5
87	788	803	818	834	849	864	879	894	909	924	6 9.0
88	939	954	969	984	*000	*015	*030	*045	*060	*075	7 10.5
89	46 090	105	120	135	150	165	180	195	210	225	8 12.0
290	240	255	270	285	300	315	330	345	359	374	9 13.5
91	389	404	419	434	449	464	479	494	509	523	14
92	538	553	568	583	598	613	627	642	657	672	1 1.4
93	687	702	716	731	746	761	776	790	805	820	2 2.8
94	835	850	864	879	894	909	923	938	953	967	3 4.2
95	982	997	*012	*026	*041	*056	*070	*085	*100	*114	4 5.6
96	47 129	144	159	173	188	202	217	232	246	261	5 7.0
97	276	290	305	319	334	349	363	378	392	407	6 8.4
98	422	436	451	465	480	494	509	524	538	553	7 9.8
99	567	582	596	611	625	640	654	669	683	698	8 11.2
300	712	727	741	756	770	784	799	813	828	842	9 12.6
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
800	47 712	727	741	756	770	784	799	813	828	842	
01	857	871	885	900	914	929	943	958	972	986	
02	48 001	015	029	044	058	073	087	101	116	130	
03	144	159	173	187	202	216	230	244	259	273	15
04	287	302	316	330	344	359	373	387	401	416	1 1.5
05	430	444	458	473	487	501	515	530	544	558	2 3.0
06	572	586	601	615	629	643	657	671	686	700	3 4.5
07	714	728	742	756	770	785	799	813	827	841	4 6.0
08	855	869	883	897	911	926	940	954	968	982	5 7.5
09	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	6 9.0
810	49 136	150	164	178	192	206	220	234	248	262	7 10.5
11	276	290	304	318	332	346	360	374	388	402	8 12.0
12	415	429	443	457	471	485	499	513	527	541	9 13.5
13	554	568	582	596	610	624	638	651	665	679	
14	693	707	721	734	748	762	776	790	803	817	
15	831	845	859	872	886	900	914	927	941	955	14
16	969	982	996	*010	*024	*037	*051	*065	*079	*092	1 1.4
17	50 106	120	133	147	161	174	188	202	215	229	2 2.8
18	243	256	270	284	297	311	325	338	352	365	3 4.2
19	379	393	406	420	433	447	461	474	488	501	4 5.6
820	515	529	542	556	569	583	596	610	623	637	5 7.0
21	651	664	678	691	705	718	732	745	759	772	6 8.4
22	786	799	813	826	840	853	866	880	893	907	7 9.8
23	920	934	947	961	974	987	*001	*014	*028	*041	8 11.2
24	51 055	068	081	095	108	121	135	148	162	175	9 12.6
25	188	202	215	228	242	255	268	282	295	308	
26	322	335	348	362	375	388	402	415	428	441	
27	455	468	481	495	508	521	534	548	561	574	13
28	587	601	614	627	640	654	667	680	693	706	1 1.3
29	720	733	746	759	772	786	799	812	825	838	2 2.6
830	851	865	878	891	904	917	930	943	957	970	3 3.9
31	983	996	*009	*022	*035	*048	*061	*075	*088	*101	4 5.2
32	52 114	127	140	153	166	179	192	205	218	231	5 6.5
33	244	257	270	284	297	310	323	336	349	362	6 7.8
34	375	388	401	414	427	440	453	466	479	492	7 9.1
35	504	517	530	543	556	569	582	595	608	621	8 10.4
36	634	647	660	673	686	699	711	724	737	750	9 11.7
37	763	776	789	802	815	827	840	853	866	879	
38	892	905	917	930	943	956	969	982	994	*007	
39	53 020	033	046	058	071	084	097	110	122	135	12
840	148	161	173	186	199	212	224	237	250	263	1 1.2
41	275	288	301	314	326	339	352	364	377	390	2 2.4
42	403	415	428	441	453	466	479	491	504	517	3 3.6
43	529	542	555	567	580	593	605	618	631	643	4 4.8
44	656	668	681	694	706	719	732	744	757	769	5 6.0
45	782	794	807	820	832	845	857	870	882	895	6 7.2
46	908	920	933	945	958	970	983	995	*008	*020	7 8.4
47	54 033	045	058	070	083	095	108	120	133	145	8 9.6
48	158	170	183	195	208	220	233	245	258	270	9 10.8
49	283	295	307	320	332	345	357	370	382	394	
850	407	419	432	444	456	469	481	494	506	518	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
850	54 407	419	432	444	456	469	481	494	506	518	
51	531	543	555	568	580	593	605	617	630	642	
52	654	667	679	691	704	716	728	741	753	765	
53	777	790	802	814	827	839	851	864	876	888	18
54	900	913	925	937	949	962	974	986	998	*011	1 1.3
55	55 023	035	047	060	072	084	096	108	121	133	2 2.6
56	145	157	169	182	194	206	218	230	242	255	3 3.9
57	267	279	291	303	315	328	340	352	364	376	4 5.2
58	388	400	413	425	437	449	461	473	485	497	5 6.5
59	509	522	534	546	558	570	582	594	606	618	6 7.8
860	630	642	654	666	678	691	703	715	727	739	7 9.1
61	751	763	775	787	799	811	823	835	847	859	8 10.4
62	871	883	895	907	919	931	943	955	967	979	9 11.7
63	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
64	56 110	122	134	146	158	170	182	194	205	217	
65	229	241	253	265	277	289	301	312	324	336	12
66	348	360	372	384	396	407	419	431	443	455	1 1.2
67	467	478	490	502	514	526	538	549	561	573	2 2.4
68	585	597	608	620	632	644	656	667	679	691	3 3.6
69	703	714	726	738	750	761	773	785	797	808	4 4.8
870	820	832	844	855	867	879	891	902	914	926	5 6.0
71	937	949	961	972	984	996	*008	*019	*031	*043	6 7.2
72	57 054	066	078	089	101	113	124	136	148	159	7 8.4
73	171	183	194	206	217	229	241	252	264	276	8 9.6
74	287	299	310	322	334	345	357	368	380	392	9 10.8
75	403	415	426	438	449	461	473	484	496	507	
76	519	530	542	553	565	576	588	600	611	623	
77	634	646	657	669	680	692	703	715	726	738	11
78	749	761	772	784	795	807	818	830	841	852	1 1.1
79	864	875	887	898	910	921	933	944	955	967	2 2.2
880	978	990	*001	*013	*024	*035	*047	*058	*070	*081	3 3.3
81	58 092	104	115	127	138	149	161	172	184	195	4 4.4
82	206	218	229	240	252	263	274	286	297	309	5 5.5
83	320	331	343	354	365	377	388	399	410	422	6 6.6
84	433	444	456	467	478	490	501	512	524	535	7 7.7
85	546	557	569	580	591	602	614	625	636	647	8 8.8
86	659	670	681	692	704	715	726	737	749	760	9 9.9
87	771	782	794	805	816	827	838	850	861	872	
88	883	894	906	917	928	939	950	961	973	984	
89	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	10
890	59 106	118	129	140	151	162	173	184	195	207	1 1.0
91	218	229	240	251	262	273	284	295	306	318	2 2.0
92	329	340	351	362	373	384	395	406	417	428	3 3.0
93	439	450	461	472	483	494	506	517	528	539	4 4.0
94	550	561	572	583	594	605	616	627	638	649	5 5.0
95	660	671	682	693	704	715	726	737	748	759	6 6.0
96	770	780	791	802	813	824	835	846	857	868	7 7.0
97	879	890	901	912	923	934	945	956	966	977	8 8.0
98	988	999	*010	*021	*032	*043	*054	*065	*076	*086	9 9.0
99	60 097	108	119	130	141	152	163	173	184	195	
400	206	217	228	239	249	260	271	282	293	304	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
400	60 206	217	228	239	249	260	271	282	293	304	
01	314	325	336	347	358	369	379	390	401	412	
02	423	433	444	455	466	477	487	498	509	520	
03	531	541	552	563	574	584	595	606	617	627	
04	638	649	660	670	681	692	703	713	724	735	
05	746	756	767	778	788	799	810	821	831	842	
06	853	863	874	885	895	906	917	927	938	949	11
07	959	970	981	991	*002	*013	*023	*034	*045	*055	1 1.1
08	61 066	077	087	098	109	119	130	140	151	162	2 2.2
09	172	183	194	204	215	225	236	247	257	268	3 3.3
410	278	289	300	310	321	331	342	352	363	374	4 4.4
11	384	395	405	416	426	437	448	458	469	479	5 5.5
12	490	500	511	521	532	542	553	563	574	584	6 6.6
13	595	606	616	627	637	648	658	669	679	690	7 7.7
14	700	711	721	731	742	752	763	773	784	794	8 8.8
15	805	815	826	836	847	857	868	878	888	899	9 9.9
16	909	920	930	941	951	962	972	982	993	*003	
17	62 014	024	034	045	055	066	076	086	097	107	
18	118	128	138	149	159	170	180	190	201	211	
19	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
21	428	439	449	459	469	480	490	500	511	521	10
22	531	542	552	562	572	583	593	603	613	624	1 1.0
23	634	644	653	663	675	685	696	706	716	726	2 2.0
24	737	747	757	767	778	788	798	808	818	829	3 3.0
25	839	849	859	870	880	890	900	910	921	931	4 4.0
26	941	951	961	972	982	992	*002	*012	*022	*033	5 5.0
27	63 043	053	063	073	083	094	104	114	124	134	6 6 0
28	144	155	165	175	185	195	205	215	225	236	7 7.0
29	246	256	266	276	286	296	306	317	327	337	8 8.0
430	347	357	367	377	387	397	407	417	428	438	9 9.0
31	448	458	468	478	488	498	508	518	528	538	
32	548	558	568	579	589	599	609	619	629	639	
33	649	659	669	679	689	699	709	719	729	739	
34	749	759	769	779	789	799	809	819	829	839	
35	849	859	869	879	889	899	909	919	929	939	
36	949	959	969	979	988	998	*008	*018	*028	*038	9
37	64 048	058	068	078	088	098	108	118	128	137	1 0.9
38	147	157	167	177	187	197	207	217	227	237	2 1.8
39	246	256	266	276	286	296	306	316	326	335	3 2.7
440	345	355	365	375	385	395	404	414	424	434	4 3.6
41	444	454	464	473	483	493	503	513	523	532	5 4.5
42	542	552	562	572	582	591	601	611	621	631	6 5.4
43	640	650	660	670	680	689	699	709	719	729	7 6.3
44	738	748	758	768	777	787	797	807	816	826	8 7.2
45	836	846	856	865	875	885	895	904	914	924	9 8.1
46	933	943	953	963	972	982	992	*002	*011	*021	
47	65 031	040	050	060	070	079	089	099	108	118	
48	128	137	147	157	167	176	186	196	205	215	
49	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
450	65 321	331	341	350	360	369	379	389	398	408	
51	418	427	437	447	456	466	475	485	495	504	
52	514	523	533	543	552	562	571	581	591	600	
53	610	619	629	639	648	658	667	677	686	696	
54	706	715	725	734	744	753	763	772	782	792	
55	801	811	820	830	839	849	858	868	877	887	
56	896	906	916	925	935	944	954	963	973	982	10
57	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	1 1.0
58	66 087	096	106	115	124	134	143	153	162	172	2 2.0
59	181	191	200	210	219	229	238	247	257	266	3 3.0
460	276	285	295	304	314	323	332	342	351	361	4 4.0
61	370	380	389	398	408	417	427	436	445	455	5 5.0
62	464	474	483	492	502	511	521	530	539	549	6 6.0
63	558	567	577	586	596	605	614	624	633	642	7 7.0
64	652	661	671	680	689	699	708	717	727	736	8 8.0
65	745	755	764	773	783	792	801	811	820	829	9 9.0
66	839	848	857	867	876	885	894	904	913	922	
67	932	941	950	960	969	978	987	997	*006	*015	
68	67 025	034	043	052	062	071	080	089	099	108	
69	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
71	302	311	321	330	339	348	357	367	376	385	9
72	394	403	413	422	431	440	449	459	468	477	1 0.9
73	486	495	504	514	523	532	541	550	560	569	2 1.8
74	578	587	596	605	614	624	633	642	651	660	3 2.7
75	669	679	688	697	706	715	724	733	742	752	4 3.6
76	761	770	779	788	797	806	815	825	834	843	5 4.5
77	852	861	870	879	888	897	906	916	925	934	6 5.4
78	943	952	961	970	979	988	997	*006	*015	*024	7 6.3
79	68 034	043	052	061	070	079	088	097	106	115	8 7.2
480	124	133	142	151	160	169	178	187	196	205	9 8.1
81	215	224	233	242	251	260	269	278	287	296	
82	305	314	323	332	341	350	359	368	377	386	
83	395	404	413	422	431	440	449	458	467	476	
84	485	494	502	511	520	529	538	547	556	565	
85	574	583	592	601	610	619	628	637	646	655	
86	664	673	681	690	699	708	717	726	735	744	8
87	753	762	771	780	789	797	806	815	824	833	1 0.8
88	842	851	860	869	878	886	895	904	913	922	2 1.6
89	931	940	949	958	966	975	984	993	*002	*011	3 2.4
490	69 020	028	037	046	055	064	073	082	090	099	4 3.2
91	108	117	126	135	144	152	161	170	179	188	5 4.0
92	197	205	214	223	232	241	249	258	267	276	6 4.8
93	285	294	302	311	320	329	338	346	355	364	7 5.6
94	373	381	390	399	408	417	425	434	443	452	8 6.4
95	461	469	478	487	496	504	513	522	531	539	9 7.2
96	548	557	566	574	583	592	601	609	618	627	
97	636	644	653	662	671	679	688	697	705	714	
98	723	732	740	749	758	767	775	784	793	801	
99	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
500	69 897	906	914	923	932	940	949	958	966	975	
01	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
02	70 070	079	088	096	105	114	122	131	140	148	
03	157	165	174	183	191	200	209	217	226	234	
04	243	252	260	269	278	286	295	303	312	321	
05	329	338	346	355	364	372	381	389	398	406	
06	415	424	432	441	449	458	467	475	484	492	9
07	501	509	518	526	535	544	552	561	569	578	1 0.9
08	586	595	603	612	621	629	638	646	655	663	2 1.8
09	672	680	689	697	706	714	723	731	740	749	3 2.7
510	757	766	774	783	791	800	808	817	825	834	4 3.6
11	842	851	859	868	876	885	893	902	910	919	5 4.5
12	927	935	944	952	961	969	978	986	995	*003	6 5.4
13	71 012	020	029	037	046	054	063	071	079	088	7 6.3
14	096	105	113	122	130	139	147	155	164	172	8 7.2
15	181	189	198	206	214	223	231	240	248	257	9 8.1
16	265	273	282	290	299	307	315	324	332	341	
17	349	357	366	374	383	391	399	408	416	425	
18	433	441	450	458	466	475	483	492	500	508	
19	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
21	684	692	700	709	717	725	734	742	750	759	8
22	767	775	784	792	800	809	817	825	834	842	1 0.8
23	850	858	867	875	883	892	900	908	917	925	2 1.6
24	933	941	950	958	966	975	983	991	999	*008	3 2.4
25	72 016	024	032	041	049	057	066	074	082	090	4 3.2
26	099	107	115	123	132	140	148	156	165	173	5 4.0
27	181	189	198	206	214	222	230	239	247	255	6 4.8
28	263	272	280	288	296	304	313	321	329	337	7 5.6
29	346	354	362	370	378	387	395	403	411	419	8 6.4
530	428	436	444	452	460	469	477	485	493	501	9 7.2
31	509	518	526	534	542	550	558	567	575	583	
32	591	599	607	616	624	632	640	648	656	665	
33	673	681	689	697	705	713	722	730	738	746	
34	754	762	770	779	787	795	803	811	819	827	
35	835	843	852	860	868	876	884	892	900	908	
36	916	925	933	941	949	957	965	973	981	989	
37	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	7
38	73 078	086	094	102	111	119	127	135	143	151	1 0.7
39	159	167	175	183	191	199	207	215	223	231	2 1.4
540	239	247	255	263	272	280	288	296	304	312	3 2.1
41	320	328	336	344	352	360	368	376	384	392	4 2.8
42	400	408	416	424	432	440	448	456	464	472	5 3.5
43	480	488	496	504	512	520	528	536	544	552	6 4.2
44	560	568	576	584	592	600	608	616	624	632	7 4.9
45	640	648	656	664	672	679	687	695	703	711	8 5.6
46	719	727	735	743	751	759	767	775	783	791	9 6.3
47	799	807	815	823	830	838	846	854	862	870	
48	878	886	894	902	910	918	926	933	941	949	
49	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
550	74 036	044	052	060	068	076	084	092	099	107	
51	115	123	131	139	147	155	162	170	178	186	
52	194	202	210	218	225	233	241	249	257	265	
53	273	280	288	296	304	312	320	327	335	343	
54	351	359	367	374	382	390	398	406	414	421	
55	429	437	445	453	461	468	476	484	492	500	
56	507	515	523	531	539	547	554	562	570	578	
57	586	593	601	609	617	624	632	640	648	656	
58	663	671	679	687	695	702	710	718	726	733	
59	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
61	896	904	912	920	927	935	943	950	958	966	8
62	974	981	989	997	*005	*012	*020	*028	*035	*043	1 0.8
63	75 051	059	066	074	082	089	097	105	113	120	2 1.6
64	128	136	143	151	159	166	174	182	189	197	3 2.4
65	205	213	220	228	236	243	251	259	266	274	4 3.2
66	282	289	297	305	312	320	328	335	343	351	5 4.0
67	358	366	374	381	389	397	404	412	420	427	6 4.8
68	435	442	450	458	465	473	481	488	496	504	7 5.6
69	511	519	526	534	542	549	557	563	572	580	8 6.4
570	587	595	603	610	618	626	633	641	648	656	9 7.2
71	664	671	679	686	694	702	709	717	724	732	
72	740	747	755	762	770	778	785	793	800	808	
73	815	823	831	838	846	853	861	868	876	884	
74	891	899	906	914	921	929	937	944	952	959	
75	967	974	982	989	997	*005	*012	*020	*027	*035	
76	76 042	050	057	065	072	080	087	093	103	110	
77	118	125	133	140	148	155	163	170	178	185	
78	193	200	208	215	223	230	238	245	253	260	
79	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
81	418	425	433	440	448	455	462	470	477	485	7
82	492	500	507	515	522	530	537	545	552	559	1 0.7
83	567	574	582	589	597	604	612	619	626	634	2 1.4
84	641	649	656	664	671	678	686	693	701	708	3 2.1
85	716	723	730	738	745	753	760	768	775	782	4 2.8
86	790	797	805	812	819	827	834	842	849	856	5 3.5
87	864	871	879	886	893	901	908	916	923	930	6 4.2
88	938	945	953	960	967	975	982	989	997	*004	7 4.9
89	77 012	019	026	034	041	048	056	063	070	078	8 5.6
590	085	093	100	107	115	122	129	137	144	151	9 6.3
91	159	166	173	181	188	195	203	210	217	225	
92	232	240	247	254	262	269	276	283	291	298	
93	305	313	320	327	335	342	349	357	364	371	
94	379	386	393	401	408	415	422	430	437	444	
95	452	459	466	474	481	488	495	503	510	517	
96	525	532	539	546	554	561	568	576	583	590	
97	597	605	612	619	627	634	641	648	656	663	
98	670	677	685	692	699	706	714	721	728	735	
99	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
600	77 815	822	830	837	844	851	859	866	873	880	
01	887	895	902	909	916	924	931	938	945	952	
02	960	967	974	981	988	996	*003	*010	*017	*025	
03	78 032	039	046	053	061	068	075	082	089	097	
04	104	111	118	125	132	140	147	154	161	168	
05	176	183	190	197	204	211	219	226	233	240	
06	247	254	262	269	276	283	290	297	305	312	8
07	319	326	333	340	347	355	362	369	376	383	1 0.8
08	390	398	405	412	419	426	433	440	447	455	2 1.6
09	462	469	476	483	490	497	504	512	519	526	3 2.4
610	533	540	547	554	561	569	576	583	590	597	4 3.2
11	604	611	618	625	633	640	647	654	661	668	5 4.0
12	675	682	689	696	704	711	718	725	732	739	6 4.8
13	746	753	760	767	774	781	789	796	803	810	7 5.6
14	817	824	831	838	845	852	859	866	873	880	8 6.4
15	888	895	902	909	916	923	930	937	944	951	9 7.2
16	958	965	972	979	986	993	*000	*007	*014	*021	
17	79 029	036	043	050	057	064	071	078	085	092	
18	099	106	113	120	127	134	141	148	155	162	
19	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
21	309	316	323	330	337	344	351	358	365	372	7
22	379	386	393	400	407	414	421	428	435	442	1 0.7
23	449	456	463	470	477	484	491	498	505	511	2 1.4
24	518	525	532	539	546	553	560	567	574	581	3 2.1
25	588	595	602	609	616	623	630	637	644	650	4 2.8
26	657	664	671	678	685	692	699	706	713	720	5 3.5
27	727	734	741	748	754	761	768	775	782	789	6 4.2
28	796	803	810	817	824	831	837	844	851	858	7 4.9
29	865	872	879	886	893	900	906	913	920	927	8 5.6
630	934	941	948	955	962	969	975	982	989	996	9 6.3
31	80 003	010	017	024	030	037	044	051	058	065	
32	072	079	085	092	099	106	113	120	127	134	
33	140	147	154	161	168	175	182	188	195	202	
34	209	216	223	229	236	243	250	257	264	271	
35	277	284	291	298	305	312	318	325	332	339	
36	346	353	359	366	373	380	387	393	400	407	6
37	414	421	428	434	441	448	455	462	468	475	1 0.6
38	482	489	496	502	509	516	523	530	536	543	2 1.2
39	550	557	564	570	577	584	591	598	604	611	3 1.8
640	618	625	632	638	645	652	659	665	672	679	4 2.4
41	686	693	699	706	713	720	726	733	740	747	5 3.0
42	754	760	767	774	781	787	794	801	808	814	6 3.6
43	821	828	835	841	848	855	862	868	875	882	7 4.2
44	889	895	902	909	916	922	929	936	943	949	8 4.8
45	956	963	969	976	983	990	996	*003	*010	*017	9 5.4
46	81 023	030	037	043	050	057	064	070	077	084	
47	090	097	104	111	117	124	131	137	144	151	
48	158	164	171	178	184	191	198	204	211	218	
49	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
650	81 291	298	305	311	318	325	331	338	345	351	
51	358	365	371	378	385	391	398	405	411	418	
52	425	431	438	445	451	458	465	471	478	485	
53	491	498	505	511	518	525	531	538	544	551	
54	558	564	571	578	584	591	598	604	611	617	
55	624	631	637	644	651	657	664	671	677	684	
56	690	697	704	710	717	723	730	737	743	750	
57	757	763	770	776	783	790	796	803	809	816	
58	823	829	836	842	849	856	862	869	875	882	
59	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
61	82 020	027	033	040	046	053	060	066	073	079	7
62	086	092	099	105	112	119	125	132	138	145	1 0.7
63	151	158	164	171	178	184	191	197	204	210	2 1.4
64	217	223	230	236	243	249	256	263	269	276	3 2.1
65	282	289	295	302	308	315	321	328	334	341	4 2.8
66	347	354	360	367	373	380	387	393	400	406	5 3.5
67	413	419	426	432	439	445	452	458	465	471	6 4.2
68	478	484	491	497	504	510	517	523	530	536	7 4.9
69	543	549	556	562	569	575	582	588	595	601	8 5.6
670	607	614	620	627	633	640	646	653	659	666	9 6 3
71	672	679	685	692	698	705	711	718	724	730	
72	737	743	750	756	763	769	776	782	789	795	
73	802	808	814	821	827	834	840	847	853	860	
74	866	872	879	885	892	898	905	911	918	924	
75	930	937	943	950	956	963	969	975	982	988	
76	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
77	83 059	065	072	078	085	091	097	104	110	117	
78	123	129	136	142	149	155	161	168	174	181	
79	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
81	315	321	327	334	340	347	353	359	366	372	6
82	378	385	391	398	404	410	417	423	429	436	1 0.6
83	442	448	455	461	467	474	480	487	493	499	2 1.2
84	506	512	518	525	531	537	544	550	556	563	3 1.8
85	569	575	582	588	594	601	607	613	620	626	4 2.4
86	632	639	645	651	658	664	670	677	683	689	5 3.0
87	696	702	708	715	721	727	734	740	746	753	6 3 6
88	759	765	771	778	784	790	797	803	809	816	7 4.2
89	822	828	835	841	847	853	860	866	872	879	8 4.8
690	885	891	897	904	910	916	923	929	935	942	9 5.4
91	948	954	960	967	973	979	985	992	998	*004	
92	84 011	017	023	029	036	042	048	055	061	067	
93	073	080	086	092	098	105	111	117	123	130	
94	136	142	148	155	161	167	173	180	186	192	
95	198	205	211	217	223	230	236	242	248	255	
96	261	267	273	280	286	292	298	305	311	317	
97	323	330	336	342	348	354	361	367	373	379	
98	386	392	398	404	410	417	423	429	435	442	
99	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
700	84 510	516	522	528	533	541	547	553	559	566	
01	572	578	584	590	597	603	609	615	621	628	
02	634	640	646	652	658	665	671	677	683	689	
03	696	702	708	714	720	726	733	739	745	751	
04	757	763	770	776	782	788	794	800	807	813	
05	819	825	831	837	844	850	856	862	868	874	
06	880	887	893	899	905	911	917	924	930	936	7
07	942	948	954	960	967	973	979	985	991	997	1 0.7
08	85 003	009	016	022	028	034	040	046	052	058	2 1.4
09	065	071	077	083	089	095	101	107	114	120	3 2.1
710	126	132	138	144	150	156	163	169	175	181	4 2.8
11	187	193	199	205	211	217	224	230	236	242	5 3.5
12	248	254	260	266	272	278	285	291	297	303	6 4.2
13	309	315	321	327	333	339	345	352	358	364	7 4.9
14	370	376	382	388	394	400	406	412	418	425	8 5.6
15	431	437	443	449	455	461	467	473	479	485	9 6.3
16	491	497	503	509	516	522	528	534	540	546	
17	552	558	564	570	576	582	588	594	600	606	
18	612	618	625	631	637	643	649	655	661	667	
19	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
21	794	800	806	812	818	824	830	836	842	848	6
22	854	860	866	872	878	884	890	896	902	908	1 0.6
23	914	920	926	932	938	944	950	956	962	968	2 1.2
24	974	980	986	992	998	*004	*010	*016	*022	*028	3 1.8
25	86 034	040	046	052	058	064	070	076	082	088	4 2.4
26	094	100	106	112	118	124	130	136	141	147	5 3.0
27	153	159	165	171	177	183	189	195	201	207	6 3.6
28	213	219	225	231	237	243	249	255	261	267	7 4.2
29	273	279	285	291	297	303	308	314	320	326	8 4.8
730	332	338	344	350	356	362	368	374	380	386	9 5.4
31	392	398	404	410	415	421	427	433	439	445	
32	451	457	463	469	475	481	487	493	499	504	
33	510	516	522	528	534	540	546	552	558	564	
34	570	576	581	587	593	599	605	611	617	623	
35	629	635	641	646	652	658	664	670	676	682	
36	688	694	700	705	711	717	723	729	735	741	5
37	747	753	759	764	770	776	782	788	794	800	1 0.5
38	806	812	817	823	829	835	841	847	853	859	2 1.0
39	864	870	876	882	888	894	900	906	911	917	3 1.5
740	923	929	935	941	947	953	958	964	970	976	4 2.0
41	982	988	994	999	*005	*011	*017	*023	*029	*035	5 2.5
42	87 040	046	052	058	064	070	075	081	087	093	6 3.0
43	099	105	111	116	122	128	134	140	146	151	7 3.5
44	157	163	169	175	181	186	192	198	204	210	8 4.0
45	216	221	227	233	239	245	251	256	262	268	9 4.5
46	274	280	286	291	297	303	309	315	320	326	
47	332	338	344	349	355	361	367	373	379	384	
48	390	396	402	408	413	419	425	431	437	442	
49	448	454	460	466	471	477	483	489	495	500	
750	506	512	518	523	529	535	541	547	552	558	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
750	87 506	512	518	523	529	535	541	547	552	558	
51	564	570	576	581	587	593	599	604	610	616	
52	622	628	633	639	645	651	656	662	668	674	
53	679	685	691	697	703	708	714	720	726	731	
54	737	743	749	754	760	766	772	777	783	789	
55	795	800	806	812	818	823	829	835	841	846	
56	852	858	864	869	875	881	887	892	898	904	
57	910	915	921	927	933	938	944	950	955	961	
58	967	973	978	984	990	996	*001	*007	*013	*018	
59	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
61	138	144	150	156	161	167	173	178	184	190	6
62	195	201	207	213	218	224	230	235	241	247	1 0.6
63	252	258	264	270	275	281	287	292	298	304	2 1.2
64	309	315	321	326	332	338	343	349	355	360	3 1.8
65	366	372	377	383	389	395	400	406	412	417	4 2.4
66	423	429	434	440	446	451	457	463	468	474	5 3.0
67	480	485	491	497	502	508	513	519	525	530	6 3.6
68	536	542	547	553	559	564	570	576	581	587	7 4.2
69	593	598	604	610	615	621	627	632	638	643	8 4.8
770	649	655	660	666	672	677	683	689	694	700	9 5.4
71	705	711	717	722	728	734	739	745	750	756	
72	762	767	773	779	784	790	795	801	807	812	
73	818	824	829	835	840	846	852	857	863	868	
74	874	880	885	891	897	902	908	913	919	925	
75	930	936	941	947	953	958	964	969	975	981	
76	986	992	997	*003	*009	*014	*020	*025	*031	*037	
77	89 042	048	053	059	064	070	076	081	087	092	
78	098	104	109	115	120	126	131	137	143	148	
79	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	
81	265	271	276	282	287	293	298	304	310	315	5
82	321	326	332	337	343	348	354	360	365	371	1 0.5
83	376	382	387	393	398	404	409	415	421	426	2 1.0
84	432	437	443	448	454	459	465	470	476	481	3 1.5
85	487	492	498	504	509	515	520	526	531	537	4 2.0
86	542	548	553	559	564	570	575	581	586	592	5 2.5
87	597	603	609	614	620	625	631	636	642	647	6 3.0
88	653	658	664	669	675	680	686	691	697	702	7 3.5
89	708	713	719	724	730	735	741	746	752	757	8 4.0
790	763	768	774	779	785	790	796	801	807	812	9 4.5
91	818	823	829	834	840	845	851	856	862	867	
92	873	878	883	889	894	900	905	911	916	922	
93	927	933	938	944	949	955	960	966	971	977	
94	982	988	993	998	*004	*009	*015	*020	*026	*031	
95	90 037	042	048	053	059	064	069	075	080	086	
96	091	097	102	108	113	119	124	129	135	140	
97	146	151	157	162	168	173	179	184	189	195	
98	200	206	211	217	222	227	233	238	244	249	
99	253	260	266	271	276	282	287	293	298	304	
800	309	314	320	325	331	336	342	347	352	358	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
800	90 309	314	320	325	331	336	342	347	352	358	
01	363	369	374	380	385	390	396	401	407	412	
02	417	423	428	434	439	445	450	455	461	466	
03	472	477	482	488	493	499	504	509	515	520	
04	526	531	536	542	547	553	558	563	569	574	
05	580	585	590	596	601	607	612	617	623	628	
06	634	639	644	650	655	660	666	671	677	682	
07	687	693	698	703	709	714	720	725	730	736	
08	741	747	752	757	763	768	773	779	784	789	
09	795	800	806	811	816	822	827	832	838	843	
810	849	854	859	863	870	875	881	886	891	897	
11	902	907	913	918	924	929	934	940	945	950	6
12	956	961	966	972	977	982	988	993	998	*004	1 0.6
13	91 009	014	020	025	030	036	041	046	052	057	2 1.2
14	062	068	073	078	084	089	094	100	105	110	3 1.8
15	116	121	126	132	137	142	148	153	158	164	4 2.4
16	169	174	180	185	190	196	201	206	212	217	5 3.0
17	222	228	233	238	243	249	254	259	265	270	6 3.6
18	275	281	286	291	297	302	307	312	318	323	7 4.2
19	328	334	339	344	350	355	360	365	371	376	8 4.8
820	381	387	392	397	403	408	413	418	424	429	9 5.4
21	434	440	445	450	455	461	466	471	477	482	
22	487	492	498	503	508	514	519	524	529	535	
23	540	545	551	556	561	566	572	577	582	587	
24	593	598	603	609	614	619	624	630	635	640	
25	645	651	656	661	666	672	677	682	687	693	
26	698	703	709	714	719	724	730	735	740	745	
27	751	756	761	766	772	777	782	787	793	798	
28	803	808	814	819	824	829	834	840	845	850	
29	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
31	960	965	971	976	981	986	991	997	*002	*007	5
32	92 012	018	023	028	033	038	044	049	054	059	1 0.5
33	065	070	075	080	085	091	096	101	106	111	2 1.0
34	117	122	127	132	137	143	148	153	158	163	3 1.5
35	169	174	179	184	189	195	200	205	210	215	4 2.0
36	221	226	231	236	241	247	252	257	262	267	5 2.5
37	273	278	283	288	293	298	304	309	314	319	6 3.0
38	324	330	335	340	345	350	355	361	366	371	7 3.5
39	376	381	387	392	397	402	407	412	418	423	8 4.0
840	428	433	438	443	449	454	459	464	469	474	9 4.5
41	480	485	490	495	500	505	511	516	521	526	
42	531	536	542	547	552	557	562	567	572	578	
43	583	588	593	598	603	609	614	619	624	629	
44	634	639	645	650	655	660	665	670	675	681	
45	686	691	696	701	706	711	716	722	727	732	
46	737	742	747	752	758	763	768	773	778	783	
47	788	793	799	804	809	814	819	824	829	834	
48	840	845	850	855	860	865	870	875	881	886	
49	891	896	901	906	911	916	921	927	932	937	
850	942	947	952	957	962	967	973	978	983	988	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
850	92 942	947	952	957	962	967	973	978	983	988	
51	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
52	93 044	049	054	059	064	069	075	080	085	090	
53	095	100	105	110	115	120	125	131	136	141	
54	146	151	156	161	166	171	176	181	186	192	
55	197	202	207	212	217	222	227	232	237	242	
56	247	252	258	263	268	273	278	283	288	293	6
57	298	303	308	313	318	323	328	334	339	344	1 0.6
58	349	354	359	364	369	374	379	384	389	394	2 1.2
59	399	404	409	414	420	425	430	435	440	445	3 1.8
860	450	455	460	465	470	475	480	485	490	495	4 2.4
61	500	505	510	515	520	526	531	536	541	546	5 3.0
62	551	556	561	566	571	576	581	586	591	596	6 3.6
63	601	606	611	616	621	626	631	636	641	646	7 4.2
64	651	656	661	666	671	676	682	687	692	697	8 4.8
65	702	707	712	717	722	727	732	737	742	747	9 5.4
66	752	757	762	767	772	777	782	787	792	797	
67	802	807	812	817	822	827	832	837	842	847	
68	852	857	862	867	872	877	882	887	892	897	
69	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
71	94 002	007	012	017	022	027	032	037	042	047	5
72	052	057	062	067	072	077	082	086	091	096	1 0.5
73	101	106	111	116	121	126	131	136	141	146	2 1.0
74	151	156	161	166	171	176	181	186	191	196	3 1.5
75	201	206	211	216	221	226	231	236	240	245	4 2.0
76	250	255	260	265	270	275	280	285	290	295	5 2.5
77	300	305	310	315	320	325	330	335	340	345	6 3.0
78	349	354	359	364	369	374	379	384	389	394	7 3.5
79	399	404	409	414	419	424	429	433	438	443	8 4.0
880	448	453	458	463	468	473	478	483	488	493	9 4.5
81	498	503	507	512	517	522	527	532	537	542	
82	547	552	557	562	567	571	576	581	586	591	
83	596	601	606	611	616	621	626	630	635	640	
84	645	650	655	660	665	670	675	680	685	689	
85	694	699	704	709	714	719	724	729	734	738	
86	743	748	753	758	763	768	773	778	783	787	4
87	792	797	802	807	812	817	822	827	832	836	1 0.4
88	841	846	851	856	861	866	871	876	880	885	2 0.8
89	890	895	900	905	910	915	919	924	929	934	3 1.2
890	939	944	949	954	959	963	968	973	978	983	4 1.6
91	988	993	998	*002	*007	*012	*017	*022	*027	*032	5 2.0
92	95 036	041	046	051	056	061	066	071	075	080	6 2.4
93	085	090	095	100	105	109	114	119	124	129	7 2.8
94	134	139	143	148	153	158	163	168	173	177	8 3.2
95	182	187	192	197	202	207	211	216	221	226	9 3.6
96	231	236	240	245	250	255	260	265	270	274	
97	279	284	289	294	299	303	308	313	318	323	
98	328	332	337	342	347	352	357	361	366	371	
99	376	381	386	390	395	400	405	410	415	419	
900	424	429	434	439	444	448	453	458	463	468	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
900	95 424	429	434	439	444	448	453	458	463	468	
01	472	477	482	487	492	497	501	506	511	516	
02	521	525	530	535	540	545	550	554	559	564	
03	569	574	578	583	588	593	598	602	607	612	
04	617	622	626	631	636	641	646	650	655	660	
05	665	670	674	679	684	689	694	698	703	708	
06	713	718	722	727	732	737	742	746	751	756	
07	761	766	770	775	780	785	789	794	799	804	
08	809	813	818	823	828	832	837	842	847	852	
09	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
11	952	957	961	966	971	976	980	985	990	995	5
12	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	1 0.5
13	96 047	052	057	061	066	071	076	080	085	090	2 1.0
14	095	099	104	109	114	118	123	128	133	137	3 1.5
15	142	147	152	156	161	166	171	175	180	185	4 2.0
16	190	194	199	204	209	213	218	223	227	232	5 2.5
17	237	242	246	251	256	261	265	270	275	280	6 3.0
18	284	289	294	298	303	308	313	317	322	327	7 3.5
19	332	336	341	346	350	355	360	365	369	374	8 4.0
920	379	384	388	393	398	402	407	412	417	421	9 4.5
21	426	431	435	440	445	450	454	459	464	468	
22	473	478	483	487	492	497	501	506	511	515	
23	520	525	530	534	539	544	548	553	558	562	
24	567	572	577	581	586	591	595	600	605	609	
25	614	619	624	628	633	638	642	647	652	656	
26	661	666	670	675	680	685	689	694	699	703	
27	708	713	717	722	727	731	736	741	745	750	
28	755	759	764	769	774	778	783	788	792	797	
29	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
31	895	900	904	909	914	918	923	928	932	937	4
32	942	946	951	956	960	965	970	974	979	984	1 0.4
33	988	993	997	*002	*007	*011	*016	*021	*025	*030	2 0.8
34	97 035	039	044	049	053	058	063	067	072	077	3 1.2
35	081	086	090	095	100	104	109	114	118	123	4 1.6
36	128	132	137	142	146	151	155	160	165	169	5 2.0
37	174	179	183	188	192	197	202	206	211	216	6 2.4
38	220	225	230	234	239	243	248	253	257	262	7 2.8
39	267	271	276	280	285	290	294	299	304	308	8 3.2
940	313	317	322	327	331	336	340	345	350	354	9 3.6
41	359	364	368	373	377	382	387	391	396	400	
42	405	410	414	419	424	428	433	437	442	447	
43	451	456	460	465	470	474	479	483	488	493	
44	497	502	506	511	516	520	525	529	534	539	
45	543	548	552	557	562	566	571	575	580	585	
46	589	594	598	603	607	612	617	621	626	630	
47	635	640	644	649	653	658	663	667	672	676	
48	681	685	690	695	699	704	708	713	717	722	
49	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
950	97 772	777	782	786	791	795	800	804	809	813	
51	818	823	827	832	836	841	845	850	855	859	
52	864	868	873	877	882	886	891	896	900	905	
53	909	914	918	923	928	932	937	941	946	950	
54	955	959	964	968	973	978	982	987	991	996	
55	98 000	005	009	014	019	023	028	032	037	041	
56	046	050	055	059	064	068	073	078	082	087	
57	091	096	100	105	109	114	118	123	127	132	
58	137	141	146	150	155	159	164	168	173	177	
59	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
61	272	277	281	286	290	295	299	304	308	313	5
62	318	322	327	331	336	340	345	349	354	358	1 0.5
63	363	367	372	376	381	385	390	394	399	403	2 1.0
64	408	412	417	421	426	430	435	439	444	448	3 1.5
65	453	457	462	466	471	475	480	484	489	493	4 2.0
66	498	502	507	511	516	520	525	529	534	538	5 2.5
67	543	547	552	556	561	565	570	574	579	583	6 3.0
68	588	592	597	601	605	610	614	619	623	628	7 3.5
69	632	637	641	646	650	655	659	664	668	673	8 4.0
970	677	682	686	691	695	700	704	709	713	717	9 4.5
71	722	726	731	735	740	744	749	753	758	762	
72	767	771	776	780	784	789	793	798	802	807	
73	811	816	820	825	829	834	838	843	847	851	
74	856	860	863	869	874	878	883	887	892	896	
75	900	905	909	914	918	923	927	932	936	941	
76	945	949	954	958	963	967	972	976	981	985	
77	989	994	998	*003	*007	*012	*016	*021	*025	*029	
78	99 034	038	043	047	052	056	061	065	069	074	
79	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
81	167	171	176	180	185	189	193	198	202	207	4
82	211	216	220	224	229	233	238	242	247	251	1 0.4
83	255	260	264	269	273	277	282	286	291	295	2 0.8
84	300	304	308	313	317	322	326	330	335	339	3 1.2
85	344	348	352	357	361	366	370	374	379	383	4 1.6
86	388	392	396	401	405	410	414	419	423	427	5 2.0
87	432	436	441	445	449	454	458	463	467	471	6 2.4
88	476	480	484	489	493	498	502	506	511	515	7 2.8
89	520	524	528	533	537	542	546	550	555	559	8 3.2
990	564	568	572	577	581	585	590	594	599	603	9 3.6
91	607	612	616	621	625	629	634	638	642	647	
92	651	656	660	664	669	673	677	682	686	691	
93	695	699	704	708	712	717	721	726	730	734	
94	739	743	747	752	756	760	765	769	774	778	
95	782	787	791	795	800	804	808	813	817	822	
96	826	830	835	839	843	848	852	856	861	865	
97	870	874	878	883	887	891	896	900	904	909	
98	913	917	922	926	930	935	939	944	948	952	
99	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
1000	000 000	043	087	130	174	217	260	304	347	391	
1001	434	477	521	564	608	651	694	738	781	824	
1002	868	911	954	998	*041	*084	*128	*171	*214	*258	
1003	001 301	344	388	431	474	517	561	604	647	690	44
1004	734	777	820	863	907	950	993	*036	*080	*123	1 4.4
1005	002 166	209	252	296	339	382	425	468	512	555	2 8.8
1006	598	641	684	727	771	814	857	900	943	986	3 13.2
1007	003 029	073	116	159	202	245	288	331	374	417	4 17.6
1008	461	504	547	590	633	676	719	762	805	848	5 22.0
1009	891	934	977	*020	*063	*106	*149	*192	*235	*278	6 26.4
1010	004 321	364	407	450	493	536	579	622	665	708	7 30.8
1011	751	794	837	880	923	966	*009	*052	*095	*138	8 35.2
1012	005 180	223	266	309	352	395	438	481	524	567	9 39.6
1013	609	652	695	738	781	824	867	909	952	995	
1014	006 038	081	124	166	209	252	295	338	380	423	
1015	466	509	552	594	637	680	723	765	808	851	43
1016	894	936	979	*022	*065	*107	*150	*193	*236	*278	1 4.3
1017	007 321	364	406	449	492	534	577	620	662	705	2 8.6
1018	748	790	833	876	918	961	*004	*046	*089	*132	3 12.9
1019	008 174	217	259	302	345	387	430	472	515	558	4 17.2
1020	600	643	685	728	770	813	856	898	941	983	5 21.5
1021	009 026	068	111	153	196	238	281	323	366	408	6 25.8
1022	451	493	536	578	621	663	706	748	791	833	7 30.1
1023	876	918	961	*003	*045	*088	*130	*173	*215	*258	8 34.4
1024	010 300	342	385	427	470	512	554	597	639	681	9 38.7
1025	724	766	809	851	893	936	978	*020	*063	*105	
1026	011 147	190	232	274	317	359	401	444	486	528	
1027	570	613	655	697	740	782	824	866	909	951	42
1028	993	*035	*078	*120	*162	*204	*247	*289	*331	*373	1 4.2
1029	012 415	458	500	542	584	626	669	711	753	795	2 8.4
1030	837	879	922	964	*006	*048	*090	*132	*174	*217	3 12.6
1031	013 259	301	343	385	427	469	511	553	596	638	4 16.8
1032	680	722	764	806	848	890	932	974	*016	*058	5 21.0
1033	014 100	142	184	226	268	310	352	395	437	479	6 25.2
1034	521	563	605	647	689	730	772	814	856	898	7 29.4
1035	940	982	*024	*066	*108	*150	*192	*234	*276	*318	8 33.6
1036	015 360	402	444	485	527	569	611	653	695	737	9 37.8
1037	779	821	863	904	946	988	*030	*072	*114	*156	
1038	016 197	239	281	323	365	407	448	490	532	574	
1039	616	657	699	741	783	824	866	908	950	992	41
1040	017 033	075	117	159	200	242	284	326	367	409	1 4.1
1041	451	492	534	576	618	659	701	743	784	826	2 8.2
1042	868	909	951	993	*034	*076	*118	*159	*201	*243	3 12.3
1043	018 284	326	368	409	451	492	534	576	617	659	4 16.4
1044	700	742	784	825	867	908	950	992	*033	*075	5 20.5
1045	019 116	158	199	241	282	324	366	407	449	490	6 24.6
1046	532	573	615	656	698	739	781	822	864	905	7 28.7
1047	947	988	*030	*071	*113	*154	*195	*237	*278	*320	8 32.8
1048	020 361	403	444	486	527	568	610	651	693	734	9 36.9
1049	775	817	858	900	941	982	*024	*065	*107	*148	
1050	021 189	231	272	313	355	396	437	479	520	561	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
1050	021 189	231	272	313	355	396	437	479	520	561	
1051	603	644	685	727	768	809	851	892	933	974	
1052	022 016	057	098	140	181	222	263	305	346	387	
1053	428	470	511	552	593	635	676	717	758	799	42
1054	841	882	923	964	*005	*047	*088	*129	*170	*211	1 4.2
1055	023 252	294	335	376	417	458	499	541	582	623	2 8.4
1056	664	705	746	787	828	870	911	952	993	*034	3 12.6
1057	024 075	116	157	198	239	280	321	363	404	445	4 16.8
1058	486	527	568	609	650	691	732	773	814	855	5 21.0
1059	896	937	978	*019	*060	*101	*142	*183	*224	*265	6 25.2
1060	025 306	347	388	429	470	511	552	593	634	674	7 29.4
1061	715	756	797	838	879	920	961	*002	*043	*084	8 33.6
1062	026 125	165	206	247	288	329	370	411	452	492	9 37.8
1063	533	574	615	656	697	737	778	819	860	901	
1064	942	982	*023	*064	*105	*146	*186	*227	*268	*309	
1065	027 350	390	431	472	513	553	594	635	676	716	41
1066	757	798	839	879	920	961	*002	*042	*083	*124	1 4.1
1067	028 164	205	246	287	327	368	409	449	490	531	2 8.2
1068	571	612	653	693	734	775	815	856	896	937	3 12.3
1069	978	*018	*059	*100	*140	*181	*221	*262	*303	*343	4 16.4
1070	029 384	424	465	506	546	587	627	668	708	749	5 20.5
1071	789	830	871	911	952	992	*033	*073	*114	*154	6 24.6
1072	030 195	235	276	316	357	397	438	478	519	559	7 28.7
1073	600	640	681	721	762	802	843	883	923	964	8 32.8
1074	031 004	045	085	126	166	206	247	287	328	368	9 36.9
1075	408	449	489	530	570	610	651	691	732	772	
1076	812	853	893	933	974	*014	*054	*095	*135	*175	
1077	032 216	256	296	337	377	417	458	498	538	578	40
1078	619	659	699	740	780	820	860	901	941	981	1 4.0
1079	033 021	062	102	142	182	223	263	303	343	384	2 8.0
1080	424	464	504	544	585	625	665	705	745	785	3 12.0
1081	826	866	906	946	986	*027	*067	*107	*147	*187	4 16.0
1082	034 227	267	308	348	388	428	468	508	548	588	5 20.0
1083	628	669	709	749	789	829	869	909	949	989	6 24.0
1084	035 029	069	109	149	190	230	270	310	350	390	7 28.0
1085	430	470	510	550	590	630	670	710	750	790	8 32.0
1086	830	870	910	950	990	*030	*070	*110	*150	*190	9 36.0
1087	036 230	269	309	349	389	429	469	509	549	589	
1088	629	669	709	749	789	828	868	908	948	988	
1089	037 028	068	108	148	187	227	267	307	347	387	39
1090	426	466	506	546	586	626	665	705	745	785	1 3.9
1091	825	865	904	944	984	*024	*064	*103	*143	*183	2 7.8
1092	038 223	262	302	342	382	421	461	501	541	580	3 11.7
1093	620	660	700	739	779	819	859	898	938	978	4 15.6
1094	039 017	057	097	136	176	216	255	295	335	374	5 19.5
1095	414	454	493	533	573	612	652	692	731	771	6 23.4
1096	811	850	890	929	969	*009	*048	*088	*127	*167	7 27.3
1097	040 207	246	286	325	365	405	444	484	523	563	8 31.2
1098	602	642	681	721	761	800	840	879	919	958	9 35.1
1099	998	*037	*077	*116	*156	*195	*235	*274	*314	*353	
1100	041 393	432	472	511	551	590	630	669	708	748	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

TABLE II

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LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

FOR

EACH MINUTE

0°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.		
0	—		—		—	0.00 000	60			
1	6.46 373	30103	6.46 373	30103	3.53 627	0.00 000	59	3476	3218	2997
2	6.76 476	17609	6.76 476	17609	3.23 524	0.00 000	58	.1 348	322	300
3	6.94 085	12494	6.94 085	12494	3.05 915	0.00 000	57	.2 695	644	599
4	7.06 579	9691	7.06 579	9691	2.93 421	0.00 000	56	.3 1043	965	899
5	7.16 270	7918	7.16 270	7918	2.83 730	0.00 000	55	.4 1390	1287	1199
6	7.24 188	6694	7.24 188	6694	2.75 812	0.00 000	54	.5 1738	1609	1498
7	7.30 882	5800	7.30 882	5800	2.69 118	0.00 000	53			
8	7.36 682	5115	7.36 682	5115	2.63 318	0.00 000	52	2802	2633	2483
9	7.41 797	4576	7.41 797	4576	2.58 203	0.00 000	51	.1 280	263	248
10	7.46 373	4139	7.46 373	4139	2.53 627	0.00 000	50	.2 560	527	497
11	7.50 512	3779	7.50 512	3779	2.49 488	0.00 000	49	.3 841	790	745
12	7.54 291	3476	7.54 291	3476	2.45 709	0.00 000	48	.4 1121	1053	993
13	7.57 767	3218	7.57 767	3219	2.42 233	0.00 000	47	.5 1401	1316	1242
14	7.60 985	2997	7.60 986	2996	2.39 014	0.00 000	46			
15	7.63 982	2802	7.63 982	2803	2.36 018	0.00 000	45	2287	2081	1848
16	7.66 784	2633	7.66 785	2633	2.33 215	0.00 000	44	.1 223	202	185
17	7.69 417	2483	7.69 418	2482	2.30 582	9.99 999	43	.2 445	404	370
18	7.71 900	2348	7.71 900	2348	2.28 100	9.99 999	42	.3 668	606	554
19	7.74 248	2227	7.74 248	2228	2.25 752	9.99 999	41	.4 891	808	739
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	40	.5 1113	1010	924
21	7.78 594	2021	7.78 595	2020	2.21 405	9.99 999	39	1704	1579	1478
22	7.80 615	1930	7.80 615	1931	2.19 385	9.99 999	38	.1 170	158	147
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37	.2 341	316	294
24	7.84 393	1773	7.84 394	1773	2.15 606	9.99 999	36	.3 511	474	442
25	7.86 166	1704	7.86 167	1704	2.13 833	9.99 999	35	.4 682	632	589
26	7.87 870	1639	7.87 871	1639	2.12 129	9.99 999	34	.5 852	789	736
27	7.89 509	1579	7.89 510	1579	2.10 490	9.99 999	33			
28	7.91 088	1524	7.91 089	1524	2.08 911	9.99 999	32	1379	1297	1223
29	7.92 612	1472	7.92 613	1473	2.07 387	9.99 998	31	.1 138	130	122
30	7.94 084	1424	7.94 086	1424	2.05 914	9.99 998	30	.2 276	259	245
31	7.95 508	1379	7.95 510	1379	2.04 490	9.99 998	29	.3 414	389	367
32	7.96 887	1336	7.96 889	1336	2.03 111	9.99 998	28	.4 552	519	489
33	7.98 223	1297	7.98 225	1297	2.01 775	9.99 998	27	.5 690	649	612
34	7.99 520	1259	7.99 522	1259	2.00 478	9.99 998	26			
35	8.00 779	1223	8.00 781	1223	1.99 219	9.99 998	25	1158	1200	1046
36	8.02 002	1190	8.02 004	1190	1.97 996	9.99 998	24	.1 116	110	105
37	8.03 192	1158	8.03 194	1159	1.96 806	9.99 997	23	.2 232	220	209
38	8.04 350	1128	8.04 353	1128	1.95 647	9.99 997	22	.3 347	330	314
39	8.05 478	1100	8.05 481	1100	1.94 519	9.99 997	21	.4 463	440	418
40	8.06 578	1072	8.06 581	1072	1.93 419	9.99 997	20	.5 579	550	523
41	8.07 650	1046	8.07 653	1047	1.92 347	9.99 997	19			
42	8.08 696	1022	8.08 700	1022	1.91 300	9.99 997	18	999	954	914
43	8.09 718	999	8.09 722	998	1.90 278	9.99 997	17	.1 100	95	91
44	8.10 717	976	8.10 720	976	1.89 280	9.99 996	16	.2 200	191	183
45	8.11 693	954	8.11 696	955	1.88 304	9.99 996	15	.3 300	286	274
46	8.12 647	934	8.12 651	934	1.87 349	9.99 996	14	.4 400	382	366
47	8.13 581	914	8.13 585	915	1.86 415	9.99 996	13	.5 500	477	457
48	8.14 495	896	8.14 500	895	1.85 500	9.99 996	12			
49	8.15 391	877	8.15 395	878	1.84 605	9.99 996	11	877	843	812
50	8.16 268	860	8.16 273	860	1.83 727	9.99 995	10	.1 88	84	81
51	8.17 128	843	8.17 133	843	1.82 867	9.99 995	9	.2 175	169	162
52	8.17 971	827	8.17 976	828	1.82 024	9.99 995	8	.3 263	253	244
53	8.18 798	812	8.18 804	812	1.81 196	9.99 995	7	.4 351	337	325
54	8.19 610	797	8.19 616	797	1.80 384	9.99 995	6	.5 438	422	406
55	8.20 407	782	8.20 413	782	1.79 587	9.99 994	5	782	755	730
56	8.21 189	769	8.21 195	769	1.78 805	9.99 994	4	.1 78	75	73
57	8.21 958	755	8.21 964	756	1.78 036	9.99 994	3	.2 156	151	146
58	8.22 713	743	8.22 720	742	1.77 280	9.99 994	2	.3 235	226	219
59	8.23 456	.730	8.23 462	730	1.76 538	9.99 994	1	.4 313	302	292
60	8.24 186		8.24 192		1.75 808	9.99 993	0	.5 391	377	365
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.		

89°

1°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.		
0	8.24 186		8.24 192		1.75 808	9.99 993	60			
1	8.24 903	717	8.24 910	718	1.75 090	9.99 993	59		717	695	673
2	8.25 609	706	8.25 616	706	1.74 384	9.99 993	58	.1	71.7	69.5	67.3
3	8.26 304	695	8.26 312	696	1.73 688	9 99 993	57	.2	143.4	139.0	134.6
4	8.26 988	684	8.26 996	684	1.73 004	9.99 992	56	.3	215.1	208.5	201.9
5	8.27 661	673	8.27 669	673	1.72 331	9.99 992	55	.4	286.8	278.0	269.2
6	8.28 324	663	8.28 332	663	1.71 668	9.99 992	54	.5	358.5	347.5	336.5
7	8.28 977	653	8.28 986	654	1.71 014	9.99 992	53				
8	8.29 621	644	8.29 629	643	1.70 371	9.99 992	52		633	634	616
9	8.30 255	634	8.30 263	634	1.69 737	9.99 991	51	.1	65.3	63.4	61.6
10	8.30 879	624	8.30 888	625	1.69 112	9.99 991	50	.2	130.6	126.8	123.2
11	8.31 495	616	8.31 505	617	1.68 495	9.99 991	49	.3	195.9	190.2	184.8
12	8.32 103	608	8.32 112	607	1.67 888	9.99 990	48	.4	261.2	253.6	246.4
13	8.32 702	599	8.32 711	599	1.67 289	9.99 990	47	.5	326.5	317.0	308.0
14	8.33 292	590	8.33 302	591	1.66 698	9.99 990	46				
15	8.33 875	583	8.33 886	584	1.66 114	9.99 990	45	′	599	583	568
16	8.34 450	575	8.34 461	575	1.65 539	9.99 989	44	.1	59.9	58.3	56.8
17	8.35 018	568	8.35 029	568	1.64 971	9.99 989	43	.2	119.8	116.6	113.6
18	8.35 578	560	8.35 590	561	1.64 410	9.99 989	42	.3	179.7	174.9	170.4
19	8.36 131	553	8.36 143	553	1.63 857	9.99 989	41	.4	239.6	233.2	227.2
20	8.36 678	547	8.36 689	546	1.63 311	9.99 988	40	.5	299.5	291.5	284.0
21	8.37 217	539	8.37 229	540	1.62 771	9.99 988	39				
22	8.37 750	533	8.37 762	533	1.62 238	9.99 988	38		553	539	526
23	8.38 276	526	8.38 289	527	1.61 711	9.99 987	37	.1	55.3	53.9	52.6
24	8.38 796	520	8.38 809	520	1.61 191	9.99 987	36	.2	110.6	107.8	105.2
25	8.39 310	514	8.39 323	514	1.60 677	9.99 987	35	.3	165.9	161.7	157.8
26	8.39 818	508	8.39 832	509	1.60 168	9.99 986	34	.4	221.2	215.6	210.4
27	8.40 320	502	8.40 334	502	1.59 666	9.99 986	33	.5	276.5	269.5	263.0
28	8.40 816	496	8.40 830	496	1.59 170	9.99 986	32				
29	8.41 307	491	8.41 321	491	1.58 679	9.99 985	31		514	502	490
30	8.41 792	485	8.41 807	486	1.58 193	9.99 985	30	.1	51.4	50.2	49
31	8.42 272	480	8.42 287	480	1.57 713	9.99 985	29	.2	102.8	100.4	98
32	8.42 746	474	8.42 762	475	1.57 238	9.99 984	28	.3	154.2	150.6	147
33	8.43 216	470	8.43 232	470	1.56 768	9.99 984	27	.4	205.6	200.8	196
34	8.43 680	464	8.43 696	464	1.56 304	2.99 984	26	.5	257.0	251.0	245
35	8.44 139	459	8.44 156	460	1.55 844	9.99 983	25		480	470	460
36	8.44 594	455	8.44 611	455	1.55 389	9.99 983	24	.1	48	47	46
37	8.45 044	450	8.45 061	450	1.54 939	9.99 983	23	.2	96	94	92
38	8.45 489	445	8.45 507	446	1.54 493	9.99 982	22	.3	144	141	138
39	8.45 930	441	8.45 948	441	1.54 052	9.99 982	21	.4	192	188	184
40	8.46 366	436	8.46 385	437	1 53 615	9.99 982	20	.5	240	235	230
41	8.46 799	433	8.46 817	432	1.53 183	9.99 981	19				
42	8.47 226	427	8.47 245	428	1.52 755	9.99 981	18		450	440	430
43	8.47 650	424	8.47 669	424	1.52 331	9.99 981	17	.1	45	44	43
44	8.48 069	419	8.48 089	420	1.51 911	9.99 980	16	.2	90	88	86
45	8.48 485	416	8.48 505	416	1.51 495	9.99 980	15	.3	135	132	129
46	8.48 896	411	8.48 917	412	1.51 083	9.99 979	14	.4	180	176	172
47	8.49 304	408	8.49 325	408	1.50 675	9.99 979	13	.5	225	220	215
48	8.49 708	404	8.49 729	404	1.50 271	9.99 979	12				
49	8.50 108	400	8.50 130	401	1.49 870	9.99 978	11		420	410	400
50	8.50 504	396	8.50 527	397	1 49 473	9.99 978	10	.1	42	41	40
51	8.50 897	393	8.50 920	393	1 49 080	9.99 977	9	.2	84	82	80
52	8.51 287	390	8.51 310	390	1 48 690	9.99 977	8	.3	126	123	120
53	8.51 673	386	8.51 696	386	1 48 304	9.99 977	7	.4	168	164	160
54	8.52 055	382	8.52 079	383	1 47 921	9.99 976	6	.5	210	205	200
55	8.52 434	379	8.52 459	380	1 47 541	9.99 976	5		390	380	370
56	8.52 810	376	8.52 835	376	1 47 165	9.99 975	4	.1	39	38	37
57	8.53 183	373	8.53 208	373	1 46 792	9.99 975	3	.2	78	76	74
58	8.53 552	369	8.53 578	370	1 46 422	9.99 974	2	.3	117	114	111
59	8.53 919	367	8.53 945	367	1.46 055	9.99 974	1	.4	156	152	148
60	8.54 282	363	8.54 308	363	1.45 692	9.99 974	0	.5	195	190	185
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.		

2°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.		
0	8 54 282		8.54 308		1.45 692	9.99 974	60			
1	8.54 642	360	8.54 669	361	1.45 331	9.99 973	59	381	350	340
2	8.54 999	357	8.55 027	358	1.44 973	9.99 973	58	.1 36	35	34
3	8.55 354	355	8.55 382	355	1.44 618	9.99 972	57	.2 72	70	68
4	8.55 705	351	8.55 734	352	1.44 266	9.99 972	56	.3 108	105	102
5	8.56 054	349	8.56 083	349	1.43 917	9.99 971	55	.4 144	140	136
6	8.56 400	346	8.56 429	346	1.43 571	9.99 971	54	.5 180	175	170
7	8.56 743	343	8.56 773	344	1.43 227	9.99 970	53	.6 216	210	204
8	8.57 084	341	8.57 114	341	1.42 886	9.99 970	52	.7 252	245	238
9	8.57 421	337	8.57 452	338	1.42 548	9.99 969	51	.8 288	280	272
10	8.57 757	336	8.57 788	336	1.42 212	9.99 969	50	.9 324	315	306
11	8.58 089	332	8.58 121	333	1.41 879	9.99 968	49	330	320	310
12	8.58 419	330	8 58 451	330	1.41 549	9.99 968	48	.1 33	32	31
13	8.58 747	328	8.58 779	328	1.41 221	9.99 967	47	.2 66	64	62
14	8.59 072	325	8.59 105	326	1.40 895	9.99 967	46	.3 99	96	93
15	8.59 395	323	8.59 428	323	1.40 572	9.99 967	45	.4 132	128	124
16	8.59 715	320	8.59 749	321	1.40 251	9.99 966	44	.5 165	160	155
17	8.60 033	318	8.60 068	319	1.39 932	9.99 966	43	.6 198	192	186
18	8.60 349	316	8.60 384	316	1.39 616	9.99 965	42	.7 231	224	217
19	8.60 662	313	8.60 698	314	1.39 302	9.99 964	41	.8 264	256	248
20	8.60 973	311	8.61 009	311	1.38 991	9.99 964	40	.9 297	288	279
21	8.61 282	309	8.61 319	310	1.38 681	9.99 963	39	300	290	285
22	8.61 589	307	8.61 626	307	1.38 374	9.99 963	38	.1 30	29	28.5
23	8.61 894	305	8.61 931	305	1.38 069	9.99 962	37	.2 60	58	57.0
24	8.62 196	302	8.62 234	303	1.37 766	9.99 962	36	.3 90	87	85.5
25	8.62 497	301	8.62 535	301	1.37 465	9.99 961	35	.4 120	116	114.0
26	8.62 795	298	8.62 834	299	1.37 166	9.99 961	34	.5 150	145	142.5
27	8.63 091	296	8.63 131	297	1.36 869	9.99 960	33	.6 180	174	171.0
28	8.63 385	294	8.63 426	295	1.36 574	9.99 960	32	.7 210	203	199.5
29	8.63 678	293	8.63 718	292	1.36 282	9.99 959	31	.8 240	232	228.0
30	8.63 968	290	8.64 009	291	1.35 991	9.99 959	30	.9 270	261	256.5
31	8.64 256	288	8.64 298	289	1.35 702	9.99 958	29	280	275	270
32	8.64 543	287	8.64 585	287	1.35 415	9.99 958	28	.1 28.0	27.5	27.0
33	8.64 827	284	8.64 870	285	1.35 130	9.99 957	27	.2 56.0	55.0	54.0
34	8.65 110	283	8.65 154	284	1.34 846	9.99 956	26	.3 84.0	82.5	81.0
35	8.65 391	281	8.65 435	281	1.34 565	9.99 956	25	.4 112.0	110.0	108.0
36	8.65 670	279	8.65 715	280	1.34 285	9.99 955	24	.5 140.0	137.5	135.0
37	8.65 947	277	8.65 993	278	1.34 007	9.99 955	23	.6 168.0	165.0	162.0
38	8.66 223	276	8.66 269	276	1.33 731	9 99 954	22	.7 196.0	192.5	189.0
39	8.66 497	274	8.66 543	274	1.33 457	9.99 954	21	.8 224.0	220.0	216.0
40	8.66 769	272	8.66 816	273	1.33 184	9.99 953	20	.9 252.0	247.5	243.0
41	8.67 039	270	8.67 087	271	1.32 913	9.99 952	19	265	260	255
42	8.67 308	269	8.67 356	269	1.32 644	9.99 952	18	.1 26.5	26.0	25.5
43	8.67 575	267	8.67 624	268	1.32 376	9.99 951	17	.2 53.0	52.0	51.0
44	8.67 841	266	8.67 890	266	1.32 110	9.99 951	16	.3 79.5	78.0	76.5
45	8.68 104	263	8.68 154	264	1.31 846	9.99 950	15	.4 106.0	104.0	102.0
46	8.68 367	263	8.68 417	263	1.31 583	9.99 949	14	.5 132.5	130.0	127.5
47	8.68 627	260	8.68 678	261	1.31 322	9.99 949	13	.6 159.0	156.0	153.0
48	8.68 886	259	8.68 938	260	1.31 062	9.99 948	12	.7 185.5	182.0	178.5
49	8.69 144	258	8.69 196	258	1.30 804	9.99 948	11	.8 212.0	208.0	204.0
50	8.69 400	256	8.69 453	257	1.30 547	9.99 947	10	.9 238.5	234.0	229.5
51	8.69 654	254	8.69 708	255	1.30 292	9.99 946	9	250	245	240
52	8.69 907	253	8.69 962	254	1.30 038	9.99 946	8	.1 25.0	24.5	24.0
53	8.70 159	252	8.70 214	252	1.29 786	9.99 945	7	.2 50.0	49.0	48.0
54	8.70 409	250	8.70 465	251	1.29 535	9.99 944	6	.3 75.0	73.5	72.0
55	8.70 658	249	8.70 714	249	1.29 286	9.99 944	5	.4 100.0	98.0	96.0
56	8.70 905	247	8.70 962	248	1.29 038	9.99 943	4	.5 125.0	122.5	120.0
57	8.71 151	246	8.71 208	246	1.28 792	9.99 942	3	.6 150.0	147.0	144.0
58	8.71 395	244	8.71 453	245	1.28 547	9.99 942	2	.7 175.0	171.5	168.0
59	8.71 638	243	8.71 697	244	1.28 303	9.99 941	1	.8 200.0	196.0	192.0
60	8.71 880	242	8.71 940	243	1.28 060	9.99 940	0	.9 225.0	220.5	216.0
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.		

3°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.			
0	8.71 880	240	8.71 940	241	1.28 060	9.99 940	60				
1	8.72 120	239	8.72 181	239	1.27 819	9.99 940	59		238	234	229
2	8.72 359	238	8.72 420	239	1.27 580	9.99 939	58	.1	23.8	23.4	22.9
3	8.72 597	237	8.72 659	237	1.27 341	9.99 938	57	.2	47.6	46.8	45.8
4	8.72 834	235	8.72 896	236	1.27 104	9.99 938	56	.3	71.4	70.2	68.7
5	8.73 069	234	8.73 132	234	1.26 868	9.99 937	55	.4	95.2	93.6	91.6
6	8.73 303	232	8.73 366	234	1.26 634	9.99 936	54	.5	119.0	117.0	114.5
7	8.73 535	232	8.73 600	232	1.26 400	9.99 936	53	.6	142.8	140.4	137.4
8	8.73 767	230	8.73 832	231	1.26 168	9.99 935	52	.7	166.6	163.8	160.3
9	8.73 997	229	8.74 063	229	1.25 937	9.99 934	51	.8	190.4	187.2	183.2
10	8.74 226	228	8.74 292	229	1.25 708	9.99 934	50	.9	214.2	210.6	206.1
11	8.74 454	226	8.74 521	227	1.25 479	9.99 933	49		225	220	216
12	8.74 680	226	8.74 748	226	1.25 252	9.99 932	48	.1	22.5	22.0	21.6
13	8.74 906	224	8.74 974	225	1.25 026	9.99 932	47	.2	45.0	44.0	43.2
14	8.75 130	223	8.75 199	224	1.24 801	9.99 931	46	.3	67.5	66.0	64.8
15	8.75 353	222	8.75 423	222	1.24 577	9.99 930	45	.4	90.0	88.0	86.4
16	8.75 575	220	8.75 645	222	1.24 355	9.99 929	44	.5	112.5	110.0	108.0
17	8.75 795	220	8.75 867	220	1.24 133	9.99 929	43	.6	135.0	132.0	129.6
18	8.76 015	219	8.76 087	219	1.23 913	9.99 928	42	.7	157.5	154.0	151.2
19	8.76 234	217	8.76 306	219	1.23 694	9.99 927	41	.8	180.0	176.0	172.8
20	8.76 451	216	8.76 525	217	1.23 475	9.99 926	40	.9	202.5	198.0	194.4
21	8.76 667	216	8.76 742	216	1.23 258	9.99 926	39		212	208	204
22	8.76 883	214	8.76 958	215	1.23 042	9.99 925	38	.1	21.2	20.8	20.4
23	8.77 097	213	8.77 173	214	1.22 827	9.99 924	37	.2	42.4	41.6	40.8
24	8.77 310	212	8.77 387	213	1.22 613	9.99 923	36	.3	63.6	62.4	61.2
25	8.77 522	211	8.77 600	211	1.22 400	9.99 923	35	.4	84.8	83.2	81.6
26	8.77 733	210	8.77 811	211	1.22 189	9.99 922	34	.5	106.0	104.0	102.0
27	8.77 943	209	8.78 022	210	1.21 978	9.99 921	33	.6	127.2	124.8	122.4
28	8.78 152	208	8.78 232	209	1.21 768	9.99 920	32	.7	148.4	145.6	142.8
29	8.78 360	208	8.78 441	208	1.21 559	9.99 920	31	.8	169.6	166.4	163.2
30	8.78 568	206	8.78 649	208	1.21 351	9.99 919	30	.9	190.8	187.2	183.6
31	8.78 774	205	8.78 855	206	1 21 145	9.99 918	29		201	197	193
32	8.78 979	204	8.79 061	206	1.20 939	9.99 917	28	.1	20.1	19.7	19.3
33	8.79 183	203	8.79 266	205	1.20 734	9 99 917	27	.2	40.2	39.4	38.6
34	8.79 386	202	8.79 470	204	1.20 530	9.99 916	26	.3	60.3	59.1	57.9
35	8.79 588	201	8.79 673	203	1.20 327	9.99 915	25	.4	80.4	78.8	77.2
36	8.79 789	201	8.79 875	202	1.20 125	9.99 914	24	.5	100.5	98.5	96.5
37	8.79 990	199	8.80 076	201	1.19 924	9.99 913	23	.6	120.6	118.2	115.8
38	8.80 189	199	8.80 277	201	1.19 723	9.99 913	22	.7	140.7	137.9	135.1
39	8.80 388	197	8.80 476	199	1.19 524	9.99 912	21	.8	160.8	157.6	154.4
40	8.80 585	197	8.80 674	198	1.19 326	9.99 911	20	.9	180.9	177.3	173.7
41	8.80 782	196	8.80 872	198	1.19 128	9.99 910	19		189	185	181
42	8.80 978	195	8.81 068	196	1.18 932	9.99 909	18	.1	18.9	18.5	18.1
43	8.81 173	194	8.81 264	196	1.18 736	9.99 909	17	.2	37.8	37.0	36.2
44	8.81 367	193	8.81 459	195	1.18 541	9.99 908	16	.3	56.7	55.5	54.3
45	8.81 560	192	8.81 653	194	1.18 347	9.99 907	15	.4	75.6	74.0	72.4
46	8.81 752	192	8.81 846	193	1.18 154	9.99 906	14	.5	94.5	92.5	90.5
47	8.81 944	190	8.82 038	192	1.17 962	9.99 905	13	.6	113.4	111.0	108.6
48	8.82 134	190	8.82 230	192	1.17 770	9.99 904	12	.7	132.3	129.5	126.7
49	8.82 324	189	8.82 420	190	1.17 580	9.99 904	11	.8	151.2	148.0	144.8
50	8.82 513	188	8.82 610	190	1.17 390	9.99 903	10	.9	170.1	166.5	162.9
51	8.82 701	187	8.82 799	189	1.17 201	9.99 902	9		4	3	2	1
52	8.82 888	187	8.82 987	188	1.17 013	9.99 901	8	.1	0.4	0.3	0.2	0.1
53	8.83 075	186	8.83 175	188	1.16 825	9.99 900	7	.2	0.8	0.6	0.4	0.2
54	8.83 261	185	8.83 361	186	1.16 639	9.99 899	6	.3	1.2	0.9	0.6	0.3
55	8.83 446	184	8.83 547	185	1.16 453	9.99 898	5	.4	1.6	1.2	0.8	0.4
56	8.83 630	183	8.83 732	184	1.16 268	9.99 898	4	.5	2.0	1.5	1.0	0.5
57	8.83 813	183	8.83 916	184	1.16 084	9.99 897	3	.6	2.4	1.8	1.2	0.6
58	8.83 996	181	8.84 100	184	1.15 900	9.99 896	2	.7	2.8	2.1	1.4	0.7
59	8.84 177	181	8.84 282	182	1.15 718	9.99 895	1	.8	3.2	2.4	1.6	0.8
60	8.84 358		8.84 464	182	1.15 536	9.99 894	0	.9	3.6	2.7	1.8	0.9
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.			

86°

4°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.			
0	8.84 358	181	8.84 464	182	1.15 536	9.99 894	60				
1	8.84 539	179	8.84 646	180	1.15 354	9.99 893	59		181	179	177
2	8.84 718	179	8.84 826	180	1.15 174	9.99 892	58	.1	18.1	17.9	17.7
3	8.84 897	178	8.85 006	179	1.14 994	9.99 891	57	.2	36.2	35.8	35.4
4	8.85 075	177	8.85 185	178	1.14 815	9.99 891	56	.3	54.3	53.7	53.1
5	8.85 252	177	8.85 363	177	1.14 637	9.99 890	55	.4	72.4	71.6	70.8
6	8.85 429	176	8.85 540	177	1.14 460	9.99 889	54	.5	90.5	89.5	88.5
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53	.6	108.6	107.4	106.2
8	8.85 780	175	8.85 893	176	1.14 107	9.99 887	52	.7	126.7	125.3	123.9
9	8.85 955	173	8.86 069	174	1.13 931	9.99 886	51	.8	144.8	143.2	141.6
10	8.86 128	173	8.86 243	174	1.13 757	9.99 885	50	.9	162.9	161.1	159.3
11	8.86 301	173	8.86 417	174	1.13 583	9.99 884	49		175	173	171
12	8.86 474	173	8.86 591	174	1.13 409	9.99 883	48	.1	17.5	17.3	17.1
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47	.2	35.0	34.6	34.2
14	8.86 816	171	8.86 935	172	1.13 065	9.99 881	46	.3	52.5	51.9	51.3
15	8.86 987	171	8.87 106	171	1.12 894	9.99 880	45	.4	70.0	69.2	68.4
16	8.87 156	169	8.87 277	171	1.12 723	9.99 879	44	.5	87.5	86.5	85.5
17	8.87 325	169	8.87 447	170	1.12 553	9.99 879	43	.6	105.0	103.8	102.6
18	8.87 494	169	8.87 616	169	1.12 384	9.99 878	42	.7	122.5	121.1	119.7
19	8.87 661	167	8.87 785	169	1.12 215	9.99 877	41	.8	140.0	138.4	136.8
20	8.87 829	168	8.87 953	168	1.12 047	9.99 876	40	.9	157.5	155.7	153.9
21	8.87 995	166	8.88 120	167	1.11 880	9.99 875	39		168	166	164
22	8.88 161	166	8.88 287	167	1.11 713	9.99 874	38	.1	16.8	16.6	16.4
23	8.88 326	165	8.88 453	166	1.11 547	9.99 873	37	.2	33.6	33.2	32.8
24	8.88 490	164	8.88 618	165	1.11 382	9.99 872	36	.3	50.4	49.8	49.2
25	8.88 654	164	8.88 783	165	1.11 217	9.99 871	35	.4	67.2	66.4	65.6
26	8.88 817	163	8.88 948	165	1.11 052	9.99 870	34	.5	84.0	83.0	82.0
27	8.88 980	163	8.89 111	163	1.10 889	9.99 869	33	.6	100.8	99.6	98.4
28	8.89 142	162	8.89 274	163	1.10 726	9.99 868	32	.7	117.6	116.2	114.8
29	8.89 304	162	8.89 437	163	1.10 563	9.99 867	31	.8	134.4	132.8	131.2
30	8.89 464	160	8.89 598	161	1.10 402	9.99 866	30	.9	151.2	149.4	147.6
31	8.89 625	161	8.89 760	162	1.10 240	9.99 865	29		162	159	157
32	8.89 784	159	8.89 920	160	1.10 080	9.99 864	28	.1	16.2	15.9	15.7
33	8.89 943	159	8.90 080	160	1.09 920	9.99 863	27	.2	32.4	31.8	31.4
34	8.90 102	159	8.90 240	160	1.09 760	9.99 862	26	.3	48.6	47.7	47.1
35	8.90 260	158	8.90 399	159	1.09 601	9.99 861	25	.4	64.8	63.6	62.8
36	8.90 417	157	8.90 557	158	1.09 443	9.99 860	24	.5	81.0	79.5	78.5
37	8.90 574	157	8.90 715	158	1.09 285	9.99 859	23	.6	97.2	95.4	94.2
38	8.90 730	156	8.90 872	157	1.09 128	9.99 858	22	.7	113.4	111.3	109.9
39	8.90 885	155	8.91 029	157	1.08 971	9.99 857	21	.8	129.6	127.2	125.6
40	8.91 040	155	8.91 185	156	1.08 815	9.99 856	20	.9	145.8	143.1	141.3
41	8.91 195	155	8.91 340	155	1.08 660	9.99 855	19		155	153	151
42	8.91 349	154	8.91 495	155	1.08 505	9.99 854	18	.1	15.5	15.3	15.1
43	8.91 502	153	8.91 650	155	1.08 350	9.99 853	17	.2	31.0	30.6	30.2
44	8.91 655	153	8.91 803	153	1.08 197	9.99 852	16	.3	46.5	45.9	45.3
45	8.91 807	152	8.91 957	154	1.08 043	9.99 851	15	.4	62.0	61.2	60.4
46	8.91 959	152	8.92 110	153	1.07 890	9.99 850	14	.5	77.5	76.5	75.5
47	8.92 110	151	8.92 262	152	1.07 738	9.99 848	13	.6	93.0	91.8	90.6
48	8.92 261	151	8.92 414	152	1.07 586	9.99 847	12	.7	108.5	107.1	105.7
49	8.92 411	150	8.92 565	151	1.07 435	9.99 846	11	.8	124.0	122.4	120.8
50	8.92 561	150	8.92 716	151	1.07 284	9.99 845	10	.9	139.5	137.7	135.9
51	8.92 710	149	8.92 866	150	1.07 134	9.99 844	9		149	147	1
52	8.92 859	149	8.93 016	150	1.06 984	9.99 843	8	.1	14.9	14.7	0.1
53	8.93 007	148	8.93 165	149	1.06 835	9.99 842	7	.2	29.8	29.4	0.2
54	8.93 154	147	8.93 313	148	1.06 687	9.99 841	6	.3	44.7	44.1	0.3
55	8.93 301	147	8.93 462	149	1.06 538	9.99 840	5	.4	59.6	58.8	0.4
56	8.93 448	147	8.93 609	147	1.06 391	9.99 839	4	.5	74.5	73.5	0.5
57	8.93 594	146	8.93 756	147	1.06 244	9.99 838	3	.6	89.4	88.2	0.6
58	8.93 740	146	8.93 903	147	1.06 097	9.99 837	2	.7	104.3	102.9	0.7
59	8.93 885	145	8.94 049	146	1.05 951	9.99 836	1	.8	119.2	117.6	0.8
60	8.94 030	145	8.94 195	146	1.05 805	9.99 834	0	.9	134.1	132.3	0.9
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.			

85°

5°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.		
0	8.94 030	144	8.94 195	145	1.05 805	9.99 834	60			
1	8.94 174	143	8.94 340	145	1.05 660	9.99 833	59	145	143	141
2	8.94 317	144	8.94 485	145	1.05 515	9.99 832	58	.1 14.5	14.3	14.1
3	8.94 461	142	8.94 630	143	1.05 370	9.99 831	57	.2 29.0	28.6	28.2
4	8.94 603	143	8.94 773	144	1.05 227	9.99 830	56	.3 43.5	42.9	42.3
5	8.94 746	141	8.94 917	143	1.05 083	9.99 829	55	.4 58.0	57.2	56.4
6	8.94 887	142	8.95 060	142	1.04 940	9.99 828	54	.5 72.5	71.5	70.5
7	8.95 029	141	8.95 202	142	1.04 798	9.99 827	53	.6 87.0	85.8	84.6
8	8.95 170	140	8.95 344	142	1.04 656	9.99 825	52	.7 101.5	100.1	98.7
9	8.95 310	140	8.95 486	141	1.04 514	9.99 824	51	.8 116.0	114.4	112.8
10	8.95 450	139	8.95 627	140	1.04 373	9.99 823	50	.9 130.5	128.7	126.9
11	8.95 589	139	8.95 767	141	1.04 233	9.99 822	49	139	138	136
12	8.95 728	139	8.95 908	139	1.04 092	9 99 821	48	.1 13.9	13.8	13.6
13	8.95 867	139	8.96 047	140	1.03 953	9.99 820	47	.2 27.8	27.6	27.2
14	8.96 005	138	8.96 187	138	1.03 813	9.99 819	46	.3 41.7	41.4	40.8
15	8.96 143	137	8.96 325	139	1.03 675	9.99 817	45	.4 55.6	55.2	54.4
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44	.5 69.5	69.0	68.0
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43	.6 83.4	82.8	81.6
18	8.96 553	136	8.96 739	138	1.03 261	9.99 814	42	.7 97.3	96.6	95.2
19	8.96 689	136	8.96 877	136	1.03 123	9.99 813	41	.8 111.2	110.4	108.8
20	8.96 825	135	8.97 013	137	1.02 987	9.99 812	40	.9 125.1	124.2	122.4
21	8.96 960	135	8.97 150	135	1.02 850	9.99 810	39	135	133	131
22	8.97 095	134	8.97 285	136	1.02 715	9.99 809	38	.1 13.5	13.3	13.1
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37	.2 27.0	26.6	26.2
24	8.97 363	133	8.97 556	135	1.02 444	9.99 807	36	.3 40.5	39.9	39.3
25	8.97 496	133	8.97 691	134	1.02 309	9.99 806	35	.4 54.0	53.2	52.4
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34	.5 67.5	66.5	65.5
27	8.97 762	132	8.97 959	133	1.02 041	9.99 803	33	.6 81.0	79.8	78.6
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32	.7 94.5	93.1	91.7
29	8.98 026	131	8.98 225	133	1.01 775	9.99 801	31	.8 108.0	106.4	104.8
30	8.98 157	131	8.98 358	133	1.01 642	9.99 800	30	.9 121.5	119.7	117.9
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29	129	128	126
32	8.98 419	131	8.98 622	132	1.01 378	9.99 797	28	.1 12.9	12.8	12.6
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27	.2 25.8	25.6	25.2
34	8.98 679	130	8.98 884	131	1.01 116	9.99 795	26	.3 38.7	38.4	37.8
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25	.4 51.6	51.2	50.4
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24	.5 64.5	64.0	63.0
37	8.99 066	129	8.99 275	130	1.00 725	9.99 791	23	.6 77.4	76.8	75.6
38	8.99 194	128	8.99 405	130	1.00 595	9.99 790	22	.7 90.3	89.6	88.2
39	8.99 322	128	8.99 534	129	1.00 466	9.99 788	21	.8 103.2	102.4	100.8
40	8.99 450	127	8.99 662	128	1.00 338	9.99 787	20	.9 116.1	115.2	113.4
41	8.99 577	127	8.99 791	129	1.00 209	9.99 786	19	125	123	122
42	8.99 704	126	8.99 919	128	1.00 081	9.99 785	18	.1 12.5	12.3	12.2
43	8.99 830	126	9.00 046	127	0.99 954	9.99 783	17	.2 25.0	24.6	24.4
44	8.99 956	126	9.00 174	128	0.99 826	9.99 782	16	.3 37.5	36.9	36.6
45	9.00 082	125	9.00 301	127	0.99 699	9.99 781	15	.4 50.0	49.2	48.8
46	9.00 207	125	9.00 427	126	0.99 573	9.99 780	14	.5 62.5	61.5	61.0
47	9.00 332	124	9.00 553	126	0.99 447	9.99 778	13	.6 75.0	73.8	73.2
48	9.00 456	125	9.00 679	126	0.99 321	9.99 777	12	.7 87.5	86.1	85.4
49	9.00 581	123	9.00 805	125	0.99 195	9.99 776	11	.8 100.0	98.4	97.6
50	9.00 704	124	9.00 930	125	0.99 070	9.99 775	10	.9 112.5	110.7	109.8
51	9.00 828	123	9.01 053	124	0.98 945	9.99 773	9	121	120	1
52	9 00 951	123	9.01 179	124	0.98 821	9.99 772	8	.1 12.1	12.0	0.1
53	9.01 074	122	9.01 303	124	0.98 697	9.99 771	7	.2 24.2	24.0	0.2
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6	.3 36.3	36.0	0.3
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5	.4 48.4	48.0	0.4
56	9.01 440	121	9.01 673	123	0.98 327	9.99 767	4	.5 60.5	60.0	0.5
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3	.6 72.6	72.0	0.6
58	9.01 682	121	9.01 918	122	0.98 082	9.99 764	2	.7 84.7	84.0	0.7
59	9.01 803	120	9.02 040	122	0.97 960	9.99 763	1	.8 96.8	96.0	0.8
60	9.01 923		9.02 162		0.97 838	9.99 761	0	.9 108.9	108.0	0.9
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.		

6°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.		
0	9.01 923		9.02 162		0.97 838	9.99 761	60			
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59	121	120	119
2	9.02 163	120	9.02 404	121	0.97 596	9.99 759	58	.1 12.1	12.0	11.9
3	9.02 283	120	9.02 525	121	0.97 475	9.99 757	57	.2 24.2	24.0	23.8
4	9.02 402	119	9.02 645	120	0.97 355	9.99 756	56	.3 36.3	36.0	35.7
5	9.02 520	118	9.02 766	121	0.97 234	9.99 755	55	.4 48.4	48.0	47.6
6	9.02 639	119	9 02 885	119	0.97 115	9.99 753	54	.5 60.5	60.0	59.5
7	9.02 757	118	9.03 005	120	0.96 995	9.99 752	53	.6 72.6	72.0	71.4
8	9.02 874	117	9 03 124	119	0.96 876	9.99 751	52	.7 84.7	84.0	83.3
9	9.02 992	118	9.03 242	118	0.96 758	9.99 749	51	.8 96.8	96.0	95.2
10	9.03 109	117	9.03 361	119	0.96 639	9.99 748	50	.9 108.9	108.0	107.1
11	9.03 226	117	9.03 479	118	0.96 521	9.99 747	49		118	117	116
12	9.03 342	116	9.03 597	118	0.96 403	9.99 745	48	.1 11.8	11.7	11.6
13	9.03 458	116	9.03 714	117	0.96 286	9.99 744	47	.2 23.6	23.4	23.2
14	9.03 574	116	9.03 832	118	0.96 168	9.99 742	46	.3 35.4	35.1	34.8
15	9.03 690	116	9.03 948	116	0.96 052	9.99 741	45	.4 47.2	46.8	46.4
16	9.03 805	115	9.04 065	117	0.95 935	9.99 740	44	.5 59.0	58.5	58.0
17	9.03 920	115	9.04 181	116	0.95 819	9.99 738	43	.6 70.8	70.2	69.6
18	9.04 034	114	9.04 297	116	0.95 703	9.99 737	42	.7 82.6	81.9	81.2
19	9.04 149	115	9.04 413	116	0.95 587	9.99 736	41	.8 94.4	93.6	92.8
20	9.04 262	113	9.04 528	115	0.95 472	9.99 734	40	.9 106.2	105.3	104.4
21	9.04 376	114	9.04 643	115	0.95 357	9.99 733	39		115	114	113
22	9.04 490	114	9.04 758	115	0.95 242	9.99 731	38	.1 11.5	11.4	11.3
23	9.04 603	113	9.04 873	115	0.95 127	9.99 730	37	.2 23.0	22.8	22.6
24	9.04 715	112	9.04 987	114	0.95 013	9.99 728	36	.3 34.5	34.2	33.9
25	9.04 828	113	9.05 101	114	0.94 899	9.99 727	35	.4 46.0	45.6	45.2
26	9.04 940	112	9.05 214	113	0.94 786	9.99 726	34	.5 57.5	57.0	56.5
27	9.05 052	112	9.05 328	114	0.94 672	9.99 724	33	.6 69.0	68.4	67.8
28	9.05 164	112	9.05 441	113	0.94 559	9.99 723	32	.7 80.5	79.8	79.1
29	9.05 275	111	9.05 553	112	0.94 447	9.99 721	31	.8 92.0	91.2	90 4
30	9.05 386	111	9.05 666	113	0.94 334	9.99 720	30	.9 103.5	102.6	101.7
31	9.05 497	111	9.05 778	112	0.94 222	9.99 718	29		112	111	110
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28	.1 11.2	11.1	11.0
33	9.05 717	110	9.06 002	112	0.93 998	9.99 716	27	.2 22.4	22.2	22.0
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26	.3 33.6	33.3	33.0
35	9.05 937	110	9.06 224	111	0.93 776	9.99 713	25	.4 44.8	44.4	44.0
36	9.06 046	109	9.06 335	111	0.93 665	9.99 711	24	.5 56.0	55.5	55.0
37	9.06 155	109	9.06 445	110	0.93 555	9.99 710	23	.6 67.2	66.6	66.0
38	9.06 264	109	9.06 556	111	0.93 444	9.99 708	22	.7 78.4	77.7	77.0
39	9.06 372	108	9.06 666	110	0.93 334	9.99 707	21	.8 89.6	88.8	88.0
40	9.06 481	109	9.06 775	109	0.93 225	9.99 705	20	.9 100.8	99.9	99.0
41	9.06 589	108	9.06 885	110	0.93 115	9.99 704	19		109	108	107
42	9.06 696	107	9.06 994	109	0.93 006	9.99 702	18	.1 10.9	10.8	10.7
43	9.06 804	108	9.07 103	109	0.92 897	9.99 701	17	.2 21.8	21.6	21.4
44	9.06 911	107	9.07 211	108	0.92 789	9.99 699	16	.3 32.7	32.4	32.1
45	9.07 018	107	9.07 320	109	0.92 680	9.99 698	15	.4 43.6	43.2	42.8
46	9.07 124	106	9.07 428	108	0.92 572	9.99 696	14	.5 54.5	54.0	53.5
47	9.07 231	107	9.07 536	108	0.92 464	9.99 695	13	.6 65.4	64.8	64.2
48	9.07 337	106	9.07 643	107	0.92 357	9.99 693	12	.7 76.3	75.6	74.9
49	9.07 442	105	9.07 751	108	0.92 249	9.99 692	11	.8 87.2	86.4	85.6
50	9.07 548	106	9.07 858	107	0.92 142	9.99 690	10	.9 98.1	97.2	96.3
51	9.07 653	105	9.07 964	106	0.92 036	9.99 689	9		106	105	104
52	9.07 758	105	9.08 071	107	0.91 929	9.99 687	8	.1 10.6	10.5	10.4
53	9.07 863	105	9.08 177	106	0.91 823	9.99 686	7	.2 21.2	21.0	20.8
54	9.07 968	105	9.08 283	106	0.91 717	9.99 684	6	.3 31.8	31.5	31.2
55	9.08 072	104	9.08 389	106	0.91 611	9.99 683	5	.4 42.4	42.0	41.6
56	9.08 176	104	9.08 495	106	0.91 505	9.99 681	4	.5 53.0	52.5	52.0
57	9.08 280	104	9.08 600	105	0.91 400	9.99 680	3	.6 63.6	63.0	62.4
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2	.7 74.2	73.5	72.8
59	9.08 486	103	9.08 810	105	0.91 190	9.99 677	1	.8 84.8	84.0	83.2
60	9.08 589	103	9.08 914	104	0.91 086	9.99 675	0	.9 95.4	94.5	93.6
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.		

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7°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.
0	9.08 589		9.08 914		0.91 086	9.99 675	60	
1	9.08 692	103	9.09 019	105	0.90 981	9.99 674	59	105 104 103
2	9.08 795	103	9.09 123	104	0.90 877	9.99 672	58	.1 10.5 10.4 10.3
3	9.08 897	102	9.09 227	104	0.90 773	9.99 670	57	.2 21.0 20.8 20.6
4	9.08 999	102	9.09 330	103	0.90 670	9.99 669	56	.3 31.5 31.2 30.9
5	9.09 101	101	9.09 434	104	0.90 566	9.99 667	55	.4 42.0 41.6 41.2
6	9.09 202	102	9.09 537	103	0.90 463	9.99 666	54	.5 52.5 52.0 51.5
7	9.09 304	101	9.09 640	103	0.90 360	9.99 664	53	.6 63.0 62.4 61.8
8	9.09 405	101	9.09 742	102	0.90 258	9.99 663	52	.7 73.5 72.8 72.1
9	9.09 506	100	9.09 845	103	0.90 155	9.99 661	51	.8 84.0 83.2 82.4
10	9.09 606	101	9.09 947	102	0.90 053	9.99 659	50	.9 94.5 93.6 92.7
11	9.09 707	100	9.10 049	101	0.89 951	9.99 658	49	102 101 100
12	9.09 807	100	9.10 150	102	0.89 850	9.99 656	48	.1 10.2 10.1 10.0
13	9.09 907	99	9.10 252	101	0.89 748	9.99 655	47	.2 20.4 20.2 20.0
14	9.10 006	100	9.10 353	101	0.89 647	9.99 653	46	.3 30.6 30.3 30.0
15	9.10 106	99	9.10 454	101	0.89 546	9.99 651	45	.4 40.8 40.4 40.0
16	9.10 205	99	9.10 555	101	0.89 445	9.99 650	44	.5 51.0 50.5 50.0
17	9.10 304	98	9.10 656	100	0.89 344	9.99 648	43	.6 61.2 60.6 60.0
18	9.10 402	99	9.10 756	100	0.89 244	9.99 647	42	.7 71.4 70.7 70.0
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41	.8 81.6 80.8 80.0
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	40	.9 91.8 90.9 90.0
21	9.10 697	98	9.11 056	100	0.88 944	9.99 642	39	99 98
22	9.10 795	98	9.11 155	99	0.88 845	9.99 640	38	.1 9.9 9.8
23	9.10 893	98	9.11 254	99	0.88 746	9.99 638	37	.2 19.8 19.6
24	9.10 990	97	9.11 353	99	0.88 647	9.99 637	36	.3 29.7 29.4
25	9.11 087	97	9.11 452	99	0.88 548	9.99 635	35	.4 39.6 39.2
26	9.11 184	97	9.11 551	99	0.88 449	9.99 633	34	.5 49.5 49.0
27	9.11 281	97	9.11 649	98	0.88 351	9.99 632	33	.6 59.4 58.8
28	9.11 377	96	9.11 747	98	0.88 253	9.99 630	32	.7 69.3 68.6
29	9.11 474	97	9.11 845	98	0.88 155	9.99 629	31	.8 79.2 78.4
30	9.11 570	96	9.11 943	98	0.88 057	9.99 627	30	.9 89.1 88.2
31	9.11 666	96	9.12 040	97	0.87 960	9.99 625	29	97 96 95
32	9.11 761	95	9.12 138	98	0.87 862	9.99 624	28	.1 9.7 9.6 9.5
33	9.11 857	96	9.12 235	97	0.87 765	9.99 622	27	.2 19.4 19.2 19.0
34	9.11 952	95	9.12 332	97	0.87 668	9.99 620	26	.3 29.1 28.8 28.5
35	9.12 047	95	9.12 428	96	0.87 572	9.99 618	25	.4 38.8 38.4 38.0
36	9.12 142	95	9.12 525	97	0.87 475	9.99 617	24	.5 48.5 48.0 47.5
37	9.12 236	94	9.12 621	96	0.87 379	9.99 615	23	.6 58.2 57.6 57.0
38	9.12 331	95	9.12 717	96	0.87 283	9.99 613	22	.7 67.9 67.2 66.5
39	9.12 425	94	9.12 813	96	0.87 187	9.99 612	21	.8 77.6 76.8 76.0
40	9.12 519	94	9.12 909	96	0.87 091	9.99 610	20	.9 87.3 86.4 85.5
41	9.12 612	93	9.13 004	95	0.86 996	9.99 608	19	94 93 92
42	9.12 706	94	9.13 099	95	0.86 901	9.99 607	18	.1 9.4 9.3 9.2
43	9.12 799	93	9.13 194	95	0.86 806	9.99 605	17	.2 18.8 18.6 18.4
44	9.12 892	93	9.13 289	95	0.86 711	9.99 603	16	.3 28.2 27.9 27.6
45	9.12 985	93	9.13 384	95	0.86 616	9.99 601	15	.4 37.6 37.2 36.8
46	9.13 078	93	9.13 478	94	0.86 522	9.99 600	14	.5 47.0 46.5 46.0
47	9.13 171	92	9.13 573	95	0.86 427	9.99 598	13	.6 56.4 55.8 55.2
48	9.13 263	92	9.13 667	94	0.86 333	9.99 596	12	.7 65.8 65.1 64.4
49	9.13 355	92	9.13 761	94	0.86 239	9.99 595	11	.8 75.2 74.4 73.6
50	9.13 447	92	9.13 854	93	0.86 146	9.99 593	10	.9 84.6 83.7 82.8
51	9.13 539	92	9 13 948	94	0.86 052	9.99 591	9	91 90 1
52	9.13 630	91	9.14 041	93	0.85 959	9.99 589	8	.1 9.1 9.0 0.2
53	9.13 722	92	9.14 134	93	0.85 866	9.99 588	7	.2 18.2 18.0 0.4
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6	.3 27.3 27.0 0.6
55	9 13 904	91	9.14 320	93	0.85 680	9.99 584	5	.4 36.4 36.0 0.8
56	9.13 994	90	9.14 412	92	0.85 588	9.99 582	4	.5 45.5 45.0 1.0
57	9.14 085	91	9.14 504	92	0.85 496	9.99 581	3	.6 54.6 54.0 1.2
58	9.14 175	90	9.14 597	93	0.85 403	9.99 579	2	.7 63.7 63.0 1.4
59	9.14 266	91	9.14 688	91	0.85 312	9.99 577	1	.8 72.8 72.0 1.6
60	9.14 356	90	9.14 780	92	0.85 220	9.99 575	0	.9 81.9 81.0 1.8
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.

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8°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.			
0	9.14 356		9.14 780		0.85 220	9.99 575	60				
1	9.14 445	89	9.14 872	92	0.85 128	9.99 574	59		92	91	90
2	9.14 535	90	9.14 963	91	0.85 037	9.99 572	58	.1	9.2	9.1	9.0
3	9.14 624	89	9.15 054	91	0.84 946	9.99 570	57	.2	18.4	18.2	18.0
4	9.14 714	90	9.15 145	91	0.84 855	9.99 568	56	.3	27.6	27.3	27.0
5	9.14 803	89	9.15 236	91	0.84 764	9.99 566	55	.4	36.8	36.4	36.0
6	9.14 891	88	9.15 327	91	0.84 673	9.99 565	54	.5	46.0	45.5	45.0
7	9.14 980	89	9.15 417	90	0.84 583	9.99 563	53	.6	55.2	54.6	54.0
8	9.15 069	89	9.15 508	91	0.84 492	9.99 561	52	.7	64.4	63.7	63.0
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51	.8	73.6	72.8	72.0
10	9.15 245	88	9.15 688	90	0.84 312	9.99 557	50	.9	82.8	81.9	81.0
11	9.15 333	88	9.15 777	89	0.84 223	9.99 556	49			89	88
12	9.15 421	88	9.15 867	90	0.84 133	9.99 554	48		.1	8.9	8.8
13	9.15 508	87	9.15 956	89	0.84 044	9.99 552	47		.2	17.8	17.6
14	9.15 596	88	9.16 046	90	0.83 954	9.99 550	46		.3	26.7	26.4
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	45		.4	35.6	35.2
16	9.15 770	87	9.16 224	89	0.83 776	9.99 546	44		.5	44.5	44.0
17	9.15 857	87	9.16 312	88	0.83 688	9.99 545	43		.6	53.4	52.8
18	9.15 944	87	9.16 401	89	0.83 599	9.99 543	42		.7	62.3	61.6
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41		.8	71.2	70.4
20	9.16 116	86	9.16 577	88	0.83 423	9.99 539	40		.9	80.1	79.2
21	9.16 203	87	9.16 665	88	0.83 335	9.99 537	39			87	86
22	9.16 289	86	9.16 753	88	0.83 247	9.99 535	38		.1	8.7	8.6
23	9.16 374	85	9.16 841	88	0.83 159	9.99 533	37		.2	17.4	17.2
24	9.16 460	86	9.16 928	87	0.83 072	9.99 532	36		.3	26.1	25.8
25	9.16 545	85	9.17 016	88	0.82 984	9.99 530	35		.4	34.8	34.4
26	9.16 631	86	9.17 103	87	0.82 897	9.99 528	34		.5	43.5	43.0
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33		.6	52.2	51.6
28	9.16 801	85	9.17 277	87	0.82 723	9.99 524	32		.7	60.9	60.2
29	9.16 886	85	9.17 363	86	0.82 637	9.99 522	31		.8	69.6	68.8
30	9.16 970	84	9.17 450	87	0.82 550	9.99 520	30		.9	78.3	77.4
31	9.17 055	85	9.17 536	86	0.82 464	9.99 518	29			85	84
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28		.1	8.5	8.4
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27		.2	17.0	16.8
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26		.3	25.5	25.2
35	9.17 391	84	9.17 880	85	0.82 120	9.99 511	25		.4	34.0	33.6
36	9.17 474	83	9.17 965	86	0.82 035	9.99 509	24		.5	42.5	42.0
37	9.17 558	84	9.18 051	85	0.81 949	9.99 507	23		.6	51.0	50.4
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22		.7	59.5	58.8
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21		.8	68.0	67.2
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	20		.9	76.5	75.6
41	9.17 890	83	9.18 391	85	0.81 609	9.99 499	19			83	82
42	9.17 973	83	9.18 475	84	0.81 525	9.99 497	18		1	8.3	8.2
43	9.18 055	82	9.18 560	85	0.81 440	9.99 495	17		.2	16.6	16.4
44	9.18 137	82	9.18 644	84	0.81 356	9.99 494	16		.3	24.9	24.6
45	9.18 220	83	9.18 728	84	0.81 272	9.99 492	15		.4	33.2	32.8
46	9.18 302	82	9.18 812	84	0.81 188	9.99 490	14		.5	41.5	41.0
47	9.18 383	81	9.18 896	84	0.81 104	9.99 488	13		.6	49.8	49.2
48	9.18 465	82	9.18 979	83	0.81 021	9.99 486	12		.7	58.1	57.4
49	9.18 547	82	9.19 063	84	0.80 937	9.99 484	11		.8	66.4	65.6
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	10		.9	74.7	73.8
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9			81	80	2
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8	.1	8.1	8.0	0.2
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7	.2	16.2	16.0	0.4
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6	.3	24.3	24.0	0.6
55	9.19 033	80	9.19 561	83	0.80 439	9.99 472	5	.4	32.4	32.0	0.8
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4	.5	40.5	40.0	1.0
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3	.6	48.6	48.0	1.2
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2	.7	56.7	56.0	1.4
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1	.8	64.8	64.0	1.6
60	9.19 433		9.19 971	82	0.80 029	9.99 462	0	.9	72.9	72.0	1.8
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.			

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9°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.		
0	9.19 433	80	9.19 971	82	0.80 029	9.99 462	60			
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59		82	81	80
2	9.19 592	80	9.20 134	82	0.79 866	9.99 458	58	.1	8.2	8.1	8.0
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57	2	16.4	16.2	16.0
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56	.3	24.6	24.3	24.0
5	9.19 830	79	9.20 378	82	0.79 622	9.99 452	55	.4	32.8	32.4	32.0
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54	.5	41.0	40.5	40.0
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53	.6	49.2	48.6	48.0
8	9.20 067	78	9.20 621	80	0.79 379	9.99 446	52	.7	57.4	56.7	56.0
9	9.20 145	78	9.20 701	81	0.79 299	9.99 444	51	.8	65.6	64.8	64.0
10	9.20 223	79	9.20 782	80	0.79 218	9.99 442	50	.9	73.8	72.9	72.0
11	9.20 302	78	9.20 862	80	0.79 138	9.99 440	49			79	78
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48	.1		7.9	7.8
13	9.20 458	77	9.21 022	80	0.78 978	9.99 436	47	.2		15.8	15.6
14	9.20 535	78	9.21 102	80	0.78 898	9.99 434	46	.3		23.7	23.4
15	9.20 613	78	9.21 182	79	0.78 818	9.99 432	45	.4		31.6	31.2
16	9.20 691	77	9.21 261	80	0.78 739	9.99 429	44	.5		39.5	39.0
17	9.20 768	77	9.21 341	79	0.78 659	9.99 427	43	.6		47.4	46.8
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42	.7		55.3	54.6
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41	.8		63.2	62.4
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	40	.9		71.1	70.2
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39			77	76
22	9.21 153	77	9.21 736	79	0.78 264	9.99 417	38	.1		7.7	7.6
23	9.21 229	76	9.21 814	78	0.78 186	9.99 415	37	.2		15.4	15.2
24	9.21 306	77	9.21 893	79	0.78 107	9.99 413	36	.3		23.1	22.8
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35	.4		30.8	30.4
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34	.5		38.5	38.0
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33	.6		46.2	45.6
28	9.21 610	76	9.22 205	78	0.77 795	9.99 404	32	.7		53.9	53.2
29	9.21 685	75	9.22 283	78	0.77 717	9.99 402	31	.8		61.6	60.8
30	9.21 761	76	9.22 361	78	0.77 639	9.99 400	30	.9		69.3	68.4
31	9.21 836	75	9.22 438	77	0.77 562	9.99 398	29			75	74
32	9.21 912	76	9.22 516	78	0.77 484	9.99 396	28	.1		7.5	7.4
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27	.2		15.0	14.8
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26	.3		22.5	22.2
35	9.22 137	75	9.22 747	77	0.77 253	9.99 390	25	.4		30.0	29.6
36	9.22 211	74	9.22 824	77	0.77 176	9.99 388	24	.5		37.5	37.0
37	9.22 286	75	9.22 901	77	0.77 099	9.99 385	23	.6		45.0	44.4
38	9.22 361	75	9.22 977	76	0.77 023	9.99 383	22	.7		52.5	51.8
39	9.22 435	74	9.23 054	77	0.76 946	9.99 381	21	.8		60.0	59.2
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	20	.9		67.5	66.6
41	9.22 583	74	9.23 206	76	0.76 794	9.99 377	19			73	72
42	9.22 657	74	9.23 283	77	0.76 717	9.99 375	18	.1		7.3	7.2
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17	.2		14.6	14.4
44	9.22 805	74	9.23 435	76	0.76 565	9.99 370	16	.3		21.9	21.6
45	9.22 878	73	9.23 510	75	0.76 490	9.99 368	15	.4		29.2	28.8
46	9.22 952	74	9.23 586	76	0.76 414	9.99 366	14	.5		36.5	36.0
47	9.23 025	73	9.23 661	75	0.76 339	9.99 364	13	.6		43.8	43.2
48	9.23 098	73	9.23 737	76	0.76 263	9.99 362	12	.7		51.1	50.4
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11	.8		58.4	57.6
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	10	.9		65.7	64.8
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9		71	3	2
52	9.23 390	73	9.24 037	75	0.75 963	9.99 353	8	.1	7.1	0.3	0.2
53	9.23 462	72	9.24 112	75	0.75 888	9.99 351	7	.2	14.2	0.6	0.4
54	9.23 535	73	9.24 186	74	0.75 814	9.99 348	6	.3	21.3	0.9	0.6
55	9.23 607	72	9.24 261	75	0.75 739	9.99 346	5	.4	28.4	1.2	0.8
56	9.23 679	72	9.24 335	74	0.75 665	9.99 344	4	.5	35.5	1.5	1.0
57	9.23 752	73	9.24 410	75	0.75 590	9.99 342	3	.6	42.6	1.8	1.2
58	9.23 823	71	9.24 484	74	0.75 516	9.99 340	2	.7	49.7	2.1	1.4
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1	.8	56.8	2.4	1.6
60	9.23 967	72	9.24 632	74	0.75 368	9.99 335	0	.9	63.9	2.7	1.8
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.		

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10°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.		
0	9.23 967	72	9.24 632	74	0.75 368	9.99 335	60			
1	9.24 039	71	9.24 706	73	0.75 294	9.99 333	59		74	73
2	9.24 110	71	9.24 779	74	0.75 221	9.99 331	58	.1	7.4	7.3
3	9.24 181	72	9.24 853	73	0.75 147	9.99 328	57	.2	14.8	14.6
4	9 24 253	71	9.24 926	74	0.75 074	9.99 326	56	.3	22.2	21.9
5	9.24 324	71	9.25 000	73	0.75 000	9.99 324	55	.4	29.6	29.2
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	54	.5	37.0	36.5
7	9.24 466	70	9.25 146	73	0.74 854	9.99 319	53	.6	44.4	43.8
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	52	.7	51.8	51.1
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	51	.8	59.2	58.4
10	9.24 677	71	9.25 365	72	0.74 635	9.99 313	50	.9	66.6	65.7
11	9.24 748	70	9.25 437	73	0.74 563	9.99 310	49		72	71
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	48	.1	7.2	7.1
13	9.24 888	70	9.25 582	73	0.74 418	9.99 306	47	.2	14.4	14.2
14	9.24 958	70	9.25 655	72	0.74 345	9.99 304	46	.3	21.6	21 3
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	45	.4	28.8	28 4
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	44	.5	36.0	35 5
17	9.25 168	70	9.25 871	72	0.74 129	9.99 297	43	.6	43.2	42 6
18	9.25 237	69	9.25 943	72	0.74 057	9.99 294	42	.7	50.4	49.7
19	9.25 307	70	9.26 015	72	0.73 985	9.99 292	41	.8	57.6	56.8
20	9.25 376	69	9.26 086	71	0.73 914	9.99 290	40	.9	64.8	63.9
21	9.25 445	69	9.26 158	72	0.73 842	9.99 288	39		70	69
22	9.25 514	69	9.26 229	71	0.73 771	9.99 285	38	.1	7.0	6.9
23	9.25 583	69	9.26 301	72	0.73 699	9.99 283	37	.2	14.0	13.8
24	9.25 652	69	9.26 372	71	0.73 628	9.99 281	36	.3	21.0	20.7
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	35	.4	28.0	27.6
26	9.25 790	69	9 26 514	71	0.73 486	9.99 276	34	.5	35.0	34.5
27	9.25 858	68	9.26 585	71	0.73 415	9.99 274	33	.6	42.0	41.4
28	9.25 927	69	9.26 655	70	0.73 345	9.99 271	32	.7	49.0	48.3
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	31	.8	56.0	55.2
30	9.26 063	68	9.26 797	71	0.73 203	9.99 267	30	.9	63.0	62.1
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	29		68	67
32	9.26 199	68	9.26 937	70	0.73 063	9.99 262	28	.1	6.8	6.7
33	9.26 267	68	9.27 008	71	0.72 992	9.99 260	27	.2	13.6	13.4
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	26	.3	20.4	20.1
35	9.26 403	67	9.27 148	70	0.72 852	9.99 255	25	.4	27.2	26.8
36	9.26 470	68	9.27 218	70	0.72 782	9.99 252	24	.5	34.0	33.5
37	9.26 538	67	9.27 288	70	0.72 712	9.99 250	23	.6	40.8	40.2
38	9.26 605	67	9.27 357	69	0.72 643	9.99 248	22	.7	47.6	46.9
39	9.26 672	67	9.27 427	70	0.72 573	9.99 245	21	.8	54.4	53.6
40	9.26 739	67	9.27 496	69	0.72 504	9.99 243	20	.9	61.2	60.3
41	9.26 806	67	9.27 566	70	0.72 434	9.99 241	19		66	65
42	9.26 873	67	9.27 635	69	0.72 365	9.99 238	18	.1	6.6	6.5
43	9.26 940	67	9.27 704	69	0.72 296	9.99 236	17	.2	13.2	13.0
44	9.27 007	67	9.27 773	69	0.72 227	9.99 233	16	.3	19.8	19.5
45	9.27 073	66	9.27 842	69	0.72 158	9.99 231	15	.4	26.4	26.0
46	9.27 140	67	9.27 911	69	0.72 089	9.99 229	14	.5	33.0	32.5
47	9.27 206	66	9.27 980	69	0.72 020	9.99 226	13	.6	39.6	39.0
48	9.27 273	67	9.28 049	69	0.71 951	9.99 224	12	.7	46.2	45.5
49	9.27 339	66	9.28 117	68	0.71 883	9.99 221	11	.8	52.8	52.0
50	9.27 405	66	9.28 186	69	0.71 814	9.99 219	10	.9	59.4	58.5
51	9.27 471	66	9.28 254	68	0.71 746	9.99 217	9		3	2
52	9.27 537	66	9.28 323	69	0.71 677	9.99 214	8	.1	0.3	0.2
53	9.27 602	65	9.28 391	68	0.71 609	9.99 212	7	.2	0.6	0.4
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	6	.3	0.9	0.6
55	9.27 734	66	9.28 527	68	0.71 473	9.99 207	5	.4	1.2	0.8
56	9.27 799	65	9.28 595	67	0.71 405	9.99 204	4	.5	1.5	1.0
57	9.27 864	65	9.28 662	68	0.71 338	9.99 202	3	.6	1.8	1.2
58	9.27 930	66	9.28 730	68	0.71 270	9.99 200	2	.7	2.1	1.4
59	9.27 995	65	9.28 798	68	0.71 202	9.99 197	1	.8	2.4	1.6
60	9.28 060	65	9.28 865	67	0.71 135	9.99 195	0	.9	2.7	1.8
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.		

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11°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.
0	9.28 060		9.28 865		0.71 135	9.99 195	60	
1	9.28 125	65	9.28 933	68	0.71 067	9.99 192	59	68 | 67
2	9.28 190	65	9.29 000	67	0.71 000	9.99 190	58	.1 | 6.8 | 6.7
3	9.28 254	64	9.29 067	67	0.70 933	9.99 187	57	.2 | 13.6 | 13.4
4	9.28 319	65	9.29 134	67	0.70 866	9.99 185	56	.3 | 20 4 | 20 1
5	9.28 384	64	9.29 201	67	0.70 799	9.99 182	55	.4 | 27 2 | 26 8
6	9 28 448	64	9.29 268	67	0.70 732	9.99 180	54	.5 | 34 0 | 33 5
7	9.28 512	65	9.29 335	67	0.70 665	9.99 177	53	.6 | 40.8 | 40.2
8	9.28 577	64	9.29 402	66	0.70 598	9.99 175	52	.7 | 47.6 | 46.9
9	9.28 641	64	9.29 468	67	0.70 532	9.99 172	51	.8 | 54.4 | 53.6
10	9.28 705	64	9.29 535	66	0.70 465	9.99 170	50	.9 | 61 2 | 60.3
11	9.28 769	64	9.29 601	67	0.70 399	9.99 167	49	66 | 65
12	9.28 833	63	9.29 668	66	0.70 332	9.99 165	48	.1 | 6.6 | 6.5
13	9.28 896	64	9.29 734	66	0.70 266	9.99 162	47	.2 | 13.2 | 13.0
14	9.28 960	64	9.29 800	66	0.70 200	9 99 160	46	.3 | 19 8 | 19.5
15	9.29 024	63	9.29 866	66	0.70 134	9.99 157	45	.4 | 26.4 | 26.0
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	44	.5 | 33.0 | 32.5
17	9.29 150	64	9.29 998	66	0.70 002	9.99 152	43	.6 | 39.6 | 39.0
18	9.29 214	63	9.30 064	66	0.69 936	9.99 150	42	.7 | 46.2 | 45.5
19	9.29 277	63	9.30 130	65	0.69 870	9.99 147	41	.8 | 52.8 | 52.0
20	9.29 340	63	9.30 195	66	0.69 805	9.99 145	40	.9 | 59.4 | 58.5
21	9.29 403	63	9.30 261	65	0.69 739	9.99 142	39	64 | 63
22	9.29 466	63	9.30 326	65	0.69 674	9.99 140	38	.1 | 6.4 | 6.3
23	9.29 529	62	9.30 391	66	0.69 609	9.99 137	37	.2 | 12.8 | 12.6
24	9.29 591	63	9.30 457	65	0.69 543	9.99 135	36	.3 | 19.2 | 18.9
25	9.29 654	62	9.30 522	65	0.69 478	9.99 132	35	.4 | 25.6 | 25.2
26	9.29 716	63	9.30 587	65	0.69 413	9.99 130	34	.5 | 32.0 | 31.5
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	33	.6 | 38.4 | 37.8
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	32	.7 | 44.8 | 44.1
29	9.29 903	63	9.30 782	64	0.69 218	9.99 122	31	.8 | 51.2 | 50.4
30	9.29 966	62	9.30 846	65	0.69 154	9.99 119	30	.9 | 57.6 | 56.7
31	9.30 028	62	9.30 911	64	0.69 089	9.99 117	29	62 | 61
32	9.30 090	61	9.30 975	65	0.69 025	9.99 114	28	.1 | 6.2 | 6.1
33	9.30 151	62	9.31 040	64	0.68 960	9.99 112	27	.2 | 12.4 | 12.2
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	26	.3 | 18.6 | 18.3
35	9.30 275	61	9.31 168	65	0.68 832	9.99 106	25	.4 | 24.8 | 24.4
36	9.30 336	62	9.31 233	64	0.68 767	9.99 104	24	.5 | 31.0 | 30.5
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	23	.6 | 37.2 | 36.6
38	9.30 459	62	9.31 361	64	0.68 639	9.99 099	22	.7 | 43.4 | 42.7
39	9.30 521	61	9.31 425	64	0.68 575	9.99 096	21	.8 | 49.6 | 48.8
40	9.30 582	61	9.31 489	63	0.68 511	9.99 093	20	.9 | 55.8 | 54.9
41	9.30 643	61	9.31 552	64	0.68 448	9.99 091	19	60 | 59
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	18	.1 | 6.0 | 5.9
43	9.30 765	62	9.31 679	64	0.68 321	9.99 086	17	.2 | 12.0 | 11.8
44	9.30 826	61	9.31 743	63	0.68 257	9.99 083	16	.3 | 18.0 | 17.7
45	9.30 887	60	9.31 806	64	0.68 194	9.99 080	15	.4 | 24.0 | 23.6
46	9.30 947	61	9.31 870	63	0.68 130	9.99 078	14	.5 | 30.0 | 29.5
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	13	.6 | 36.0 | 35.4
48	9.31 068	61	9.31 996	63	0.68 004	9.99 072	12	.7 | 42.0 | 41.3
49	9.31 129	60	9.32 059	63	0.67 941	9.99 070	11	.8 | 48.0 | 47.2
50	9.31 189	61	9.32 122	63	0.67 878	9.99 067	10	.9 | 54.0 | 53.1
51	9.31 250	60	9.32 185	63	0.67 815	9.99 064	9	3 | 2
52	9.31 310	60	9.32 248	63	0.67 752	9.99 062	8	.1 | 0.3 | 0.2
53	9.31 370	60	9.32 311	63	0.67 689	9.99 059	7	.2 | 0.6 | 0.4
54	9.31 430	60	9.32 373	62	0.67 627	9.99 056	6	.3 | 0.9 | 0.6
55	9.31 490	59	9.32 436	63	0.67 564	9.99 054	5	.4 | 1.2 | 0.8
56	9.31 549	60	9.32 498	62	0.67 502	9.99 051	4	.5 | 1.5 | 1.0
57	9.31 609	60	9.32 561	62	0.67 439	9.99 048	3	.6 | 1.8 | 1.2
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	2	.7 | 2.1 | 1.4
59	9.31 728	60	9.32 685	62	0.67 315	9.99 043	1	.8 | 2.4 | 1.6
60	9.31 788		9.32 747		0.67 253	9.99 040	0	.9 | 2.7 | 1.8
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.

12°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.
0	9.31 788		9.32 747		0.67 253	9.99 040	60	
1	9.31 847	59	9.32 810	63	0.67 190	9.99 038	59	63 62
2	9.31 907	60	9.32 872	62	0.67 128	9.99 035	58	.1 6.3 6.2
3	9.31 966	59	9.32 933	61	0.67 067	9.99 032	57	.2 12.6 12.4
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	56	.3 18.9 18.6
5	9.32 084	59	9.33 057	62	0.66 943	9.99 027	55	.4 25.2 24.8
6	9.32 143	59	9 33 119	61	0.66 881	9.99 024	54	.5 31.5 31.0
7	9.32 202	59	9.33 180	62	0.66 820	9.99 022	53	.6 37.8 37.2
8	9.32 261	58	9.33 242	61	0.66 758	9.99 019	52	.7 44.1 43.4
9	9.32 319	59	9.33 303	62	0.66 697	9.99 016	51	.8 50.4 49.6
10	9.32 378	59	9.33 365	61	0.66 635	9.99 013	50	.9 56.7 55.8
11	9.32 437	58	9.33 426	61	0.66 574	9.99 011	49	61 60
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	48	.1 6.1 6.0
13	9.32 553	59	9.33 548	61	0.66 452	9.99 005	47	.2 12.2 12.0
14	9.32 612	58	9.33 609	61	0.66 391	9.99 002	46	.3 18.3 18.0
15	9 32 670	58	9.33 670	61	0.66 330	9.99 000	45	.4 24.4 24.0
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	44	.5 30.5 30.0
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	43	.6 36.6 36.0
18	9.32 844	58	9.33 853	60	0.66 147	9.98 991	42	.7 42.7 42.0
19	9.32 902	58	9.33 913	61	0.66 087	9.98 989	41	.8 48.8 48.0
20	9.32 960	58	9.33 974	60	0.66 026	9.98 986	40	.9 54.9 54.0
21	9.33 018	57	9.34 034	61	0.65 966	9.98 983	39	59
22	9.33 075	58	9.34 095	60	0.65 905	9.98 980	38	.1 5.9
23	9.33 133	57	9.34 155	60	0.65 845	9.98 978	37	.2 11.8
24	9.33 190	58	9.34 215	61	0.65 785	9.98 975	36	.3 17.7
25	9.33 248	57	9.34 276	60	0.65 724	9.98 972	35	.4 23.6
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	34	.5 29.5
27	9.33 362	58	9.34 396	60	0.65 604	9.98 967	33	.6 35.4
28	9.33 420	57	9.34 456	60	0.65 544	9.98 964	32	.7 41.3
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	31	.8 47.2
30	9.33 534	57	9.34 576	59	0.65 424	9.98 958	30	.9 53.1
31	9.33 591	56	9.34 635	60	0.65 365	9.98 955	29	58 57
32	9.33 647	57	9.34 695	60	0.65 305	9.98 953	28	1 5.8 5.7
33	9.33 704	57	9.34 755	59	0.65 245	9.98 950	27	2 11.6 11.4
34	9.33 761	57	9.34 814	60	0.65 186	9.98 947	26	3 17.4 17.1
35	9.33 818	56	9.34 874	59	0.65 126	9.98 944	25	4 23.2 22.8
36	9.33 874	57	9.34 933	59	0.65 067	9.98 941	24	5 29.0 28.5
37	9.33 931	56	9.34 992	59	0.65 008	9.98 938	23	6 34.8 34.2
38	9.33 987	56	9.35 051	59	0.64 949	9.98 936	22	7 40.6 39.9
39	9.34 043	56	9.35 111	60	0.64 889	9.98 933	21	8 46.4 45.6
40	9.34 100	57	9.35 170	59	0.64 830	9.98 930	20	9 52.2 51.3
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	19	56 55
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	18	.1 5.6 5.5
43	9.34 268	56	9.35 347	59	0.64 653	9.98 921	17	.2 11.2 11.0
44	9.34 324	56	9.35 405	58	0.64 595	9.98 919	16	.3 16.8 16.5
45	9.34 380	56	9.35 464	59	0 64 536	9.98 916	15	.4 22.4 22.0
46	9.34 436	55	9.35 523	59	0.64 477	9.98 913	14	.5 28.0 27.5
47	9.34 491	56	9.35 581	58	0.64 419	9.98 910	13	.6 33.6 33.0
48	9.34 547	55	9.35 640	59	0.64 360	9.98 907	12	.7 39.2 38.5
49	9.34 602	56	9.35 698	58	0.64 302	9.98 904	11	.8 44.8 44.0
50	9.34 658	55	9.35 757	59	0.64 243	9.98 901	10	.9 50.4 49.5
51	9.34 713	56	9.35 815	58	0.64 185	9.98 898	9	3 2
52	9.34 769	55	9.35 873	58	0.64 127	9.98 896	8	.1 0.3 0.2
53	9.34 824	55	9 35 931	58	0.64 069	9.98 893	7	.2 0.6 0.4
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	6	.3 0.9 0.6
55	9.34 934	55	9.36 047	58	0.63 953	9.98 887	5	.4 1.2 0.8
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	4	.5 1.5 1.0
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3	.6 1.8 1.2
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	2	.7 2.1 1.4
59	9.35 154	55	9.36 279	58	0.63 721	9.98 875	1	.8 2.4 1.6
60	9.35 209	55	9.36 336	57	0.63 664	9.98 872	0	.9 2.7 1.8
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.

13°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.		Prop. Pts.
0	9.35 209		9.36 336		0.63 664	9.98 872	60	
1	9.35 263	54	9.36 394	58	0.63 606	9.98 869	59	58 57
2	9.35 318	55	9.36 452	58	0.63 548	9.98 867	58	.1 5.8 5.7
3	9.35 373	55	9.36 509	57	0.63 491	9.98 864	57	.2 11.6 11.4
4	9.35 427	54	9.36 566	57	0.63 434	9.98 861	56	.3 17.4 17.1
5	9.35 481	54	9.36 624	58	0.63 376	9.98 858	55	.4 23.2 22.8
6	9.35 536	55	9.36 681	57	0.63 319	9.98 855	54	.5 29.0 28.5
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	53	.6 34.8 34.2
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	52	.7 40.6 39.9
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	51	.8 46.4 45.6
10	9.35 752	54	9.36 909	57	0.63 091	9.98 843	50	.9 52.2 51.3
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	49	56 55
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	48	.1 5.6 5.5
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	47	.2 11.2 11.0
14	9.35 968	54	9.37 137	57	0.62 863	9.98 831	46	.3 16.8 16.5
15	9.36 022	53	9.37 193	56	0.62 807	9.98 828	45	.4 22.4 22.0
16	9.36 075	54	9.37 250	57	0.62 750	9.98 825	44	.5 28.0 27.5
17	9.36 129	53	9.37 306	56	0.62 694	9.98 822	43	.6 33.6 33.0
18	9.36 182	54	9.37 363	57	0.62 637	9.98 819	42	.7 39.2 38.5
19	9.36 236	53	9.37 419	56	0.62 581	9.98 816	41	.8 44.8 44.0
20	9.36 289	53	9.37 476	57	0.62 524	9.98 813	40	.9 50.4 49.5
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	39	54
22	9.36 395	53	9.37 588	56	0.62 412	9.98 807	38	.1 5.4
23	9.36 449	54	9.37 644	56	0.62 356	9.98 804	37	.2 10.8
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	36	.3 16.2
25	9.36 555	53	9.37 756	56	0.62 244	9.98 798	35	.4 21.6
26	9.36 608	53	9.37 812	56	0.62 188	9.98 795	34	.5 27.0
27	9.36 660	52	9.37 868	56	0.62 132	9.98 792	33	.6 32.4
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	32	.7 37.8
29	9.36 766	53	9.37 980	56	0.62 020	9.98 786	31	.8 43.2
30	9.36 819	53	9.38 035	55	0.61 965	9.98 783	30	.9 48.6
31	9.36 871	52	9.38 091	56	0.61 909	9.98 780	29	53 52
32	9.36 924	53	9.38 147	56	0.61 853	9.98 777	28	.1 5.3 5.2
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	27	.2 10.6 10.4
34	9.37 028	52	9.38 257	55	0.61 743	9.98 771	26	.3 15.9 15.6
35	9.37 081	53	9.38 313	56	0.61 687	9.98 768	25	.4 21.2 20.8
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	24	.5 26.5 26.0
37	9.37 185	52	9.38 423	55	0.61 577	9.98 762	23	.6 31.8 31.2
38	9.37 237	52	9.38 479	56	0.61 521	9.98 759	22	.7 37.1 36.4
39	9.37 289	52	9.38 534	55	0.61 466	9.98 756	21	.8 42.4 41.6
40	9.37 341	52	9.38 589	55	0.61 411	9.98 753	20	.9 47.7 46.8
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	19	51 4
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	18	.1 5.1 0.4
43	9.37 497	52	9.38 754	55	0.61 246	9.98 743	17	.2 10.2 0.8
44	9.37 549	52	9.38 808	54	0.61 192	9.98 740	16	.3 15.3 1.2
45	9.37 600	51	9.38 863	55	0.61 137	9.98 737	15	.4 20.4 1.6
46	9.37 652	52	9.38 918	55	0.61 082	9.98 734	14	.5 25.5 2.0
47	9.37 703	51	9.38 972	54	0.61 028	9.98 731	13	.6 30.6 2.4
48	9.37 755	52	9.39 027	55	0.60 973	9.98 728	12	.7 35.7 2.8
49	9.37 806	51	9.39 082	55	0.60 918	9.98 725	11	.8 40.8 3.2
50	9.37 858	52	9.39 136	54	0.60 864	9.98 722	10	.9 45.9 3.6
51	9.37 909	51	9.39 190	54	0.60 810	9.98 719	9	3 2
52	9.37 960	51	9.39 245	55	0.60 755	9.98 715	8	.1 0.3 0.2
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	7	.2 0.6 0.4
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	6	.3 0.9 0.6
55	9.38 113	51	9.39 407	54	0.60 593	9.98 706	5	.4 1.2 0.8
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	4	.5 1.5 1.0
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3	.6 1.8 1.2
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	2	.7 2.1 1.4
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	1	.8 2.4 1.6
60	9.38 368	51	9.39 677	54	0.60 323	9.98 690	0	.9 2.7 1.8
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	′	Prop. Pts.

76°

14°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.38 368		9.39 677		0.60 323	9.98 690		60	
1	9.38 418	50	9.39 731	54	0.60 269	9.98 687	3	59	
2	9.38 469	51	9.39 785	54	0.60 215	9.98 684	3	58	
3	9.38 519	50	9.39 838	53	0.60 162	9.98 681	3	57	54 53
4	9.38 570	51	9.39 892	54	0.60 108	9.98 678	3	56	.1 5.4 5.3
5	9.38 620	50	9.39 945	53	0.60 055	9.98 675	3	55	.2 10.8 10.6
6	9.38 670	50	9.39 999	54	0.60 001	9.98 671	4	54	.3 16.2 15.9
7	9.38 721	51	9.40 052	53	0.59 948	9.98 668	3	53	.4 21.6 21.2
8	9.38 771	50	9.40 106	54	0.59 894	9.98 665	3	52	.5 27.0 26.5
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51	.6 32.4 31.8
10	9.38 871	50	9.40 212	53	0.59 788	9.98 659	3	50	.7 37.8 37.1
11	9.38 921	50	9.40 266	54	0.59 734	9.98 656	3	49	.8 43.2 42.4
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	4	48	.9 48.6 47.7
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47	
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46	
15	9.39 121	49	9.40 478	53	0.59 522	9.98 643	3	45	52 51
16	9.39 170	50	9.40 531	53	0.59 469	9.98 640	3	44	.1 5.2 5.1
17	9.39 220	50	9.40 584	53	0.59 416	9.98 636	4	43	.2 10.4 10.2
18	9.39 270	49	9.40 636	52	0.59 364	9.98 633	3	42	.3 15.6 15.3
19	9.39 319	50	9.40 689	53	0.59 311	9.98 630	3	41	.4 20.8 20.4
20	9.39 369	49	9.40 742	53	0.59 258	9.98 627	3	40	.5 26.0 25.5
21	9.39 418	49	9.40 795	53	0.59 205	9.98 623	4	39	.6 31.2 30.6
22	9.39 467	49	9.40 847	52	0.59 153	9.98 620	3	38	.7 36.4 35.7
23	9.39 517	50	9.40 900	53	0.59 100	9.98 617	3	37	.8 41.6 40.8
24	9.39 566	49	9.40 952	52	0.59 048	9.98 614	3	36	.9 46.8 45.9
25	9.39 615	49	9.41 005	53	0.58 995	9.98 610	4	35	
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34	50 49
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33	.1 5.0 4.9
28	9.39 762	49	9.41 161	52	0.58 839	9.98 601	3	32	.2 10.0 9.8
29	9.39 811	49	9.41 214	53	0.58 786	9.98 597	4	31	.3 15.0 14.7
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	30	.4 20.0 19.6
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29	.5 25.0 24.5
32	9.39 958	49	9.41 370	52	0.58 630	9.98 588	3	28	.6 30.0 29.4
33	9.40 006	48	9.41 422	52	0.58 578	9.98 584	4	27	.7 35.0 34.3
34	9.40 055	49	9.41 474	52	0.58 526	9.98 581	3	26	.8 40.0 39.2
35	9.40 103	48	9.41 526	52	0.58 474	9.98 578	3	25	.9 45.0 44.1
36	9.40 152	49	9.41 578	52	0.58 422	9.98 574	4	24	
37	9.40 200	48	9.41 629	51	0.58 371	9.98 571	3	23	
38	9.40 249	49	9.41 681	52	0.58 319	9.98 568	3	22	48 47
39	9.40 297	48	9.41 733	52	0.58 267	9.98 565	3	21	.1 4.8 4.7
40	9.40 346	49	9.41 784	51	0.58 216	9.98 561	4	20	.2 9.6 9.4
41	9.40 394	48	9.41 836	52	0.58 164	9.98 558	3	19	.3 14.4 14.1
42	9.40 442	48	9.41 887	51	0.58 113	9.98 555	3	18	.4 19.2 18.8
43	9.40 490	48	9.41 939	52	0.58 061	9.98 551	4	17	.5 24.0 23.5
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16	.6 28.8 28.2
45	9.40 586	48	9.42 041	51	0.57 959	9.98 545	3	15	.7 33.6 32.9
46	9.40 634	48	9.42 093	52	0.57 907	9.98 541	4	14	.8 38.4 37.6
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13	.9 43.2 42.3
48	9.40 730	48	9.42 195	51	0.57 805	9.98 535	3	12	
49	9.40 778	48	9.42 246	51	0.57 754	9.98 531	4	11	
50	9.40 825	47	9.42 297	51	0.57 703	9.98 528	3	10	4 3
51	9.40 873	48	9.42 348	51	0.57 652	9.98 525	3	9	.1 0.4 0.3
52	9.40 921	48	9.42 399	51	0.57 601	9.98 521	4	8	.2 0.8 0.6
53	9.40 968	47	9.42 450	51	0.57 550	9.98 518	3	7	.3 1.2 0.9
54	9.41 016	48	9.42 501	51	0.57 499	9.98 515	3	6	.4 1.6 1.2
55	9.41 063	47	9.42 552	51	0.57 448	9.98 511	4	5	.5 2.0 1.5
56	9.41 111	48	9.42 603	51	0.57 397	9.98 508	3	4	.6 2.4 1.8
57	9.41 158	47	9.42 653	50	0.57 347	9.98 505	3	3	.7 2.8 2.1
58	9.41 205	47	9.42 704	51	0.57 296	9.98 501	4	2	.8 3.2 2.4
59	9.41 252	47	9.42 755	51	0.57 245	9.98 498	3	1	.9 3.6 2.7
60	9.41 300	48	9.42 805	50	0.57 195	9.98 494	4	0	
′	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

75°

15°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.41 300	47	9.42 805	52	0.57 195	9.98 404	3	60	
1	9.41 347	47	9.42 856	50	0.57 144	9.98 491	3	59	
2	9.41 394	47	9.42 906	51	0.57 094	9.98 488	4	58	
3	9.41 441	47	9.42 957	50	0.57 043	9.98 484	4	57	52 50
4	9.41 488	47	9.43 007	50	0.56 993	9.98 481	3	56	.1 5.1 5.0
5	9.41 535	47	9.43 057	51	0.56 943	9.98 477	4	55	.2 10.2 10.0
6	9.41 582	46	9.43 108	50	0.56 892	9.98 474	3	54	.3 15.3 15.0
7	9.41 628	47	9.43 158	50	0.56 842	9.98 471	3	53	.4 20.4 20.0
8	9.41 675	47	9.43 208	50	0.56 792	9.98 467	4	52	.5 25.5 25.0
9	9.41 722	46	9.43 258	50	0.56 742	9.98 464	3	51	.6 30.6 30.0
10	9.41 768	47	9.43 308	50	0.56 692	9.98 460	4	50	.7 35.7 35.0
11	9.41 815	46	9.43 358	50	0.56 642	9.98 457	3	49	.8 40.8 40.0
12	9.41 861	47	9.43 408	50	0.56 592	9.98 453	4	48	.9 45.9 45.0
13	9.41 908	46	9.43 458	50	0.56 542	9.98 450	3	47	
14	9.41 954	47	9.43 508	50	0.56 492	9.98 447	3	46	
15	9.42 001	46	9.43 558	49	0.56 442	9.98 443	4	45	49 48
16	9.42 047	46	9.43 607	50	0.56 393	9.98 440	3	44	.1 4.9 4.8
17	9.42 093	47	9.43 657	50	0.56 343	9.98 436	4	43	.2 9.8 9.6
18	9.42 140	46	9.43 707	49	0.56 293	9.98 433	3	42	.3 14.7 14.4
19	9.42 186	46	9.43 756	50	0.56 244	9.98 429	4	41	.4 19.6 19.2
20	9.42 232	46	9.43 806	49	0.56 194	9.98 426	3	40	.5 24.5 24.0
21	9.42 278	46	9.43 855	50	0.56 145	9.98 422	4	39	.6 29.4 28.8
22	9.42 324	46	9.43 905	49	0.56 095	9.98 419	3	38	.7 34.3 33.6
23	9.42 370	46	9.43 954	50	0.56 046	9.98 415	4	37	.8 39.2 38.4
24	9.42 416	45	9.44 004	49	0.55 996	9.98 412	3	36	.9 44.1 43.2
25	9.42 461	46	9.44 053	49	0.55 947	9.98 409	3	35	
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	4	34	47 46
27	9.42 553	46	9.44 151	50	0.55 849	9.98 402	3	33	.1 4.7 4.6
28	9.42 599	45	9.44 201	49	0.55 799	9.98 398	4	32	.2 9.4 9.2
29	9.42 644	46	9.44 250	49	0.55 750	9.98 395	3	31	.3 14.1 13.8
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	4	30	.4 18.8 18.4
31	9.42 735	46	9.44 348	49	0.55 652	9.98 388	3	29	.5 23.5 23.0
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	4	28	.6 28.2 27.6
33	9.42 826	46	9.44 446	49	0.55 554	9.98 381	3	27	.7 32.9 32.2
34	9.42 872	45	9.44 495	49	0.55 505	9.98 377	4	26	.8 37.6 36.8
35	9.42 917	45	9.44 544	48	0.55 456	9.98 373	4	25	.9 42.3 41.4
36	9.42 962	46	9.44 592	49	0.55 408	9.98 370	3	24	
37	9.43 008	45	9.44 641	49	0.55 359	9.98 366	4	23	
38	9.43 053	45	9.44 690	48	0.55 310	9.98 363	3	22	45 44
39	9.43 098	45	9.44 738	49	0.55 262	9.98 359	4	21	.1 4.5 4.4
40	9.43 143	45	9.44 787	49	0.55 213	9.98 356	3	20	.2 9.0 8.8
41	9.43 188	45	9.44 836	48	0.55 164	9.98 352	4	19	.3 13.5 13.2
42	9.43 233	45	9.44 884	49	0.55 116	9.98 349	3	18	.4 18.0 17.6
43	9.43 278	45	9.44 933	48	0.55 067	9.98 345	4	17	.5 22.5 22.0
44	9.43 323	44	9.44 981	48	0.55 019	9.98 342	3	16	.6 27.0 26.4
45	9.43 367	45	9 45 029	48	0.54 971	9.98 338	4	15	.7 31.5 30.8
46	9.43 412	45	9.45 078	49	0.54 922	9.98 334	4	14	.8 36.0 35.2
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	3	13	.9 40.5 39.6
48	9.43 502	45	9.45 174	48	0.54 826	9.98 327	4	12	
49	9.43 546	44	9.45 222	48	0.54 778	9.98 324	3	11	
50	9.43 591	45	9.45 271	49	0.54 729	9.98 320	4	10	4 3
51	9.43 635	44	9.45 319	48	0.54 681	9.98 317	3	9	.1 0.4 0.3
52	9.43 680	45	9.45 367	48	0.54 633	9.98 313	4	8	.2 0.8 0.6
53	9.43 724	44	9.45 415	48	0.54 585	9.98 309	4	7	.3 1.2 0.9
54	9.43 769	45	9.45 463	48	0.54 537	9.98 306	3	6	.4 1.6 1.2
55	9.43 813	44	9.45 511	48	0.54 489	9.98 302	4	5	.5 2.0 1.5
56	9.43 857	44	9.45 559	48	0.54 441	9.98 299	3	4	.6 2.4 1.8
57	9.43 901	44	9.45 606	47	0.54 394	9.98 295	4	3	.7 2.8 2.1
58	9.43 946	45	9.45 654	48	0.54 346	9.98 291	4	2	.8 3.2 2.4
59	9.43 990	44	9.45 702	48	0.54 298	9.98 288	3	1	.9 3.6 2.7
60	9.44 034	44	9.45 750	48	0.54 250	9.98 284	4	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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16°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.44 034		9.45 750		0.54 250	9.98 284		60	
1	9 44 078	44	9.45 797	47	0.54 203	9.98 281	3	59	
2	9.44 122	44	9.45 845	48	0.54 155	9.98 277	4	58	
3	9 44 166	44	9.45 892	47	0.54 108	9.98 273	4	57	48 47
4	9 44 210	44	9.45 940	48	0.54 060	9.98 270	3	56	.1 4.8 4.7
		43		47			4		.2 9.6 9.4
5	9.44 253	44	9.45 987	48	0.54 013	9.98 266	4	55	.3 14.4 14.1
6	9.44 297	44	9.46 035	47	0.53 965	9.98 262	4	54	.4 19.2 18.8
7	9.44 341	44	9 46 082	48	0.53 918	9.98 259	3	53	.5 24.0 23.5
8	9.44 385	43	9.46 130	47	0.53 870	9.98 255	4	52	.6 28.8 28.2
9	9.44 428	44	9.46 177	47	0.53 823	9.98 251	4	51	.7 33.6 32.9
10	9.44 472	44	9.46 224	47	0.53 776	9.98 248	3	50	.8 38 4 37.6
11	9.44 516	43	9.46 271	48	0.53 729	9.98 244	4	49	.9 43.2 42.3
12	9.44 559	43	9.46 319	47	0.53 681	9.98 240	4	48	
13	9.44 602	44	9.46 366	47	0.53 634	9.98 237	3	47	
14	9.44 646	43	9.46 413	47	0.53 587	9.98 233	4	46	46 45
15	9 44 689	44	9.46 460	47	0.53 540	9.98 229	4	45	.1 4.6 4.5
16	9.44 733	43	9.46 507	47	0.53 493	9.98 226	3	44	.2 9.2 9.0
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	4	43	.3 13.8 13.5
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	4	42	.4 18.4 18.0
19	9.44 862	43	9.46 648	46	0.53 352	9.98 215	3	41	.5 23.0 22.5
20	9.44 905	43	9.46 694	47	0.53 306	9.98 211	4	40	.6 27.6 27.0
21	9.44 948	44	9.46 741	47	0.53 259	9.98 207	4	39	.7 32.2 31.5
22	9.44 992	43	9.46 788	47	0.53 212	9.98 204	3	38	.8 36.8 36.0
23	9.45 035	42	9.46 835	46	0.53 165	9.98 200	4	37	.9 41.4 40.5
24	9.45 077	43	9.46 881	47	0.53 119	9.98 196	4	36	
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	4	35	
26	9.45 163	43	9.46 975	46	0.53 025	9.98 189	3	34	44 43
27	9.45 206	43	9.47 021	47	0.52 979	9.98 185	4	33	.1 4.4 4.3
28	9.45 249	43	9.47 068	46	0.52 932	9.98 181	4	32	.2 8.8 8.6
29	9.45 292	43	9.47 114	46	0.52 886	9.98 177	4	31	.3 13.2 12.9
30	9.45 334	42	9.47 160	46	0.52 840	9.98 174	3	30	.4 17.6 17.2
31	9.45 377	43	9.47 207	47	0.52 793	9.98 170	4	29	.5 22.0 21.5
32	9.45 419	42	9.47 253	46	0.52 747	9.98 166	4	28	.6 26.4 25.8
33	9.45 462	43	9.47 299	46	0.52 701	9.98 162	4	27	.7 30.8 30 1
34	9.45 504	42	9.47 346	47	0.52 654	9.98 159	3	26	.8 35.2 34.4
		43		46			4		.9 39.6 38.7
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	4	25	
36	9.45 589	43	9.47 438	46	0.52 562	9.98 151	4	24	
37	9.45 632	42	9 47 484	46	0.52 516	9.98 147	4	23	
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	3	22	42 41
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	4	21	.1 4.2 4.1
40	9.45 758	43	9.47 622	46	0.52 378	9.98 136	4	20	.2 8.4 8.2
41	9.45 801	42	9.47 668	46	0.52 332	9.98 132	4	19	.3 12.6 12.3
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	3	18	.4 16.8 16.4
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	4	17	.5 21.0 20.5
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16	.6 25.2 24 6
45	9.45 969	42	9.47 852	46	0.52 148	9.98 117	4	15	.7 29.4 28.7
46	9.46 011	42	9.47 897	45	0.52 103	9.98 113	4	14	.8 33.6 32 8
47	9.46 053	42	9.47 943	46	0.52 057	9.98 110	3	13	.9 37.8 36 9
48	9.46 095	42	9.47 989	46	0.52 011	9.98 106	4	12	
49	9.46 136	41	9.48 035	46	0.51 965	9.98 102	4	11	
50	9.46 178	42	9.48 080	45	0.51 920	9.98 098	4	10	4 3
51	9.46 220	42	9.48 126	46	0.51 874	9.98 094	4	9	.1 0.4 0.3
52	9.46 262	42	9.48 171	45	0.51 829	9.98 090	4	8	.2 0.8 0.6
53	9.46 303	41	9.48 217	46	0.51 783	9.98 087	3	7	.3 1.2 0.9
54	9.46 345	42	9.48 262	45	0.51 738	9.98 083	4	6	.4 1.6 1.2
		41		45			4		.5 2.0 1.5
55	9.46 386	42	9.48 307	46	0.51 693	9.98 079	4	5	.6 2.4 1.8
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4	.7 2.8 2.1
57	9.46 469	42	9.48 398	45	0.51 602	9.98 071	4	3	.8 3.2 2.4
58	9.46 511	41	9.48 443	45	0.51 557	9.98 067	4	2	.9 3.6 2 7
59	9.46 552	42	9.48 489	46	0.51 511	9.98 063	4	1	
60	9.46 594		9.48 534	45	0.51 466	9.98 060	3	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

73°

17°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.46 594		9.48 534		0.51 466	9.98 060		60	
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59	
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58	
3	9 46 717	41	9.48 669	45	0.51 331	9.98 048	4	57	45 44
4	9.46 758	41	9.48 714	45	0.51 286	9.98 044	4	56	.1 4.5 4.4
5	9.46 800	42	9.48 759	45	0.51 241	9.98 040	4	55	.2 9.0 8.8
6	9 46 841	41	9.48 804	45	0.51 196	9.98 036	4	54	.3 13.5 13.2
7	9 46 882	41	9.48 849	45	0.51 151	9.98 032	4	53	.4 18.0 17.6
8	9 46 923	41	9.48 894	45	0.51 106	9.98 029	3	52	.5 22.5 22.0
9	9 46 964	41	9.48 939	45	0.51 061	9.98 025	4	51	.6 27.0 26.4
10	9.47 005	40	9.48 984	45	0 51 016	9.98 021	4	50	.7 31.5 30.8
11	9.47 045	41	9.49 029	44	0.50 971	9.98 017	4	49	.8 36.0 35.2
12	9.47 086	41	9.49 073	44	0.50 927	9.98 013	4	48	.9 40.5 39.6
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47	
14	9.47 168	41	9 49 163	45	0.50 837	9.98 005	4	46	43 42
15	9.47 209	40	9.49 207	44	0.50 793	9.98 001	4	45	.1 4.3 4.2
16	9 47 249	41	9.49 252	45	0.50 748	9.97 997	4	44	.2 8.6 8.4
17	9 47 290	40	9.49 296	44	0.50 704	9.97 993	4	43	.3 12.9 12.6
18	9.47 330	41	9.49 341	45	0.50 659	9.97 989	4	42	.4 17.2 16 8
19	9.47 371	40	9.49 385	44	0.50 615	9.97 986	3	41	.5 21.5 21.0
20	9.47 411	41	9.49 430	45	0.50 570	9.97 982	4	40	.6 25.8 25.2
21	9.47 452	40	9.49 474	44	0 50 526	9.97 978	4	39	.7 30.1 29.4
22	9.47 492	41	9.49 519	45	0.50 481	9 97 974	4	38	.8 34.4 33.6
23	9.47 533	40	9.49 563	44	0 50 437	9.97 970	4	37	.9 38.7 37.8
24	9.47 573	40	9.49 607	44	0.50 393	9.97 966	4	36	
25	9.47 613	41	9.49 652	45	0 50 348	9.97 962	4	35	41 40
26	9.47 654	40	9.49 696	44	0.50 304	9.97 958	4	34	.1 4.1 4.0
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33	.2 8.2 8.0
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32	.3 12.3 12.0
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31	.4 16.4 16.0
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	30	.5 20.5 20.0
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29	.6 24.6 24.0
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28	.7 28.7 28.0
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27	.8 32.8 32.0
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26	.9 36.9 36.0
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25	
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24	
37	9.48 094	40	9.50 180	44	0.49 820	9.97 914	4	23	
38	9.48 133	39	9.50 223	43	0.49 777	9.97 910	4	22	39 5
39	9.48 173	40	9.50 267	44	0.49 733	9 97 906	4	21	.1 3.9 0.5
40	9.48 213	40	9.50 311	44	0 49 689	9.97 902	4	20	.2 7.8 1.0
41	9 48 252	39	9.50 355	44	0.49 645	9.97 898	4	19	.3 11.7 1.5
42	9.48 292	40	9.50 398	43	0 49 602	9.97 894	4	18	.4 15.6 2.0
43	9.48 332	40	9.50 442	44	0.49 558	9.97 890	4	17	.5 19.5 2.5
44	9.48 371	39	9.50 485	43	0.49 515	9.97 886	4	16	.6 23 4 3.0
45	9 48 411	40	9.50 529	44	0.49 471	9.97 882	4	15	.7 27.3 3.5
46	9.48 450	39	9.50 572	43	0.49 428	9.97 878	4	14	.8 31.2 4.0
47	9.48 490	40	9.50 616	44	0.49 384	9.97 874	4	13	.9 35.1 4.5
48	9.48 529	39	9.50 659	43	0.49 341	9.97 870	4	12	
49	9.48 568	39	9.50 703	44	0.49 297	9.97 866	4	11	
50	9.48 607	39	9.50 746	43	0.49 254	9.97 861	5	10	4 3
51	9.48 647	40	9.50 789	43	0.49 211	9.97 857	4	9	.1 0.4 0.3
52	9.48 686	39	9.50 833	44	0.49 167	9 97 853	4	8	.2 0.8 0.6
53	9.48 725	39	9 50 876	43	0.49 124	9.97 849	4	7	.3 1.2 0.9
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6	.4 1.6 1.2
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5	.5 2.0 1.5
56	9.48 842	39	9.51 005	43	0 48 995	9.97 837	4	4	.6 2.4 1 8
57	9 48 881	39	9.51 048	43	0 48 952	9.97 833	4	3	.7 2.8 2.1
58	9.48 920	39	9.51 092	44	0.48 908	9.97 829	4	2	.8 3.2 2.4
59	9.48 959	39	9.51 135	43	0 48 865	9.97 825	4	1	.9 3.6 2.7
60	9.48 998	39	9 51 178	43	0.48 822	9.97 821	4	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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18°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.48 998		9.51 178		0.48 822	9.97 821		60	
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	4	59	
2	9.49 076	39	9.51 264	43	0.48 736	9.97 812	5	58	
3	9.49 115	39	9.51 306	42	0.48 694	9.97 808	4	57	43 42
4	9.49 153	38	9.51 349	43	0.48 651	9.97 804	4	56	.1 4.3 4.2
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	55	.2 8.6 8.4
6	9.49 231	38	9.51 435	43	0.48 565	9.97 796	4	54	.3 12.9 12.6
7	9.49 269	39	9.51 478	42	0.48 522	9.97 792	4	53	.4 17.2 16.8
8	9.49 308	39	9.51 520	43	0.48 480	9.97 788	4	52	.5 21.5 21.0
9	9.49 347	38	9.51 563	43	0.48 437	9.97 784	4	51	.6 25.8 25.2
10	9.49 385	39	9.51 606	42	0.48 394	9.97 779	5	50	.7 30.1 29.4
11	9.49 424	38	9.51 648	43	0.48 352	9.97 775	4	49	.8 34.4 33.6
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48	.9 38.7 37.8
13	9.49 500	39	9.51 734	42	0.48 266	9.97 767	4	47	
14	9 49 539	38	9.51 776	43	0 48 224	9.97 763	4	46	41
15	9 49 577	38	9.51 819	42	0.48 181	9.97 759	4	45	.1 4.1
16	9.49 615	39	9.51 861	42	0.48 139	9.97 754	5	44	.2 8.2
17	9.49 654	38	9.51 903	43	0.48 097	9.97 750	4	43	.3 12.3
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42	.4 16.4
19	9.49 730	38	9.51 988	43	0.48 012	9.97 742	4	41	.5 20.5
20	9.49 768	38	9.52 031	42	0.47 969	9.97 738	4	40	.6 24.6
21	9.49 806	38	9.52 073	42	0.47 927	9.97 734	4	39	.7 28.7
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	5	38	.8 32.8
23	9.49 882	38	9.52 157	43	0.47 843	9.97 725	4	37	.9 36.9
24	9.49 920	38	9.52 200	42	0.47 800	9.97 721	4	36	
25	9.49 958	38	9.52 242	42	0.47 758	9.97 717	4	35	
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	4	34	39 38
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	5	33	.1 3.9 3.8
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32	.2 7.8 7.6
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31	.3 11.7 11.4
30	9.50 148	37	9.52 452	42	0.47 548	9.97 696	4	30	.4 15.6 15.2
31	9.50 185	38	9.52 494	42	0.47 506	9.97 691	5	29	.5 19.5 19.0
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28	.6 23.4 22.8
33	9.50 261	37	9.52 578	42	0.47 422	9.97 683	4	27	.7 27.3 26.6
34	9.50 298	38	9.52 620	41	0.47 380	9.97 679	4	26	.8 31.2 30.4
35	9 50 336	38	9.52 661	42	0.47 339	9.97 674	5	25	.9 35.1 34.2
36	9 50 374	37	9.52 703	42	0.47 297	9.97 670	4	24	
37	9.50 411	38	9.52 745	42	0.47 255	9.97 666	4	23	
38	9 50 449	37	9.52 787	42	0.47 213	9.97 662	4	22	37 36
39	9.50 486	37	9.52 829	41	0.47 171	9.97 657	5	21	.1 3.7 3.6
40	9.50 523	38	9.52 870	42	0.47 130	9.97 653	4	20	.2 7.4 7.2
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19	.3 11.1 10.8
42	9.50 598	37	9.52 953	42	0.47 047	9.97 645	4	18	.4 14.8 14.4
43	9.50 635	37	9.52 995	42	0.47 005	9.97 640	5	17	.5 18.5 18.0
44	9.50 673	38	9.53 037	41	0.46 963	9.97 636	4	16	.6 22.2 21.6
45	9.50 710	37	9.53 078	42	0.46 922	9.97 632	4	15	.7 25.9 25.2
46	9.50 747	37	9.53 120	41	0.46 880	9.97 628	4	14	.8 29.6 28.8
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	5	13	.9 33.3 32.4
48	9.50 821	37	9.53 202	42	0.46 798	9.97 619	4	12	
49	9.50 858	37	9.53 244	41	0.46 756	9.97 615	4	11	
50	9.50 896	38	9.53 285	42	0.46 715	9.97 610	5	10	5 4
51	9.50 933	37	9.53 327	41	0.46 673	9.97 606	4	9	.1 0.5 0.4
52	9.50 970	37	9.53 368	41	0.46 632	9.97 602	4	8	.2 1.0 0.8
53	9.51 007	37	9.53 409	41	0.46 591	9.97 597	5	7	.3 1.5 1.2
54	9.51 043	36	9.53 450	41	0.46 550	9.97 593	4	6	.4 2.0 1.6
55	9.51 080	37	9.53 492	41	0.46 508	9.97 589	4	5	.5 2 5 2.0
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	5	4	.6 3.0 2.4
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3	.7 3.5 2.8
58	9.51 191	37	9.53 615	41	0.46 385	9.97 576	4	2	.8 4.0 3.2
59	9.51 227	36	9.53 656	41	0.46 344	9.97 571	5	1	.9 4.5 3.6
60	9.51 264	37	9.53 697	42	0.46 303	9.97 567	4	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

19°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.51 264		9.53 697		0.46 303	9.97 567		60	
1	9.51 301	37	9.53 738	41	0.46 262	9.97 563	4	59	
2	9.51 338	37	9.53 779	41	0.46 221	9.97 558	5	58	
3	9.51 374	36	9.53 820	41	0.46 180	9.97 554	4	57	41 40
4	9.51 411	37	9.53 861	41	0.46 139	9.97 550	4	56	.1 4.1 4.0
5	9.51 447	36	9.53 902	41	0.46 098	9.97 545	5	55	.2 8.2 8.0
6	9.51 484	37	9.53 943	41	0.46 057	9.97 541	4	54	.3 12.3 12.0
7	9.51 520	36	9.53 984	41	0.46 016	9.97 536	5	53	.4 16.4 16.0
8	9.51 557	37	9.54 025	41	0.45 975	9.97 532	4	52	.5 20.5 20.0
9	9.51 593	36	9.54 065	40	0.45 935	9.97 528	4	51	.6 24.6 24.0
10	9.51 629	36	9.54 106	41	0.45 894	9.97 523	5	50	.7 28.7 28.0
11	9.51 666	37	9.54 147	41	0.45 853	9.97 519	4	49	.8 32.8 32 0
12	9.51 702	36	9.54 187	40	0.45 813	9.97 515	4	48	.9 36.9 36.0
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	5	47	
14	9.51 774	36	9.54 269	41	0.45 731	9.97 506	4	46	39
15	9.51 811	37	9.54 309	40	0.45 691	9.97 501	5	45	.1 3.9
16	9.51 847	36	9.54 350	41	0.45 650	9.97 497	4	44	.2 7.8
17	9.51 883	36	9.54 390	40	0.45 610	9.97 492	5	43	.3 11.7
18	9.51 919	36	9.54 431	41	0.45 569	9.97 488	4	42	.4 15 6
19	9.51 955	36	9.54 471	40	0.45 529	9.97 484	4	41	.5 19.5
20	9.51 991	36	9.54 512	41	0.45 488	9.97 479	5	40	.6 23.4
21	9.52 027	36	9.54 552	40	0.45 448	9.97 475	4	39	.7 27.3
22	9.52 063	36	9.54 593	41	0.45 407	9.97 470	5	38	.8 31.2
23	9.52 099	36	9 54 633	40	0.45 367	9.97 466	4	37	.9 35.1
24	9.52 135	36	9.54 673	40	0.45 327	9.97 461	5	36	
25	9.52 171	36	9.54 714	41	0.45 286	9.97 457	4	35	37 36
26	9.52 207	35	9.54 754	40	0.45 246	9.97 453	4	34	.1 3.7 3.6
27	9.52 242	36	9.54 794	40	0.45 206	9.97 448	5	33	.2 7.4 7.2
28	9.52 278	36	9.54 835	41	0.45 165	9.97 444	4	32	.3 11.1 10.8
29	9.52 314	36	9.54 875	40	0.45 125	9.97 439	5	31	.4 14.8 14.4
30	9.52 350	35	9.54 915	40	0.45 085	9.97 435	4	30	.5 18.5 18.0
31	9.52 385	36	9 54 955	40	0.45 045	9.97 430	5	29	.6 22.2 21.6
32	9.52 421	35	9 54 995	40	0.45 005	9.97 426	4	28	.7 25.9 25.2
33	9.52 456	36	9.55 035	40	0.44 965	9.97 421	5	27	.8 29.6 28.8
34	9.52 492	35	9.55 075	40	0.44 925	9.97 417	4	26	.9 33.3 32.4
35	9.52 527	36	9.55 115	40	0.44 885	9.97 412	5	25	
36	9.52 563	35	9.55 155	40	0.44 845	9.97 408	4	24	
37	9.52 598	36	9.55 195	40	0.44 805	9.97 403	5	23	35 34
38	9.52 634	35	9.55 235	40	0.44 765	9.97 399	4	22	.1 3.5 3.4
39	9.52 669	36	9.55 275	40	0.44 725	9 97 394	5	21	.2 7.0 6.8
40	9.52 705	35	9.55 315	40	0.44 685	9.97 390	4	20	.3 10.5 10.2
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	5	19	.4 14.0 13.6
42	9.52 775	36	9.55 395	39	0.44 605	9.97 381	4	18	.5 17.5 17.0
43	9.52 811	35	9.55 434	40	0.44 566	9.97 376	5	17	.6 21.0 20 4
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	4	16	.7 24.5 23.8
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	5	15	.8 28 0 27.2
46	9.52 916	35	9 55 554	40	0.44 446	9.97 363	4	14	.9 31.5 30.6
47	9.52 951	35	9 55 593	39	0.44 407	9.97 358	5	13	
48	9 52 986	35	9.55 633	40	0.44 367	9.97 353	5	12	
49	9.53 021	35	9.55 673	40	0.44 327	9.97 349	4	11	
50	9 53 056	35	9.55 712	39	0.44 288	9.97 344	5	10	5 4
51	9 53 092	36	9.55 752	40	0 44 248	9.97 340	4	9	.1 0.5 0.4
52	9.53 126	34	9.55 791	39	0.44 209	9.97 335	5	8	.2 1.0 0.8
53	9.53 161	35	9.55 831	40	0.44 169	9.97 331	4	7	.3 1.5 1.2
54	9.53 196	35	9 55 870	39	0.44 130	9.97 326	5	6	.4 2.0 1.6
55	9.53 231	35	9.55 910	40	0.44 090	9.97 322	4	5	.5 2.5 2.0
56	9.53 266	35	9.55 949	39	0.44 051	9.97 317	5	4	.6 3.0 2.4
57	9.53 301	35	9 55 989	40	0.44 011	9.97 312	5	3	.7 3.5 2.8
58	9.53 336	35	9.56 028	39	0.43 972	9.97 308	4	2	.8 4.0 3.2
59	9.53 370	34	9.56 067	39	0.43 933	9.97 303	5	1	.9 4.5 3.6
60	9.53 405	35	9.56 107	40	0.43 893	9.97 299	4	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

70°

20°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.53 405		9.56 107		0.43 893	9.97 299		60	
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59	
2	9.53 475	35	9.56 185	39	0.43 815	9.97 289	5	58	
3	9.53 509	34	9.56 224	39	0.43 776	9.97 285	4	57	40 39
4	9.53 544	35	9.56 264	40	0.43 736	9.97 280	5	56	1 4.0 3.9
5	9.53 578	34	9.56 303	39	0.43 697	9.97 276	4	55	2 8.0 7.8
6	9.53 613	35	9.56 342	39	0.43 658	9.97 271	5	54	3 12.0 11.7
7	9.53 647	34	9.56 381	39	0.43 619	9.97 266	5	53	4 16.0 15.6
8	9.53 682	35	9.56 420	39	0.43 580	9.97 262	4	52	5 20.0 19.5
9	9.53 716	34	9.56 459	39	0.43 541	9.97 257	5	51	6 24.0 23.4
10	9.53 751	35	9.56 498	39	0.43 502	9.97 252	5	50	7 28.0 27.3
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	4	49	8 32.0 31.2
12	9.53 819	34	9.56 576	39	0.43 424	9.97 243	5	48	9 36.0 35.1
13	9.53 854	35	9.56 615	39	0.43 385	9.97 238	5	47	
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	4	46	38 37
15	9.53 922	34	9.56 693	39	0.43 307	9.97 229	5	45	1 3.8 3.7
16	9.53 957	35	9.56 732	39	0.43 268	9.97 224	5	44	2 7.6 7.4
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	4	43	3 11.4 11.1
18	9.54 025	34	9.56 810	39	0.43 190	9.97 215	5	42	4 15.2 14.8
19	9.54 059	34	9.56 849	39	0.43 151	9.97 210	5	41	5 19.0 18.5
20	9.54 093	34	9.56 887	38	0.43 113	9.97 206	4	40	6 22.8 22.2
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39	7 26.6 25.9
22	9.54 161	34	9.56 965	39	0.43 035	9.97 196	5	38	8 30.4 29.6
23	9.54 195	34	9.57 004	39	0.42 996	9.97 192	4	37	9 34.2 33.3
24	9.54 229	34	9.57 042	38	0.42 958	9.97 187	5	36	
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	5	35	
26	9.54 297	34	9.57 120	39	0.42 880	9.97 178	4	34	35
27	9.54 331	34	9.57 158	38	0.42 842	9.97 173	5	33	1 3.5
28	9.54 365	34	9.57 197	39	0.42 803	9.97 168	5	32	2 7.0
29	9.54 399	34	9.57 235	38	0.42 765	9.97 163	5	31	3 10.5
30	9.54 433	34	9.57 274	39	0.42 726	9.97 159	4	30	4 14.0
31	9.54 466	33	9.57 312	38	0.42 688	9.97 154	5	29	5 17.5
32	9.54 500	34	9.57 351	39	0.42 649	9.97 149	5	28	6 21.0
33	9.54 534	34	9.57 389	38	0.42 611	9.97 145	4	27	7 24.5
34	9.54 567	33	9.57 428	39	0.42 572	9.97 140	5	26	8 28.0
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	5	25	9 31.5
36	9.54 635	34	9.57 504	38	0.42 496	9.97 130	5	24	
37	9.54 668	33	9.57 543	39	0.42 457	9.97 126	4	23	
38	9.54 702	34	9.57 581	38	0.42 419	9.97 121	5	22	34 33
39	9.54 735	33	9.57 619	38	0.42 381	9.97 116	5	21	1 3.4 3.3
40	9.54 769	34	9.57 658	39	0.42 342	9.97 111	5	20	2 6.8 6.6
41	9.54 802	33	9.57 696	38	0.42 304	9.97 107	4	19	3 10.2 9.9
42	9.54 836	34	9.57 734	38	0.42 266	9.97 102	5	18	4 13.6 13.2
43	9.54 869	33	9.57 772	38	0.42 228	9.97 097	5	17	5 17.0 16.5
44	9.54 903	34	9.57 810	38	0.42 190	9.97 092	5	16	6 20.4 19.8
45	9.54 936	33	9.57 849	39	0.42 151	9.97 087	5	15	7 23.8 23.1
46	9.54 969	33	9.57 887	38	0.42 113	9.97 083	4	14	8 27.2 26.4
47	9.55 003	34	9.57 925	38	0.42 075	9.97 078	5	13	9 30.6 29.7
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12	
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11	
50	9.55 102	33	9.58 039	38	0.41 961	9.97 063	5	10	5 4
51	9.55 136	34	9.58 077	38	0.41 923	9.97 059	4	9	1 0.5 0.4
52	9.55 169	33	9.58 115	38	0.41 885	9.97 054	5	8	2 1.0 0.8
53	9.55 202	33	9.58 153	38	0.41 847	9.97 049	5	7	3 1.5 1.2
54	9.55 235	33	9.58 191	38	0.41 809	9.97 044	5	6	4 2.0 1.6
55	9.55 268	33	9.58 229	38	0.41 771	9.97 039	5	5	5 2.5 2.0
56	9.55 301	33	9.58 267	38	0.41 733	9.97 035	4	4	6 3.0 2.4
57	9.55 334	33	9.58 304	37	0.41 696	9.97 030	5	3	7 3.5 2.8
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2	8 4.0 3.2
59	9.55 400	33	9.58 380	38	0.41 620	9.97 020	5	1	9 4.5 3.6
60	9.55 433	33	9.58 418	38	0.41 582	9.97 015	5	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

69°

21°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.55 433		9.58 418		0.41 582	9.97 015		60	
1	9.55 466	33	9.58 455	37	0.41 545	9.97 010	5	59	
2	9.55 499	33	9.58 493	38	0.41 507	9.97 005	5	58	
3	9.55 532	33	9.58 531	38	0.41 469	9.97 001	4	57	38 37
4	9.55 564	32	9.58 569	38	0.41 431	9.96 996	5	56	.1 3.8 3.7
		33		37			5		.2 7.6 7.4
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	55	.3 11.4 11.1
6	9.55 630	33	9.58 644	37	0.41 356	9.96 986	5	54	.4 15.2 14.8
7	9.55 663	32	9.58 681	38	0.41 319	9.96 981	5	53	.5 19.0 18.5
8	9.55 695	33	9.58 719	38	0.41 281	9.96 976	5	52	.6 22.8 22.2
9	9.55 728	33	9.58 757	37	0.41 243	9.96 971	5	51	.7 26.6 25.9
10	9.55 761	32	9.58 794	38	0.41 206	9.96 966	5	50	.8 30.4 29.6
11	9.55 793	33	9.58 832	37	0.41 168	9.96 962	4	49	.9 34.2 33.3
12	9.55 826	32	9.58 869	38	0.41 131	9.96 957	5	48	
13	9.55 858	33	9.58 907	37	0.41 093	9.96 952	5	47	
14	9.55 891	32	9.58 944	37	0.41 056	9.96 947	5	46	36 33
15	9.55 923	33	9.58 981	38	0.41 019	9.96 942	5	45	.1 3.6 3.3
16	9.55 956	32	9.59 019	37	0.40 981	9.96 937	5	44	.2 7.2 6.6
17	9.55 988	33	9.59 056	38	0.40 944	9.96 932	5	43	.3 10.8 9.9
18	9.56 021	32	9.59 094	37	0.40 906	9.96 927	5	42	.4 14.4 13.2
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41	.5 18.0 16.5
20	9.56 085	33	9.59 168	37	0.40 832	9.96 917	5	40	.6 21.6 19.8
21	9.56 118	32	9.59 205	38	0.40 795	9.96 912	5	39	.7 25.2 23.1
22	9.56 150	32	9.59 243	37	0.40 757	9.96 907	5	38	.8 28.8 26.4
23	9.56 182	33	9.59 280	37	0.40 720	9.96 903	4	37	.9 32.4 29.7
24	9.56 215	32	9.59 317	37	0.40 683	9.96 898	5	36	
25	9.56 247	32	9.59 354	37	0.40 646	9.96 893	5	35	
26	9.56 279	32	9.59 391	38	0 40 609	9.96 888	5	34	32
27	9.56 311	32	9.59 429	37	0.40 571	9.96 883	5	33	.1 3.2
28	9.56 343	32	9.59 466	37	0.40 534	9 96 878	5	32	.2 6.4
29	9.56 375	33	9.59 503	37	0.40 497	9.96 873	5	31	.3 9.6
30	9.56 408	32	9.59 540	37	0.40 460	9.96 868	5	30	.4 12.8
31	9.56 440	32	9.59 577	37	0.40 423	9.96 863	5	29	.5 16.0
32	9.56 472	32	9.59 614	37	0.40 386	9.96 858	5	28	.6 19.2
33	9.56 504	32	9.59 651	37	0.40 349	9.96 853	5	27	.7 22.4
34	9.56 536	32	9.59 688	37	0.40 312	9.96 848	5	26	.8 25.6
35	9.56 568	32	9.59 725	37	0.40 275	9 96 843	5	25	.9 28.8
36	9.56 599	31	9.59 762	37	0.40 238	9.96 838	5	24	
37	9.56 631	32	9.59 799	36	0.40 201	9.96 833	5	23	
38	9.56 663	32	9.59 835	37	0.40 165	9.96 828	5	22	31 6
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21	.1 3.1 0.6
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	20	.2 6.2 1.2
41	9.56 759	31	9.59 946	37	0.40 054	9.96 813	5	19	.3 9.3 1.8
42	9.56 790	32	9.59 983	37	0 40 017	9.96 808	5	18	.4 12.4 2.4
43	9.56 822	32	9.60 019	36	0.39 981	9.96 803	5	17	.5 15.5 3.0
44	9.56 854	32	9.60 056	37	0.39 944	9.96 798	5	16	.6 18.6 3.6
45	9.56 886	32	9.60 093	37	0.39 907	9.96 793	5	15	.7 21.7 4.2
46	9.56 917	31	9.60 130	37	0.39 870	9.96 788	5	14	.8 24.8 4.8
47	9.56 949	32	9.60 166	36	0.39 834	9.96 783	5	13	.9 27.9 5.4
48	9.56 980	31	9.60 203	37	0.39 797	9.96 778	5	12	
49	9.57 012	32	9.60 240	37	0.39 760	9.96 772	6	11	
50	9.57 044	32	9.60 276	36	0.39 724	9.96 767	5	10	5 4
51	9.57 075	31	9.60 313	37	0.39 687	9.96 762	5	9	1 0.5 0.4
52	9.57 107	32	9.60 349	36	0.39 651	9.96 757	5	8	.2 1.0 0.8
53	9.57 138	31	9.60 386	37	0.39 614	9.96 752	5	7	.3 1.5 1.2
54	9.57 169	31	9.60 422	36	0.39 578	9.96 747	5	6	.4 2.0 1.6
55	9.57 201	32	9.60 459	37	0 39 541	9.96 742	5	5	.5 2.5 2.0
56	9 57 232	31	9.60 495	36	0.39 505	9.96 737	5	4	.6 3.0 2.4
57	9 57 264	32	9.60 532	37	0.39 468	9.96 732	5	3	.7 3.5 2.8
58	9 57 295	31	9.60 568	36	0.39 432	9.96 727	5	2	.8 4.0 3.2
59	9 57 326	31	9.60 605	37	0.39 395	9.96 722	5	1	.9 4.5 3.6
60	9.57 358	32	9.60 641	36	0.39 359	9.96 717	5	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

22°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.57 358		9.60 641		0.39 359	9.96 717		60	
1	9.57 389	31	9.60 677	36	0.39 323	9.96 711	6	59	
2	9.57 420	31	9.60 714	37	0.39 286	9.96 706	5	58	
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57	37 36
4	9.57 482	31	9.60 786	36	0.39 214	9.96 696	5	56	.1 3.7 3.6
		32		37			5		.2 7.4 7.2
5	9.57 514	31	9.60 823	36	0.39 177	9.96 691	5	55	.3 11.1 10.8
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54	.4 14.8 14.4
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53	.5 18.5 18.0
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	5	52	.6 22.2 21.6
9	9.57 638	31	9.60 967	36	0.39 033	9.96 670	6	51	.7 25.9 25.2
		31		37			5		.8 29.6 28.8
10	9.57 669	31	9.61 004	36	0 38 996	9.96 665	5	50	.9 33.3 32.4
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49	
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48	
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47	
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46	35
		31		36			5		.1 3.5
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	6	45	.2 7.0
16	9.57 855	30	9.61 220	36	0.38 780	9.96 634	5	44	.3 10.5
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43	.4 14.0
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42	.5 17.5
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41	.6 21.0
		30		36			5		.7 24.5
20	9.57 978	30	9.61 364	36	0.38 636	9.96 614	6	40	.8 28.0
21	9 58 008	31	9.61 400	36	0.38 600	9.96 608	5	39	.9 31.5
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38	
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37	
24	9.58 101	30	9.61 508	36	0.38 492	9.96 593	5	36	
		31		36			5		
25	9.58 131	31	9.61 544	35	0.38 456	9.96 588	6	35	32 31
26	9.58 162	30	9.61 579	36	0.38 421	9.96 582	5	34	.1 3.2 3.1
27	9.58 192	31	9.61 615	36	0.38 385	9.96 577	5	33	.2 6.4 6.2
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32	.3 9.6 9.3
29	9.58 253	31	9.61 687	36	0.38 313	9.96 567	5	31	.4 12.8 12.4
		30		35			5		.5 16.0 15.5
30	9.58 284	30	9.61 722	36	0.38 278	9.96 562	6	30	.6 19.2 18.6
31	9.58 314	31	9.61 758	36	0.38 242	9.96 556	5	29	.7 22.4 21.7
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28	.8 25.6 24.8
33	9.58 375	31	9.61 830	36	0.38 170	9.96 546	5	27	.9 28.8 27.9
34	9.58 406	30	9.61 865	35	0.38 135	9.96 541	6	26	
		31		36			5		
35	9.58 436	31	9.61 901	35	0.38 099	9.96 535	5	25	
36	9.58 467	30	9.61 936	36	0.38 064	9.96 530	5	24	
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23	30 29
38	9.58 527	30	9.62 008	35	0.37 992	9.96 520	5	22	.1 3.0 2.9
39	9.58 557	30	9.62 043	36	0.37 957	9.96 514	6	21	.2 6.0 5.8
		31		35			5		.3 9.0 8.7
40	9.58 588	30	9.62 079	35	0.37 921	9.96 509	5	20	.4 12.0 11.6
41	9.58 618	30	9.62 114	36	0.37 886	9.96 504	6	19	.5 15.0 14 5
42	9.58 648	30	9.62 150	35	0.37 850	9.96 498	5	18	.6 18.0 17.4
43	9.58 678	31	9.62 185	36	0.37 815	9.96 493	5	17	.7 21.0 20.3
44	9.58 709	30	9.62 221	35	0.37 779	9.96 488	5	16	.8 24.0 23.2
		30		36			5		.9 27.0 26.1
45	9 58 739	30	9.62 256	36	0.37 744	9.96 483	6	15	
46	9.58 769	30	9.62 292	35	0.37 708	9.96 477	5	14	
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13	
48	9.58 829	30	9.62 362	36	0.37 638	9.96 467	5	12	
49	9.58 859	30	9.62 398	35	0.37 602	9.96 461	6	11	6 5
		30		35			5		.1 0.6 0.5
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	10	.2 1.2 1.0
51	9.58 919	30	9.62 468	36	0.37 532	9.96 451	6	9	.3 1.8 1.5
52	9.58 949	30	9.62 504	35	0.37 496	9.96 445	5	8	.4 2.4 2.0
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7	.5 3.0 2.5
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	6	6	.6 3.6 3.0
		30		35			5		.7 4.2 3.5
55	9.59 039	30	9.62 609	36	0.37 391	9.96 429	5	5	.8 4.8 4.0
56	9.59 069	29	9.62 645	35	0.37 355	9.96 424	5	4	.9 5.4 4.5
57	9.59 098	30	9.62 680	35	0.37 320	9.96 419	6	3	
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	5	2	
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1	
60	9.59 188		9.62 785		0.37 215	9.96 403		0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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23°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9 59 188		9.62 785		0.37 215	9.96 403		60	
1	9.59 218	30	9.62 820	35	0.37 180	9.96 397	6	59	
2	9.59 247	29	9.62 855	35	0.37 145	9.96 392	5	58	
3	9.59 277	30	9.62 890	35	0.37 110	9.96 387	5	57	36 35
4	9.59 307	30	9.62 926	36	0.37 074	9.96 381	6	56	.1 3.6 3.5
5	9.59 336	29	9.62 961	35	0.37 039	9.96 376	5	55	.2 7.2 7.0
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	6	54	.3 10.8 10.5
7	9.59 396	30	9.63 031	35	0.36 969	9.96 365	5	53	.4 14.4 14.0
8	9.59 425	29	9.63 066	35	0.36 934	9.96 360	5	52	.5 18.0 17.5
9	9.59 455	30	9.63 101	35	0.36 899	9.96 354	6	51	.6 21.6 21.0
10	9.59 484	29	9.63 135	34	0.36 865	9.96 349	5	50	.7 25.2 24.5
11	9.59 514	30	9.63 170	35	0.36 830	9.96 343	6	49	.8 28.8 28.0
12	9.59 543	29	9.63 205	35	0.36 795	9.96 338	5	48	.9 32.4 31.5
13	9.59 573	30	9.63 240	35	0.36 760	9.96 333	5	47	
14	9.59 602	29	9.63 275	35	0.36 725	9.96 327	6	46	
15	9.59 632	30	9.63 310	35	0.36 690	9.96 322	5	45	34
16	9.59 661	29	9.63 345	34	0.36 655	9.96 316	6	44	.1 3.4
17	9.59 690	29	9.63 379	35	0.36 621	9.96 311	5	43	.2 6.8
18	9.59 720	30	9.63 414	35	0.36 586	9.96 305	6	42	.3 10 2
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	5	41	.4 13.6
20	9.59 778	29	9.63 484	35	0.36 516	9.96 294	6	40	.5 17.0
21	9.59 808	30	9.63 519	35	0.36 481	9.96 289	5	39	.6 20.4
22	9.59 837	29	9.63 553	34	0.36 447	9.96 284	5	38	.7 23.8
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	6	37	.8 27.2
24	9.59 895	29	9.63 623	35	0.36 377	9.96 273	5	36	.9 30.6
25	9.59 924	29	9.63 657	34	0.36 343	9.96 267	6	35	
26	9.59 954	30	9.63 692	35	0.36 308	9.96 262	5	34	30 29
27	9.59 983	29	9.63 726	34	0.36 274	9.96 256	6	33	.1 3.0 2.9
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	5	32	.2 6.0 5.8
29	9 60 041	29	9.63 796	35	0.36 204	9.96 245	6	31	.3 9.0 8.7
30	9.60 070	29	9.63 830	34	0.36 170	9.96 240	5	30	.4 12.0 11.6
31	9.60 099	29	9.63 865	35	0.36 135	9.96 234	6	29	.5 15.0 14.5
32	9.60 128	29	9.63 899	34	0.36 101	9 96 229	5	28	.6 18.0 17 4
33	9.60 157	29	9.63 934	35	0.36 066	9.96 223	6	27	.7 21.0 20 3
34	9.60 186	29	9.63 968	34	0.36 032	9.96 218	5	26	.8 24.0 23 2
35	9.60 215	29	9.64 003	35	0.35 997	9.96 212	6	25	.9 27.0 26 1
36	9.60 244	29	9.64 037	34	0.35 963	9.96 207	5	24	
37	9.60 273	29	9.64 072	35	0.35 928	9.96 201	6	23	
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	5	22	28
39	9.60 331	29	9 64 140	34	0.35 860	9.96 190	6	21	.1 2.8
40	9.60 359	28	9.64 175	35	0.35 825	9.96 185	5	20	.2 5.6
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	6	19	.3 8.4
42	9.60 417	29	9.64 243	34	0.35 757	9.96 174	5	18	.4 11.2
43	9.60 446	29	9.64 278	35	0.35 722	9.96 168	6	17	.5 14.0
44	9.60 474	28	9.64 312	34	0.35 688	9.96 162	6	16	.6 16.8
45	9.60 503	29	9 64 346	34	0.35 654	9.96 157	5	15	.7 19.6
46	9.60 532	29	9 64 381	35	0.35 619	9.96 151	6	14	.8 22.4
47	9.60 561	29	9.64 415	34	0.35 585	9.96 146	5	13	.9 25.2
48	9.60 589	28	9.64 449	34	0.35 551	9.96 140	6	12	
49	9.60 618	29	9.64 483	34	0.35 517	9.96 135	5	11	
50	9.60 646	28	9.64 517	34	0.35 483	9.96 129	6	10	6 5
51	9.60 675	29	9.64 552	35	0.35 448	9.96 123	6	9	.1 0.6 0.5
52	9.60 704	29	9.64 586	34	0.35 414	9.96 118	5	8	.2 1.2 1.0
53	9.60 732	28	9.64 620	34	0.35 380	9.96 112	6	7	.3 1.8 1.5
54	9.60 761	29	9.64 654	34	0.35 346	9.96 107	5	6	.4 2.4 2.0
55	9.60 789	28	9.64 688	34	0.35 312	9.96 101	6	5	.5 3.0 2.5
56	9.60 818	29	9.64 722	34	0.35 278	9.96 095	6	4	.6 3.6 3.0
57	9.60 846	28	9.64 756	34	0.35 244	9.96 090	5	3	.7 4.2 3.5
58	9.60 875	29	9.64 790	34	0.35 210	9.96 084	6	2	.8 4.8 4.0
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	5	1	.9 5.4 4.5
60	9.60 931	28	9.64 858	34	0.35 142	9.96 073	6	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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24°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.60 931		9.64 858		0.35 142	9.96 073		60	
1	9.60 960	29	9.64 892	34	0.35 108	9.96 067	6	59	
2	9.60 988	28	9.64 926	34	0.35 074	9.96 062	5	58	
3	9.61 016	28	9.64 960	34	0.35 040	9.96 056	6	57	34 33
4	9.61 045	29	9.64 994	34	0.35 006	9.96 050	6	56	.1 3.4 3.3
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	5	55	.2 6.8 6.6
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	6	54	.3 10.2 9.9
7	9.61 129	28	9.65 096	34	0.34 904	9.96 034	5	53	.4 13 6 13.2
8	9.61 158	29	9.65 130	34	0.34 870	9.96 028	6	52	.5 17 0 16 5
9	9.61 186	28	9.65 164	34	0.34 836	9.96 022	6	51	.6 20.4 19 8
10	9.61 214	28	9.65 197	33	0.34 803	9.96 017	5	50	.7 23.8 23 1
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49	.8 27.2 26 4
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	6	48	.9 30.6 29.7
13	9.61 298	28	9.65 299	34	0.34 701	9.96 000	5	47	
14	9.61 326	28	9.65 333	34	0.34 667	9.95 994	6	46	29
15	9.61 354	28	9.65 366	33	0.34 634	9.95 988	6	45	.1 2.9
16	9.61 382	28	9.65 400	34	0.34 600	9.95 982	6	44	.2 5.8
17	9.61 411	29	9.65 434	34	0.34 566	9.95 977	5	43	.3 8.7
18	9.61 438	27	9.65 467	33	0.34 533	9.95 971	6	42	.4 11.6
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	6	41	.5 14.5
20	9.61 494	28	9.65 535	34	0.34 465	9.95 960	5	40	.6 17.4
21	9.61 522	28	9.65 568	33	0.34 432	9.95 954	6	39	.7 20.3
22	9.61 550	28	9.65 602	34	0.34 398	9.95 948	6	38	.8 23.2
23	9.61 578	28	9.65 636	34	0.34 364	9.95 942	6	37	.9 26.1
24	9.61 606	28	9.65 669	33	0.34 331	9.95 937	5	36	
25	9.61 634	28	9.65 703	34	0.34 297	9.95 931	6	35	
26	9.61 662	28	9.65 736	33	0.34 264	9.95 925	6	34	28
27	9.61 689	27	9.65 770	34	0.34 230	9.95 920	5	33	.1 2.8
28	9.61 717	28	9.65 803	33	0.34 197	9.95 914	6	32	.2 5.6
29	9.61 745	28	9.65 837	34	0.34 163	9.95 908	6	31	.3 8.4
30	9.61 773	27	9.65 870	33	0.34 130	9.95 902	6	30	.4 11.2
31	9.61 800	28	9.65 904	34	0.34 096	9.95 897	5	29	.5 14.0
32	9.61 828	28	9.65 937	33	0.34 063	9.95 891	6	28	.6 16.8
33	9.61 856	27	9.65 971	34	0.34 029	9.95 885	6	27	.7 19.6
34	9.61 883	28	9.66 004	33	0.33 996	9.95 879	6	26	.8 22.4
35	9.61 911	28	9.66 038	34	0.33 962	9.95 873	6	25	.9 25.2
36	9.61 939	27	9.66 071	33	0.33 929	9.95 868	5	24	
37	9.61 966	28	9.66 104	33	0.33 896	9.95 862	6	23	
38	9.61 994	27	9.66 138	34	0.33 862	9.95 856	6	22	27
39	9.62 021	28	9.66 171	33	0.33 829	9.95 850	6	21	.1 2.7
40	9.62 049	27	9.66 204	33	0.33 796	9.95 844	6	20	.2 5.4
41	9.62 076	28	9.66 238	34	0.33 762	9.95 839	5	19	.3 8.1
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18	.4 10.8
43	9.62 131	28	9.66 304	33	0.33 696	9.95 827	6	17	.5 13.5
44	9.62 159	27	9.66 337	33	0.33 663	9.95 821	6	16	.6 16.2
45	9.62 186	28	9.66 371	34	0.33 629	9.95 815	6	15	.7 18 9
46	9.62 214	27	9.66 404	33	0.33 596	9.95 810	5	14	.8 21.6
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13	.9 24.3
48	9.62 268	28	9.66 470	33	0.33 530	9.95 798	6	12	
49	9.62 296	27	9.66 503	33	0.33 497	9.95 792	6	11	6 5
50	9.62 323	27	9.66 537	34	0.33 463	9.95 786	6	10	.1 0.6 0.5
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	6	9	.2 1.2 1.0
52	9.62 377	28	9.66 603	33	0.33 397	9.95 775	5	8	.3 1.8 1.5
53	9.62 405	27	9.66 636	33	0.33 364	9.95 769	6	7	.4 2.4 2.0
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6	.5 3.0 2.5
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5	.6 3.6 3.0
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4	.7 4.2 3.5
57	9.62 513	27	9.66 768	33	0.33 232	9.95 745	6	3	.8 4.8 4.0
58	9.62 541	28	9.66 801	33	0.33 199	9.95 739	6	2	.9 5.4 4.5
59	9.62 568	27	9.66 834	33	0.33 166	9.95 733	6	1	
60	9.62595	27	9.66 867	33	0.33 133	9.95 728	5	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

65°

25°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.62 595	27	9.66 867	33	0.33 133	9.95 728	6	60	
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59	
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58	
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57	33 32
4	9.62 703	27	9.66 999	33	0.33 001	9 95 704	6	56	.1 3.3 3.2
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55	.2 6.6 6.4
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54	.3 9.9 9.6
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53	.4 13.2 12.8
8	9.62 811	27	9 67 131	32	0.32 869	9.95 680	6	52	.5 16 5 16.0
9	9.62 838	27	9.67 163	33	0.32 837	9.95 674	6	51	.6 19.8 19.2
10	9.62 865	27	9.67 196	33	0.32 804	9.95 668	5	50	.7 23 1 22.4
11	9.62 892	26	9.67 229	33	0.32 771	9.95 663	6	49	.8 26.4 25.6
12	9.62 918	27	9.67 262	33	0.32 738	9.95 657	6	48	.9 29.7 28.8
13	9.62 945	27	9.67 295	33	0.32 705	9.95 651	6	47	
14	9.62 972	27	9.67 327	32	0.32 673	9.95 645	6	46	27
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	45	.1 2.7
16	9.63 026	26	9.67 393	33	0.32 607	9.95 633	6	44	.2 5.4
17	9.63 052	27	9.67 426	33	0.32 574	9.95 627	6	43	3 8.1
18	9.63 079	27	9.67 458	32	0.32 542	9.95 621	6	42	.4 10.8
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41	.5 13.5
20	9.63 133	26	9.67 524	33	0.32 476	9.95 609	6	40	.6 16.2
21	9.63 159	27	9.67 556	32	0.32 444	9.95 603	6	39	.7 18.9
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38	.8 21.6
23	9.63 213	26	9.67 622	33	0.32 378	9.95 591	6	37	.9 24.3
24	9.63 239	27	9.67 654	32	0.32 346	9.95 585	6	36	
25	9.63 266	26	9.67 687	33	0.32 313	9.95 579	6	35	26
26	9.63 292	27	9.67 719	32	0.32 281	9.95 573	6	34	.1 2.6
27	9.63 319	26	9.67 752	33	0.32 248	9.95 567	6	33	.2 5.2
28	9.63 345	27	9.67 785	33	0.32 215	9.95 561	6	32	.3 7.8
29	9.63 372	26	9.67 817	32	0.32 183	9.95 555	6	31	.4 10.4
30	9.63 398	27	9.67 850	33	0.32 150	9.95 549	6	30	.5 13.0
31	9.63 425	26	9.67 882	32	0.32 118	9.95 543	6	29	.6 15.6
32	9.63 451	27	9.67 915	33	0.32 085	9.95 537	6	28	.7 18 2
33	9.63 478	26	9.67 947	32	0.32 053	9.95 531	6	27	.8 20 8
34	9.63 504	27	9.67 980	33	0.32 020	9.95 525	6	26	.9 23.4
35	9.63 531	26	9.68 012	32	0.31 988	9.95 519	6	25	
36	9.63 557	26	9 68 044	32	0.31 956	9.95 513	6	24	
37	9 63 583	27	9.68 077	33	0.31 923	9.95 507	6	23	
38	9.63 610	26	9.68 109	32	0.31 891	9.95 500	7	22	7
39	9.63 636	26	9.68 142	33	0.31 858	9.95 494	6	21	.1 0.7
40	9.63 662	27	9.68 174	32	0.31 826	9.95 488	6	20	.2 1.4
41	9.63 689	26	9.68 206	32	0 31 794	9.95 482	6	19	.3 2.1
42	9.63 715	26	9.68 239	33	0.31 761	9.95 476	6	18	.4 2.8
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17	.5 3.5
44	9.63 767	26	9.68 303	32	0.31 697	9.95 464	6	16	.6 4 2
45	9.63 794	27	9.68 336	33	0.31 664	9.95 458	6	15	.7 4.9
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14	.8 5.6
47	9 63 846	26	9.68 400	32	0.31 600	9.95 446	6	13	.9 6.3
48	9 63 872	26	9.68 432	32	0.31 568	9.95 440	6	12	
49	9.63 898	26	9.68 465	33	0.31 535	9.95 434	6	11	
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	7	10	6 5
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9	.1 0.6 0.5
52	9 63 976	26	9.68 561	32	0.31 439	9.95 415	6	8	.2 1.2 1.0
53	9.64 002	26	9.68 593	32	0.31 407	9.95 409	6	7	.3 1 8 1.5
54	9.64 028	26	9.68 626	33	0.31 374	9.95 403	6	6	.4 2 4 2 0
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	5	.5 3.0 2.5
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	6	4	.6 3.6 3.0
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	7	3	.7 4.2 3.5
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2	.8 4.8 4.0
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1	.9 5.4 4.5
60	9.64 184		9.68 818		0.31 182	9.95 366		0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

64°

26°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	60	
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59	
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58	
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	7	57	32 31
4	9.64 288	25	9.68 946	32	0.31 054	9.95 341	6	56	.1 3.2 3.1
5	9.64 313	26	9.68 978	32	0.31 022	9.95 335	6	55	.2 6.4 6.2
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54	.3 9.6 9.3
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53	.4 12.8 12.4
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	7	52	.5 16.0 15.5
9	9.64 417	25	9.69 106	32	0.30 894	9.95 310	6	51	.6 19.2 18.6
10	9.64 442	26	9.69 138	32	0.30 862	9.95 304	6	50	.7 22.4 21.7
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49	.8 25.6 24.8
12	9.64 494	26	9.69 202	32	0.30 798	9.95 292	6	48	.9 28.8 27.9
13	9.64 519	25	9.69 234	32	0.30 766	9.95 286	6	47	
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	7	46	26
15	9.64 571	25	9.69 298	32	0.30 702	9.95 273	6	45	.1 2.6
16	9.64 596	26	9.69 329	31	0.30 671	9.95 267	6	44	.2 5.2
17	9.64 622	26	9.69 361	32	0.30 639	9.95 261	6	43	.3 7.8
18	9.64 647	25	9.69 393	32	0.30 607	9.95 254	7	42	.4 10.4
19	9.64 673	26	9.69 425	32	0.30 575	9.95 248	6	41	.5 13.0
20	9.64 698	25	9.69 457	32	0.30 543	9.95 242	6	40	.6 15.6
21	9.64 724	26	9.69 488	31	0.30 512	9.95 236	6	39	.7 18.2
22	9.64 749	25	9.69 520	32	0.30 480	9.95 229	7	38	.8 20.8
23	9.64 775	26	9.69 552	32	0.30 448	9.95 223	6	37	.9 23.4
24	9.64 800	25	9.69 584	32	0.30 416	9.95 217	6	36	
25	9.64 826	26	9.69 615	31	0.30 385	9.95 211	6	35	25
26	9.64 851	25	9.69 647	32	0.30 353	9.95 204	7	34	.1 2.5
27	9.64 877	26	9.69 679	32	0.30 321	9.95 198	6	33	.2 5.0
28	9.64 902	25	9.69 710	31	0.30 290	9.95 192	6	32	.3 7.5
29	9.64 927	25	9.69 742	32	0.30 258	9.95 185	7	31	.4 10.0
30	9.64 953	26	9.69 774	32	0.30 226	9.95 179	6	30	.5 12.5
31	9.64 978	25	9.69 805	31	0.30 195	9.95 173	6	29	.6 15.0
32	9.65 003	25	9.69 837	32	0.30 163	9.95 167	6	28	.7 17.5
33	9.65 029	26	9.69 868	31	0.30 132	9.95 160	7	27	.8 20.0
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26	.9 22.5
35	9.65 079	25	9.69 932	32	0.30 068	9.95 148	6	25	
36	9.65 104	26	9.69 963	31	0.30 037	9.95 141	7	24	
37	9.65 130	25	9.69 995	32	0.30 005	9.95 135	6	23	
38	9.65 155	25	9.70 026	31	0.29 974	9.95 129	6	22	24
39	9.65 180	25	9.70 058	32	0.29 942	9.95 122	7	21	.1 2.4
40	9.65 205	25	9.70 089	31	0.29 911	9.95 116	6	20	.2 4.8
41	9.65 230	25	9.70 121	32	0.29 879	9.95 110	6	19	.3 7.2
42	9.65 255	25	9.70 152	31	0.29 848	9.95 103	7	18	.4 9.6
43	9.65 281	26	9.70 184	32	0.29 816	9.95 097	6	17	.5 12.0
44	9.65 306	25	9.70 215	31	0.29 785	9.95 090	7	16	.6 14.4
45	9.65 331	25	9.70 247	32	0.29 753	9.95 084	6	15	.7 16.8
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	6	14	.8 19.2
47	9.65 381	25	9.70 309	31	0.29 691	9.95 071	7	13	.9 21.6
48	9.65 406	25	9.70 341	32	0.29 659	9.95 065	6	12	
49	9 65 431	25	9.70 372	31	0.29 628	9.95 059	6	11	
50	9.65 456	25	9.70 404	32	0.29 596	9.95 052	7	10	7 6
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	6	9	.1 0.7 0.6
52	9.65 506	25	9.70 466	31	0.29 534	9.95 039	7	8	.2 1.4 1.2
53	9.65 531	25	9.70 498	32	0.29 502	9.95 033	6	7	.3 2.1 1.8
54	9.65 556	25	9.70 529	31	0.29 471	9.95 027	6	6	.4 2.8 2.4
55	9.65 580	24	9.70 560	31	0.29 440	9.95 020	7	5	.5 3.5 3.0
56	9.65 605	25	9.70 592	32	0.29 408	9.95 014	6	4	.6 4.2 3.6
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	7	3	.7 4.9 4.2
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2	.8 5.6 4.8
59	9.65 680	25	9.70 685	31	0.29 315	9.94 995	6	1	.9 6.3 5.4
60	9.65 705	25	9.70 717	32	0.29 283	9.94 988	7	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

63°

27°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.65 705		9.70 717		0.29 283	9.94 988		60	
1	9.65 729	24	9.70 748	31	0.29 252	9.94 982	6	59	
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	7	58	
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	6	57	32 31
4	9.65 804	25	9.70 841	31	0.29 159	9.94 962	7	56	1 3.2 3.1
5	9.65 828	24	9.70 873	32	0.29 127	9.94 956	6	55	2 6.4 6.2
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	7	54	3 9.6 9.3
7	9.65 878	25	9.70 935	31	0.29 065	9.94 943	6	53	4 12.8 12.4
8	9.65 902	24	9.70 966	31	0.29 034	9.94 936	7	52	5 16 0 15 5
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	6	51	6 19.2 18 6
10	9.65 952	25	9.71 028	31	0.28 972	9 94 923	7	50	7 22.4 21.7
11	9.65 976	24	9.71 059	31	0.28 941	9.94 917	6	49	8 25.6 24.8
12	9.66 001	25	9.71 090	31	0.28 910	9.94 911	6	48	9 28.8 27.9
13	9.66 025	24	9.71 121	31	0.28 879	9.94 904	7	47	
14	9.66 050	25	9.71 153	32	0.28 847	9.94 898	6	46	30
15	9.66 075	25	9.71 184	31	0.28 816	9.94 891	7	45	.1 3.0
16	9.66 099	24	9.71 215	31	0.28 785	9.94 885	6	44	.2 6.0
17	9.66 124	25	9.71 246	31	0.28 754	9.94 878	7	43	.3 9.0
18	9.66 148	24	9.71 277	31	0.28 723	9.94 871	7	42	.4 12.0
19	9.66 173	25	9.71 308	31	0.28 692	9.94 865	6	41	.5 15.0
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	7	40	6 18.0
21	9.66 221	24	9.71 370	31	0.28 630	9.94 852	6	39	.7 21.0
22	9 66 246	25	9.71 401	31	0.28 599	9.94 845	7	38	.8 24.0
23	9 66 270	24	9.71 431	30	0.28 569	9.94 839	6	37	.9 27.0
24	9.66 295	25	9.71 462	31	0.28 538	9.94 832	7	36	
25	9.66 319	24	9.71 493	31	0.28 507	9.94 826	6	35	25 24
26	9.66 343	24	9.71 524	31	0.28 476	9.94 819	7	34	.1 2.5 2.4
27	9.66 368	25	9.71 555	31	0.28 445	9.94 813	6	33	.2 5.0 4.8
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32	.3 7.5 7.2
29	9.66 416	24	9.71 617	31	0.28 383	9.94 799	7	31	.4 10.0 9.6
30	9.66 441	25	9 71 648	31	0.28 352	9.94 793	6	30	.5 12.5 12.0
31	9.66 465	24	9.71 679	31	0.28 321	9.94 786	7	29	.6 15.0 14.4
32	9.66 489	24	9.71 709	30	0.28 291	9.94 780	6	28	.7 17.5 16.8
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	7	27	.8 20.0 19.2
34	9.66 537	24	9.71 771	31	0.28 229	9.94 767	6	26	.9 22.5 21.6
35	9.66 562	25	9.71 802	31	0.28 198	9.94 760	7	25	
36	9.66 586	24	9.71 833	31	0.28 167	9.94 753	7	24	
37	9.66 610	24	9.71 863	30	0.28 137	9.94 747	6	23	23
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	7	22	.1 2.3
39	9.66 658	24	9.71 925	31	0.28 075	9.94 734	6	21	.2 4.6
40	9.66 682	24	9.71 955	30	0.28 045	9.94 727	7	20	.3 6.9
41	9.66 706	24	9.71 986	31	0.28 014	9.94 720	7	19	.4 9.2
42	9.66 731	25	9.72 017	31	0.27 983	9.94 714	6	18	.5 11.5
43	9.66 755	24	9.72 048	31	0.27 952	9.94 707	7	17	.6 13.8
44	9.66 779	24	9.72 078	30	0.27 922	9.94 700	7	16	.7 16.1
45	9.66 803	24	9.72 109	31	0 27 891	9.94 694	6	15	.8 18.4
46	9.66 827	24	9.72 140	31	0.27 860	9.94 687	7	14	.9 20 7
47	9.66 851	24	9.72 170	30	0.27 830	9.94 680	7	13	
48	9.66 875	24	9.72 201	31	0.27 799	9.94 674	6	12	
49	9.66 899	24	9.72 231	30	0.27 769	9.94 667	7	11	7 6
50	9.66 922	23	9.72 262	31	0.27 738	9.94 660	7	10	.1 0.7 0.6
51	9.66 946	24	9.72 293	31	0.27 707	9.94 654	6	9	.2 1.4 1.2
52	9.66 970	24	9.72 323	30	0.27 677	9.94 647	7	8	.3 2.1 1.8
53	9.66 994	24	9.72 354	31	0.27 646	9.94 640	7	7	.4 2.8 2.4
54	9.67 018	24	9.72 384	30	0.27 616	9.94 634	6	6	.5 3.5 3.0
55	9.67 042	24	9.72 415	31	0.27 585	9.94 627	7	5	.6 4.2 3.6
56	9.67 066	24	9 72 445	30	0.27 555	9.94 620	7	4	.7 4.9 4.2
57	9.67 090	24	9.72 476	31	0.27 524	9.94 614	6	3	.8 5.6 4.8
58	9.67 113	23	9.72 506	30	0.27 494	9.94 607	7	2	.9 6.3 5.4
59	9.67 137	24	9.72 537	31	0.27 463	9.94 600	7	1	
60	9.67 161	24	9.72 567	30	0.27 433	9.94 593	7	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

62°

28°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.67 161	24	9.72 567	31	0.27 433	9.94 593	6	60	
1	9.67 185	23	9.72 598	30	0.27 402	9.94 587	7	59	
2	9.67 208	24	9.72 628	31	0.27 372	9.94 580	7	58	
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	6	57	31 30
4	9.67 256	24	9.72 689	31	0.27 311	9.94 567	7	56	.1 3.1 3.0
5	9.67 280	23	9 72 720	30	0.27 280	9.94 560	7	55	.2 6.2 6.0
6	9.67 303	24	9.72 750	30	0.27 250	9.94 553	7	54	.3 9 3 9 0
7	9.67 327	23	9.72 780	31	0.27 220	9.94 546	7	53	.4 12.4 12 0
8	9.67 350	24	9.72 811	30	0.27 189	9.94 540	5	52	.5 15.5 15 0
9	9.67 374	24	9.72 841	31	0.27 159	9.94 533	7	51	.6 18.6 18 0
10	9.67 398	23	9.72 872	30	0.27 128	9.94 526	7	50	.7 21.7 21.0
11	9.67 421	24	9.72 902	30	0.27 098	9.94 519	7	49	.8 24.8 24.0
12	9.67 445	23	9.72 932	31	0.27 068	9.94 513	6	48	.9 27.9 27.0
13	9.67 468	24	9.72 963	30	0.27 037	9.94 506	7	47	
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46	29
15	9 67 515	24	9.73 023	31	0.26 977	9.94 492	7	45	.1 2.9
16	9.67 539	23	9 73 054	30	0.26 946	9.94 485	7	44	.2 5.8
17	9.67 562	24	9.73 084	30	0.26 916	9.94 479	6	43	.3 8.7
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42	.4 11.6
19	9 67 609	24	9.73 144	30	0.26 856	9.94 465	7	41	.5 14.5
20	9.67 633	23	9.73 175	31	0.26 825	9.94 458	7	40	.6 17.4
21	9.67 656	24	9.73 205	30	0.26 795	9.94 451	7	39	.7 20.3
22	9.67 680	23	9.73 235	30	0.26 765	9.94 445	6	38	.8 23.2
23	9 67 703	23	9.73 265	30	0.26 735	9.94 438	7	37	.9 26.1
24	9.67 726	24	9.73 295	30	0.26 705	9.94 431	7	36	
25	9 67 750	23	9.73 326	31	0.26 674	9.94 424	7	35	24 23
26	9 67 773	23	9.73 356	30	0.26 644	9.94 417	7	34	.1 2.4 2.3
27	9 67 796	24	9.73 386	30	0.26 614	9.94 410	6	33	.2 4.8 4 6
28	9 67 820	23	9.73 416	30	0.26 584	9.94 404	7	32	.3 7.2 6.9
29	9 67 843	23	9.73 446	30	0.26 554	9.94 397	7	31	.4 9.6 9.2
30	9.67 866	24	9.73 476	31	0.26 524	9.94 390	7	30	.5 12.0 11.5
31	9.67 890	23	9.73 507	30	0.26 493	9.94 383	7	29	.6 14.4 13 8
32	9.67 913	23	9.73 537	30	0.26 463	9.94 376	7	28	.7 16.8 16 1
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27	.8 19.2 18 4
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26	.9 21.6 20.7
35	9.67 982	24	9.73 627	30	0.26 373	9.94 355	6	25	
36	9 68 006	23	9.73 657	30	0.26 343	9.94 349	7	24	
37	9 68 029	23	9.73 687	30	0.26 313	9.94 342	7	23	
38	9 68 052	23	9.73 717	30	0.26 283	9.94 335	7	22	22
39	9 68 075	23	9.73 747	30	0.26 253	9.94 328	7	21	.1 2.2
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	20	.2 4.4
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19	.3 6.6
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	7	18	.4 8.8
43	9.68 167	23	9.73 867	30	0 26 133	9.94 300	7	17	.5 11.0
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16	.6 13.2
45	9.68 213	24	9.73 927	30	0.26 073	9.94 286	7	15	.7 15.4
46	9.68 237	23	9.73 957	30	0 26 043	9.94 279	7	14	.8 17.6
47	9.68 260	23	9.73 987	30	0.26 013	9.94 273	6	13	.9 19.8
48	9.68 283	22	9.74 017	30	0.25 983	9.94 266	7	12	
49	9.68 305	23	9.74 047	30	0.25 953	9.94 259	7	11	
50	9.68 328	23	9.74 077	30	0.25 923	9.94 252	7	10	7 6
51	9.68 351	23	9.74 107	30	0.25 893	9.94 245	7	9	.1 0.7 0.6
52	9.68 374	23	9.74 137	30	0.25 863	9.94 238	7	8	.2 1.4 1.2
53	9.68 397	23	9.74 166	29	0.25 834	9.94 231	7	7	.3 2.1 1 8
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6	.4 2 8 2.4
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	5	.5 3 5 3.0
56	9.68 466	23	9.74 256	30	0.25 744	9.94 210	7	4	.6 4.2 3.6
57	9.68 489	23	9.74 286	30	0.25 714	9.94 203	7	3	.7 4.9 4.2
58	9.68 512	23	9.74 316	30	0.25 684	9.94 196	7	2	.8 5.6 4.8
59	9.68 534	22	9.74 345	29	0.25 655	9.94 189	7	1	9 6.3 5.4
60	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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29°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	60	
1	9.68 580	23	9.74 405	30	0.25 595	9.94 175	7	59	
2	9.68 603	22	9.74 435	30	0.25 565	9.94 168	7	58	
3	9.68 625	23	9.74 465	29	0.25 535	9.94 161	7	57	30
4	9.68 648	23	9.74 494	30	0.25 506	9.94 154	7	56	.1 3 0
5	9.68 671	23	9.74 524	30	0.25 476	9.94 147	7	55	.2 6 0
6	9.68 694	22	9.74 554	29	0.25 446	9.94 140	7	54	.3 9 0
7	9.68 716	23	9.74 583	30	0.25 417	9.94 133	7	53	.4 12.0
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52	.5 15.0
9	9.68 762	22	9.74 643	30	0.25 357	9.94 119	7	51	.6 18.0
10	9.68 784	23	9.74 673	29	0.25 327	9.94 112	7	50	.7 21.0
11	9.68 807	22	9.74 702	30	0.25 298	9.94 105	7	49	.8 24.0
12	9.68 829	23	9.74 732	30	0.25 268	9.94 098	8	48	.9 27.0
13	9.68 852	23	9.74 762	29	0.25 238	9.94 090	7	47	
14	9.68 875	22	9.74 791	30	0.25 209	9.94 083	7	46	29
15	9.68 897	23	9.74 821	30	0.25 179	9.94 076	7	45	.1 2.9
16	9.68 920	22	9.74 851	29	0.25 149	9.94 069	7	44	.2 5.8
17	9.68 942	23	9.74 880	30	0.25 120	9.94 062	7	43	.3 8.7
18	9.68 965	22	9.74 910	29	0.25 090	9.94 055	7	42	.4 11.6
19	9.68 987	23	9.74 939	30	0.25 061	9.94 048	7	41	.5 14.5
20	9.69 010	22	9.74 969	29	0.25 031	9.94 041	7	40	.6 17.4
21	9.69 032	23	9.74 998	30	0.25 002	9.94 034	7	39	.7 20.3
22	9.69 055	22	9.75 028	30	0.24 972	9.94 027	7	38	.8 23.2
23	9.69 077	23	9.75 058	30	0.24 942	9.94 020	8	37	.9 26.1
24	9.69 100	22	9.75 087	29	0.24 913	9.94 012	7	36	
25	9.69 122	22	9.75 117	30	0.24 883	9.94 005	7	35	28
26	9.69 144	23	9.75 146	29	0.24 854	9.93 998	7	34	.1 2.3
27	9.69 167	22	9.75 176	30	0.24 824	9.93 991	7	33	.2 4.6
28	9.69 189	23	9.75 205	29	0.24 795	9.93 984	7	32	.3 6.9
29	9.69 212	22	9.75 235	30	0.24 765	9.93 977	7	31	.4 9.2
30	9.69 234	22	9.75 264	29	0.24 736	9.93 970	7	30	.5 11.5
31	9.69 256	23	9.75 294	30	0.24 706	9.93 963	8	29	.6 13.8
32	9.69 279	22	9.75 323	29	0.24 677	9.93 955	7	28	.7 16.1
33	9.69 301	22	9.75 353	30	0.24 647	9.93 948	7	27	.8 18.4
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26	.9 20.7
35	9.69 345	23	9.75 411	29	0.24 589	9.93 934	7	25	
36	9.69 368	22	9.75 441	30	0.24 559	9.93 927	7	24	
37	9.69 390	22	9.75 470	29	0.24 530	9.93 920	8	23	
38	9.69 412	22	9.75 500	30	0.24 500	9.93 912	7	22	22
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21	.1 2.2
40	9.69 456	23	9.75 558	29	0.24 442	9.93 898	7	20	.2 4.4
41	9.69 479	22	9.75 588	30	0.24 412	9.93 891	7	19	.3 6.6
42	9.69 501	22	9.75 617	29	0.24 383	9.93 884	7	18	.4 8.8
43	9.69 523	22	9.75 647	30	0.24 353	9.93 876	8	17	.5 11.0
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16	.6 13.2
45	9.69 567	22	9.75 705	29	0.24 295	9.93 862	7	15	.7 15.4
46	9.69 589	22	9.75 735	30	0.24 265	9.93 855	7	14	.8 17.6
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	8	13	.9 19.8
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12	
49	9.69 655	22	9.75 822	29	0.24 178	9.93 833	7	11	8 7
50	9.69 677	22	9.75 852	30	0.24 148	9.93 826	7	10	.1 0.8 0.7
51	9.69 699	22	9.75 881	29	0.24 119	9.93 819	7	9	.2 1.6 1.4
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	8	8	.3 2.4 2.1
53	9.69 743	22	9.75 939	29	0.24 061	9.93 804	7	7	.4 3.2 2.8
54	9.69 765	22	9.75 969	30	0.24 031	9.93 797	7	6	.5 4.0 3.5
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	8	5	.6 4.8 4.2
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4	.7 5.6 4.9
57	9.69 831	22	9.76 056	29	0.23 944	9.93 775	7	3	.8 6.4 5.6
58	9.69 853	22	9.76 086	30	0.23 914	9.93 768	7	2	.9 7.2 6.3
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	8	1	
60	9.69 897		9.76 144	29	0.23 856	9.93 753	7	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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30°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	60	
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	8	59	
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	7	58	
3	9.69 963	21	9.76 231	30	0.23 769	9.93 731	7	57	30 29
4	9.69 984	22	9.76 261	29	0.23 739	9.93 724	7	56	.1 3.0 2.9
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	8	55	.2 6.0 5.8
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54	.3 9.0 8.7
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53	.4 12.0 11.6
8	9.70 072	21	9.76 377	29	0.23 623	9.93 695	7	52	.5 15.0 14 5
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	8	51	.6 18.0 17 4
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	50	.7 21.0 20.3
11	9.70 137	22	9.76 464	29	0.23 536	9.93 673	7	49	.8 24.0 23.2
12	9.70 159	21	9.76 493	29	0.23 507	9.93 665	8	48	.9 27.0 26.1
13	9.70 180	22	9.76 522	29	0.23 478	9.93 658	7	47	
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	8	46	
15	9.70 224	21	9.76 580	29	0.23 420	9.93 643	7	45	28
16	9.70 245	22	9.76 609	29	0.23 391	9.93 636	7	44	.1 2.8
17	9.70 267	21	9.76 639	30	0.23 361	9.93 628	8	43	.2 5.6
18	9.70 288	22	9.76 668	29	0.23 332	9.93 621	7	42	.3 8.4
19	9.70 310	22	9 76 697	29	0.23 303	9.93 614	7	41	.4 11.2
20	9.70 332	21	9.76 725	28	0.23 275	9.93 606	8	40	.5 14.0
21	9.70 353	22	9.76 754	29	0.23 246	9.93 599	7	39	.6 16.8
22	9.70 375	21	9.76 783	29	0.23 217	9.93 591	8	38	.7 19.6
23	9.70 396	22	9.76 812	29	0.23 188	9.93 584	7	37	.8 22.4
24	9.70 418	21	9.76 841	29	0.23 159	9.93 577	7	36	.9 25.2
25	9.70 439	22	9.76 870	29	0.23 130	9.93 569	8	35	
26	9.70 461	21	9 76 899	29	0.23 101	9.93 562	7	34	22
27	9.70 482	22	9.76 928	29	0.23 072	9.93 554	8	33	.1 2.2
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	7	32	.2 4.4
29	9.70 525	22	9.76 986	29	0.23 014	9.93 539	8	31	.3 6.6
30	9.70 547	21	9.77 015	29	0.22 985	9.93 532	7	30	.4 8.8
31	9.70 568	22	9.77 044	29	0.22 956	9.93 525	7	29	.5 11.0
32	9.70 590	21	9.77 073	29	0.22 927	9.93 517	8	28	.6 13.2
33	9.70 611	22	9.77 101	28	0.22 899	9.93 510	7	27	.7 15.4
34	9.70 633	21	9 77 130	29	0.22 870	9.93 502	8	26	.8 17.6
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	7	25	.9 19.8
36	9.70 675	22	9.77 188	29	0.22 812	9.93 487	8	24	
37	9.70 697	21	9.77 217	29	0.22 783	9.93 480	7	23	
38	9.70 718	21	9.77 246	29	0.22 754	9.93 472	8	22	21
39	9.70 739	22	9.77 274	28	0.22 726	9.93 465	7	21	.1 2.1
40	9.70 761	21	9.77 303	29	0.22 697	9.93 457	8	20	.2 4.2
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	7	19	.3 6.3
42	9.70 803	21	9.77 361	29	0.22 639	9.93 442	8	18	.4 8.4
43	9.70 824	22	9.77 390	29	0.22 610	9.93 435	7	17	.5 10 5
44	9.70 846	21	9.77 418	28	0.22 582	9.93 427	8	16	.6 12.6
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	7	15	.7 14.7
46	9.70 888	21	9.77 476	29	0.22 524	9.93 412	8	14	.8 16 8
47	9.70 909	22	9.77 505	29	0.22 495	9.93 405	7	13	.9 18.9
48	9.70 931	21	9.77 533	28	0.22 467	9.93 397	8	12	
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	7	11	
50	9.70 973	21	9.77 591	29	0.22 409	9.93 382	8	10	8 7
51	9.70 994	21	9.77 619	28	0.22 381	9.93 375	7	9	.1 0.8 0.7
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	8	8	.2 1 6 1.4
53	9.71 036	21	9.77 677	29	0.22 323	9.93 360	7	7	.3 2.4 2.1
54	9.71 058	22	9.77 706	29	0.22 294	9.93 352	8	6	.4 3.2 2.8
55	9.71 079	21	9.77 734	28	0.22 266	9.93 344	8	5	.5 4.0 3.5
56	9.71 100	21	9.77 763	29	0.22 237	9.93 337	7	4	.6 4.8 4.2
57	9.71 121	21	9.77 791	28	0.22 209	9.93 329	8	3	.7 5.6 4 9
58	9.71 142	21	9.77 820	29	0.22 180	9.93 322	7	2	.8 6.4 5 6
59	9.71 163	21	9.77 849	29	0.22 151	9.93 314	8	1	.9 7.2 6.3
60	9.71 184		9 77 877	28	0.22 123	9.93 307	7	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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31°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.71 184	21	9.77 877	29	0.22 123	9.93 307	8	60	
1	9 71 205	21	9.77 906	29	0.22 094	9.93 299	8	59	
2	9.71 226	21	9.77 935	28	0.22 065	9.93 291	7	58	
3	9.71 247	21	9.77 963	29	0.22 037	9.93 284	8	57	29
4	9.71 268	21	9.77 992	28	0 22 008	9.93 276	8	56	.1 2.9
5	9.71 289	21	9.78 020	29	0.21 980	9.93 269	7	55	.2 5.8
6	9 71 310	21	9.78 049	28	0.21 951	9.93 261	8	54	.3 8.7
7	9 71 331	21	9.78 077	29	0.21 923	9.93 253	8	53	.4 11.6
8	9.71 352	21	9.78 106	29	0.21 894	9.93 246	7	52	.5 14.5
9	9.71 373	20	9.78 135	28	0.21 865	9.93 238	8	51	.6 17.4
10	9.71 393	21	9.78 163	29	0.21 837	9.93 230	8	50	.7 20.3
11	9.71 414	21	9.78 192	28	0.21 808	9.93 223	7	49	.8 23.2
12	9.71 435	21	9.78 220	29	0.21 780	9.93 215	8	48	.9 26.1
13	9.71 456	21	9.78 249	28	0.21 751	9.93 207	8	47	
14	9.71 477	21	9.78 277	29	0.21 723	9.93 200	7	46	
15	9.71 498	21	9.78 306	28	0.21 694	9.93 192	8	45	28
16	9.71 519	20	9.78 334	29	0.21 666	9.93 184	8	44	.1 2.8
17	9.71 539	21	9.78 363	28	0.21 637	9.93 177	7	43	.2 5.6
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42	.3 8.4
19	9.71 581	21	9.78 419	29	0.21 581	9.93 161	8	41	.4 11.2
20	9.71 602	20	9.78 448	28	0.21 552	9.93 154	7	40	.5 14.0
21	9.71 622	21	9.78 476	29	0.21 524	9.93 146	8	39	.6 16.8
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	8	38	.7 19.6
23	9.71 664	21	9.78 533	29	0.21 467	9.93 131	7	37	.8 22.4
24	9.71 685	20	9.78 562	28	0.21 438	9.93 123	8	36	.9 25.2
25	9.71 705	21	9.78 590	28	0.21 410	9.93 115	8	35	
26	9.71 726	21	9.78 618	29	0.21 382	9.93 108	7	34	21
27	9.71 747	20	9.78 647	28	0.21 353	9.93 100	8	33	.1 2.1
28	9.71 767	21	9.78 675	29	0.21 325	9.93 092	8	32	.2 4.2
29	9.71 788	21	9.78 704	28	0.21 296	9.93 084	8	31	.3 6.3
30	9.71 809	20	9.78 732	28	0.21 268	9.93 077	7	30	.4 8.4
31	9.71 829	21	9.78 760	29	0.21 240	9.93 069	8	29	.5 10.5
32	9.71 850	20	9.78 789	28	0.21 211	9.93 061	8	28	.6 12.6
33	9.71 870	21	9.78 817	28	0.21 183	9.93 053	8	27	.7 14.7
34	9.71 891	20	9.78 845	29	0.21 155	9.93 046	7	26	.8 16.8
35	9.71 911	21	9.78 874	28	0.21 126	9.93 038	8	25	.9 18.9
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	8	24	
37	9.71 952	21	9.78 930	29	0.21 070	9.93 022	8	23	
38	9.71 973	21	9.78 959	28	0.21 041	9.93 014	8	22	20
39	9.71 994	20	9.78 987	28	0.21 013	9.93 007	7	21	.1 2.0
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	20	.2 4.0
41	9.72 034	21	9.79 043	28	0.20 957	9.92 991	8	19	.3 6.0
42	9.72 055	20	9.79 072	29	0.20 928	9.92 983	8	18	.4 8.0
43	9.72 075	21	9.79 100	28	0.20 900	9.92 976	7	17	.5 10.0
44	9.72 096	20	9.79 128	28	0.20 872	9.92 968	8	16	.6 12.0
45	9.72 116	21	9.79 156	28	0.20 844	9.92 960	8	15	.7 14.0
46	9.72 137	20	9.79 185	29	0.20 815	9.92 952	8	14	.8 16.0
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	8	13	.9 18.0
48	9.72 177	21	9.79 241	28	0.20 759	9.92 936	8	12	
49	9.72 198	20	9.79 269	28	0.20 731	9.92 929	7	11	8 7
50	9.72 218	20	9.79 297	28	0.20 703	9.92 921	8	10	.1 0.8 0.7
51	9.72 238	20	9.79 326	29	0.20 674	9.92 913	8	9	.2 1.6 1.4
52	9.72 259	21	9.79 354	28	0.20 646	9.92 905	8	8	.3 2.4 2.1
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	8	7	.4 3.2 2 8
54	9.72 299	20	9.79 410	28	0.20 590	9.92 889	8	6	.5 4.0 3 5
55	9 72 320	21	9.79 438	28	0.20 562	9.92 881	8	5	.6 4.8 4 2
56	9.72 340	20	9.79 466	28	0.20 534	9.92 874	7	4	.7 5 6 4 9
57	9.72 360	20	9.79 495	29	0.20 505	9.92 866	8	3	.8 6.4 5.6
58	9.72 381	21	9.79 523	28	0.20 477	9.92 858	8	2	.9 7.2 6.3
59	9.72 401	20	9.79 551	28	0.20 449	9.92 850	8	1	
60	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	0	
′	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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32°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.72 421		9.79 579		0.20 421	9.92 842		60	
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	8	59	
2	9 72 461	20	9.79 635	28	0.20 365	9.92 826	8	58	
3	9.72 482	21	9.79 663	28	0.20 337	9.92 818	8	57	29 26
4	9.72 502	20	9.79 691	28	0.20 309	9.92 810	8	56	.1 2.9 2.8
5	9.72 522	20	9.79 719	28	0.20 281	9 92 803	7	55	.2 5.8 5.6
6	9.72 542	20	9.79 747	29	0.20 253	9 92 795	8	54	.3 8.7 8.4
7	9.72 562	20	9.79 776	28	0.20 224	9.92 787	8	53	.4 11.6 11.2
8	9.72 582	20	9.79 804	28	0.20 196	9.92 779	8	52	.5 14.5 14.0
9	9.72 602	20	9.79 832	28	0.20 168	9 92 771	8	51	.6 17.4 16.8
10	9.72 622	21	9.79 860	28	0.20 140	9.92 763	8	50	.7 20.3 19.6
11	9.72 643	20	9.79 888	28	0.20 112	9.92 755	8	49	.8 23.2 22.4
12	9.72 663	20	9.79 916	28	0.20 084	9.92 747	8	48	.9 26.1 25.2
13	9.72 683	20	9.79 944	28	0.20 056	9.92 739	8	47	
14	9.72 703	20	9.79 972	28	0.20 028	9.92 731	8	46	27
15	9.72 723	20	9.80 000	28	0.20 000	9.92 723	8	45	.1 2.7
16	9.72 743	20	9.80 028	28	0.19 972	9.92 715	8	44	.2 5.4
17	9.72 763	20	9.80 056	28	0.19 944	9.92 707	8	43	.3 8.1
18	9.72 783	20	9.80 084	28	0.19 916	9.92 699	8	42	.4 10.8
19	9.72 803	20	9.80 112	28	0.19 888	9.92 691	8	41	.5 13.5
20	9.72 823	20	9.80 140	28	0.19 860	9.92 683	8	40	.6 16.2
21	9.72 843	20	9.80 168	28	0.19 832	9.92 675	8	39	.7 18.9
22	9.72 863	20	9.80 195	27	0.19 805	9.92 667	8	38	.8 21.6
23	9.72 883	20	9.80 223	28	0.19 777	9.92 659	8	37	.9 24.3
24	9.72 902	19	9.80 251	28	0.19 749	9.92 651	8	36	
25	9.72 922	20	9.80 279	28	0.19 721	9.92 643	8	35	21 20
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34	.1 2 1 2.0
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33	.2 4.2 4.0
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32	.3 6.3 6.0
29	9.73 002	20	9.80 391	28	0.19 609	9.92 611	8	31	.4 8.4 8.0
30	9.73 022	19	9.80 419	28	0.19 581	9.92 603	8	30	.5 10.5 10.0
31	9.73 041	20	9.80 447	27	0.19 553	9.92 595	8	29	.6 12.6 12.0
32	9.73 061	20	9.80 474	28	0.19 526	9.92 587	8	28	.7 14.7 14.0
33	9.73 081	20	9.80 502	28	0.19 498	9.92 579	8	27	.8 16.8 16.0
34	9.73 101	20	9.80 530	28	0.19 470	9.92 571	8	26	.9 18.9 18.0
35	9.73 121	19	9.80 558	28	0.19 442	9.92 563	8	25	
36	9.73 140	20	9.80 586	28	0.19 414	9.92 555	8	24	
37	9.73 160	20	9.80 614	28	0.19 386	9.92 546	9	23	
38	9.73 180	20	9.80 642	28	0.19 358	9.92 538	8	22	19 9
39	9.73 200	19	9.80 669	27	0.19 331	9.92 530	8	21	.1 1.9 0.9
40	9.73 219	20	9.80 697	28	0.19 303	9.92 522	8	20	.2 3.8 1.8
41	9.73 239	20	9.80 725	28	0.19 275	9.92 514	8	19	.3 5.7 2.7
42	9.73 259	19	9.80 753	28	0.19 247	9.92 506	8	18	.4 7.6 3.6
43	9.73 278	20	9.80 781	28	0.19 219	9.92 498	8	17	.5 9.5 4.5
44	9.73 298	20	9.80 808	27	0.19 192	9.92 490	8	16	.6 11.4 5.4
45	9.73 318	19	9.80 836	28	0.19 164	9.92 482	8	15	.7 13.3 6.3
46	9.73 337	20	9.80 864	28	0.19 136	9.92 473	9	14	.8 15.2 7.2
47	9.73 357	20	9.80 892	28	0.19 108	9.92 465	8	13	.9 17.1 8.1
48	9.73 377	19	9.80 919	27	0.19 081	9.92 457	8	12	
49	9.73 396	20	9.80 947	28	0.19 053	9.92 449	8	11	
50	9.73 416	19	9.80 975	28	0.19 025	9.92 441	8	10	8 7
51	9.73 435	20	9.81 003	27	0.18 997	9.92 433	8	9	1 0.8 0.7
52	9.73 455	19	9.81 030	28	0.18 970	9.92 425	8	8	.2 1.6 1.4
53	9.73 474	20	9.81 058	28	0.18 942	9.92 416	9	7	.3 2.4 2.1
54	9.73 494	19	9.81 086	27	0.18 914	9.92 408	8	6	.4 3 2 2.8
55	9.73 513	20	9.81 113	28	0.18 887	9.92 400	8	5	.5 4.0 3 5
56	9.73 533	19	9 81 141	28	0.18 859	9.92 392	8	4	.6 4.8 4 2
57	9 73 552	20	9.81 169	28	0.18 831	9.92 384	8	3	.7 5.6 4 9
58	9.73 572	19	9.81 196	27	0.18 804	9.92 376	8	2	.8 6.4 5.6
59	9.73 591	20	9 81 224	28	0.18 776	9.92 367	9	1	.9 7.2 6.3
60	9.73 611		9.81 252	28	0.18 748	9.92 359	8	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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33°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9·73 611	19	9·81 252	27	0·18 748	9·92 359	8	60	
1	9·73 630	20	9·81 279	28	0·18 721	9·92 351	8	59	
2	9·73 650	19	9·81 307	28	0·18 693	9·92 343	8	58	
3	9·73 669	20	9·81 335	27	0·18 665	9·92 335	9	57	28 27
4	9·73 689	19	9·81 362	28	0·18 638	9·92 326	8	56	.1 2.8 2.7
5	9·73 708	19	9·81 390	28	0·18 610	9·92 318	8	55	.2 5.6 5.4
6	9·73 727	20	9·81 418	27	0·18 582	9·92 310	8	54	.3 8.4 8.1
7	9·73 747	19	9·81 445	28	0·18 555	9·92 302	9	53	.4 11.2 10.8
8	9·73 766	19	9·81 473	27	0·18 527	9·92 293	8	52	.5 14.0 13.5
9	9·73 785	20	9·81 500	28	0·18 500	9·92 285	8	51	.6 16.8 16.2
10	9·73 805	19	9·81 528	28	0·18 472	9·92 277	8	50	.7 19.6 18.9
11	9·73 824	19	9·81 556	27	0·18 444	9·92 269	9	49	.8 22.4 21.6
12	9·73 843	20	9·81 583	28	0·18 417	9·92 260	8	48	.9 25.2 24.3
13	9·73 863	19	9·81 611	27	0·18 389	9·92 252	8	47	
14	9·73 882	19	9·81 638	28	0·18 362	9·92 244	9	46	20
15	9·73 901	20	9·81 666	27	0·18 334	9·92 235	8	45	.1 2.0
16	9·73 921	19	9·81 693	28	0·18 307	9·92 227	8	44	.2 4.0
17	9·73 940	19	9·81 721	27	0·18 279	9·92 219	8	43	.3 6.0
18	9·73 959	19	9·81 748	28	0·18 252	9·92 211	9	42	.4 8.0
19	9·73 978	19	9·81 776	27	0·18 224	9·92 202	8	41	.5 10.0
20	9·73 997	20	9·81 803	28	0·18 197	9·92 194	8	40	.6 12.0
21	9·74 017	19	9·81 831	27	0·18 169	9·92 186	9	39	.7 14.0
22	9·74 036	19	9·81 858	28	0·18 142	9·92 177	8	38	.8 16.0
23	9·74 055	19	9·81 886	27	0·18 114	9·92 169	8	37	.9 18.0
24	9·74 074	19	9·81 913	28	0·18 087	9·92 161	9	36	
25	9·74 093	20	9·81 941	27	0·18 059	9·92 152	8	35	19
26	9·74 113	19	9·81 968	28	0·18 032	9·92 144	8	34	.1 1.9
27	9·74 132	19	9·81 996	27	0·18 004	9·92 136	9	33	.2 3.8
28	9·74 151	19	9·82 023	28	0·17 977	9·92 127	8	32	.3 5.7
29	9·74 170	19	9·82 051	27	0·17 949	9·92 119	8	31	.4 7.6
30	9·74 189	19	9·82 078	28	0·17 922	9·92 111	9	30	.5 9.5
31	9·74 208	19	9·82 106	27	0·17 894	9·92 102	8	29	.6 11.4
32	9·74 227	19	9·82 133	28	0·17 867	9·92 094	8	28	.7 13.3
33	9·74 246	19	9·82 161	27	0·17 839	9·92 086	9	27	.8 15.2
34	9·74 265	19	9·82 188	27	0·17 812	9·92 077	8	26	.9 17.1
35	9·74 284	19	9·82 215	28	0·17 785	9·92 069	9	25	
36	9·74 303	19	9·82 243	27	0·17 757	9·92 060	8	24	
37	9·74 322	19	9·82 270	28	0·17 730	9·92 052	8	23	18
38	9·74 341	19	9·82 298	27	0·17 702	9·92 044	9	22	.1 1.8
39	9·74 360	19	9·82 325	27	0·17 675	9·92 035	8	21	2 3.6
40	9·74 379	19	9·82 352	28	0·17 648	9·92 027	9	20	.3 5.4
41	9·74 398	19	9·82 380	27	0·17 620	9·92 018	8	19	.4 7.2
42	9·74 417	19	9·82 407	28	0·17 593	9·92 010	8	18	.5 9.0
43	9·74 436	19	9·82 435	27	0·17 565	9·92 002	8	17	.6 10.8
44	9·74 455	19	9·82 462	27	0·17 538	9·91 993	9	16	.7 12.6
45	9·74 474	19	9·82 489	28	0·17 511	9·91 985	8	15	.8 14.4
46	9·74 493	19	9·82 517	27	0·17 483	9·91 976	9	14	.9 16.2
47	9·74 512	19	9·82 544	27	0·17 456	9·91 968	8	13	
48	9·74 531	18	9·82 571	28	0·17 429	9·91 959	9	12	
49	9·74 549	19	9·82 599	27	0·17 401	9·91 951	8	11	9 8
50	9·74 568	19	9·82 626	27	0·17 374	9·91 942	9	10	.1 0.9 0.8
51	9·74 587	19	9·82 653	28	0·17 347	9·91 934	8	9	.2 1.8 1.6
52	9·74 606	19	9·82 681	27	0·17 319	9·91 925	9	8	.3 2.7 2.4
53	9·74 625	19	9·82 708	27	0·17 292	9·91 917	8	7	.4 3.6 3.2
54	9·74 644	18	9·82 735	27	0·17 265	9·91 908	9	6	.5 4.5 4.0
55	9·74 662	19	9·82 762	28	0·17 238	9·91 900	8	5	.6 5.4 4.8
56	9·74 681	19	9·82 790	27	0·17 210	9·91 891	9	4	.7 6.3 5.6
57	9·74 700	19	9·82 817	27	0·17 183	9·91 883	8	3	.8 7.2 6.4
58	9·74 719	19	9·82 844	27	0·17 156	9·91 874	9	2	.9 8.1 7.2
59	9·74 737	18	9·82 871	27	0·17 129	9·91 866	8	1	
60	9·74 756	19	9·82 899	28	0·17 101	9·91 857	9	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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34°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.74 756	19	9.82 899	27	0.17 101	9.91 857	8	60	
1	9.74 775	19	9.82 926	27	0.17 074	9.91 849	9	59	
2	9.74 794	18	9.82 953	27	0.17 047	9.91 840	8	58	
3	9.74 812	19	9.82 980	28	0.17 020	9.91 832	9	57	28 27
4	9.74 831	19	9.83 008	27	0.16 992	9.91 823	8	56	.1 2.8 2.7
5	9.74 850	18	9.83 035	27	0.16 965	9.91 815	9	55	.2 5.6 5.4
6	9.74 868	19	9.83 062	27	0.16 938	9.91 806	8	54	.3 8.4 8.1
7	9.74 887	19	9.83 089	28	0.16 911	9.91 798	9	53	.4 11.2 10.8
8	9.74 906	18	9.83 117	27	0.16 883	9.91 789	8	52	.5 14.0 13.5
9	9.74 924	19	9.83 144	27	0.16 856	9.91 781	9	51	.6 16.8 16.2
10	9.74 943	18	9.83 171	27	0.16 829	9.91 772	9	50	.7 19.6 18.9
11	9.74 961	19	9.83 198	27	0.16 802	9.91 763	9	49	.8 22.4 21.6
12	9.74 980	19	9.83 225	27	0.16 775	9.91 755	8	48	.9 25.2 24.3
13	9.74 999	18	9.83 252	28	0.16 748	9.91 746	9	47	
14	9.75 017	19	9.83 280	27	0.16 720	9.91 738	8	46	26
15	9.75 036	18	9.83 307	27	0.16 693	9.91 729	9	45	.1 2.6
16	9.75 054	19	9.83 334	27	0.16 666	9.91 720	9	44	.2 5.2
17	9.75 073	18	9.83 361	27	0.16 639	9.91 712	8	43	.3 7.8
18	9.75 091	19	9.83 388	27	0.16 612	9.91 703	9	42	.4 10.4
19	9.75 110	18	9.83 415	27	0.16 585	9.91 695	8	41	.5 13.0
20	9.75 128	19	9.83 442	28	0.16 558	9.91 686	9	40	.6 15.6
21	9.75 147	18	9.83 470	27	0.16 530	9.91 677	9	39	.7 18.2
22	9.75 165	19	9.83 497	27	0.16 503	9.91 669	8	38	.8 20.8
23	9.75 184	18	9.83 524	27	0.16 476	9.91 660	9	37	.9 23.4
24	9.75 202	19	9.83 551	27	0 16 449	9.91 651	9	36	
25	9.75 221	18	9.83 578	27	0.16 422	9.91 643	8	35	29
26	9.75 239	19	9.83 605	27	0.16 395	9.91 634	9	34	.1 1.9
27	9.75 258	18	9.83 632	27	0.16 368	9.91 625	9	33	.2 3.8
28	9.75 276	18	9.83 659	27	0.16 341	9.91 617	8	32	.3 5.7
29	9.75 294	19	9.83 686	27	0.16 314	9.91 608	9	31	.4 7.6
30	9.75 313	18	9.83 713	27	0.16 287	9.91 599	9	30	.5 9.5
31	9.75 331	19	9.83 740	28	0.16 260	9.91 591	8	29	.6 11.4
32	9 75 350	18	9.83 768	27	0.16 232	9 91 582	9	28	.7 13.3
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	9	27	.8 15.2
34	9.75 386	19	9.83 822	27	0.16 178	9.91 565	8	26	.9 17.1
35	9.75 405	18	9.83 849	27	0.16 151	9.91 556	9	25	
36	9.75 423	18	9.83 876	27	0.16 124	9 91 547	9	24	
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	9	23	
38	9.75 459	19	9.83 930	27	0.16 070	9.91 530	8	22	18
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	9	21	.1 1.8
40	9.75 496	18	9.83 984	27	0 16 016	9.91 512	9	20	.2 3.6
41	9.75 514	19	9.84 011	27	0.15 989	9.91 504	8	19	.3 5.4
42	9.75 533	18	9.84 038	27	0.15 962	9.91 495	9	18	.4 7.2
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	9	17	.5 9.0
44	9.75 569	18	9.84 092	27	0.15 908	9.91 477	9	16	.6 10.8
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	8	15	.7 12.6
46	9.75 605	19	9.84 146	27	0.15 854	9.91 460	9	14	.8 14.4
47	9 75 624	18	9.84 173	27	0.15 827	9.91 451	9	13	.9 16.2
48	9.75 642	18	9.84 200	27	0.15 800	9.91 442	9	12	
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	9	11	
50	9.75 678	18	9.84 254	26	0.15 746	9.91 425	8	10	9 8
51	9.75 696	18	9.84 280	27	0.15 720	9.91 416	9	9	.1 0.9 0.8
52	9.75 714	18	9.84 307	27	0.15 693	9.91 407	9	8	.2 1.8 1.6
53	9.75 733	19	9.84 334	27	0.15 666	9.91 398	9	7	.3 2 7 2.4
54	9.75 751	18	9.84 361	27	0.15 639	9.91 389	9	6	.4 3 6 3.2
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	8	5	.5 4.5 4 0
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4	.6 5.4 4.8
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3	.7 6.3 5 6
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	9	2	.8 7.2 6.4
59	9.75 841	18	9.84 496	27	0.15 504	9.91 345	9	1	.9 8.1 7.2
60	9.75 859		9.84 523		0.15 477	9.91 336	9	0	
′	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.		Prop. Pts.

55°

35°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.75 859	18	9.84 523	27	0.15 477	9.91 336	8	60	
1	9.75 877	18	9.84 550	26	0.15 450	9.91 328	9	59	
2	9.75 895	18	9.84 576	27	C.15 424	9.91 319	9	58	
3	9.75 913	18	9.84 603	27	C.15 397	9.91 310	9	57	27 26
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56	.1 2.7 2.6
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	55	.2 5.4 5.2
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54	.3 8.1 7.8
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	9	53	.4 10.8 10.4
8	9.76 003	18	9.84 738	26	0.15 262	9.91 266	8	52	.5 13.5 13.0
9	9.76 021	18	9.84 764	27	0.15 236	9.91 257	9	51	.6 16.2 15.6
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	50	.7 18.9 18.2
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49	.8 21.6 20.8
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48	.9 24.3 23.4
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47	
14	9.76 111	18	9.84 899	26	0.15 101	9.91 212	9	46	18
15	9.76 129	17	9.84 925	27	0.15 075	9.91 203	9	45	.1 1.8
16	9.76 146	18	9.84 952	27	0.15 048	9.91 194	9	44	.2 3.6
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43	.3 5.4
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42	.4 7.2
19	9.76 200	18	9.85 033	27	0.14 967	9.91 167	9	41	.5 9.0
20	9.76 218	18	9.85 059	26	0.14 941	9.91 158	9	40	.6 10.8
21	9.76 236	17	9.85 086	27	0.14 914	9.91 149	9	39	.7 12.6
22	9.76 253	18	9.85 113	27	0.14 887	9.91 141	8	38	.8 14.4
23	9.76 271	18	9.85 140	27	0.14 860	9.91 132	9	37	.9 16.2
24	9.76 289	18	9.85 166	26	0.14 834	9.91 123	9	36	
25	9.76 307	17	9.85 193	27	0.14 807	9.91 114	9	35	17
26	9.76 324	18	9.85 220	27	0.14 780	9.91 105	9	34	.1 1.7
27	9.76 342	18	9.85 247	27	0.14 753	9 91 096	9	33	.2 3.4
28	9.76 360	18	9.85 273	26	0.14 727	9.91 087	9	32	.3 5.1
29	9.76 378	17	9.85 300	27	0.14 700	9.91 078	9	31	.4 6.8
30	9.76 395	18	9.85 327	27	0.14 673	9.91 069	9	30	.5 8.5
31	9.76 413	18	9.85 354	26	0.14 646	9.91 060	9	29	.6 10.2
32	9.76 431	17	9.85 380	27	0.14 620	9.91 051	9	28	.7 11.9
33	9.76 448	18	9.85 407	27	0.14 593	9.91 042	9	27	.8 13.6
34	9.76 466	18	9.85 434	26	0.14 566	9.91 033	10	26	.9 15.3
35	9.76 484	17	9.85 460	27	0.14 540	9.91 023	9	25	
36	9.76 501	18	9.85 487	27	0.14 513	9.91 014	9	24	
37	9.76 519	18	9.85 514	26	0.14 486	9.91 005	9	23	
38	9.76 537	17	9.85 540	27	0.14 460	9.90 996	9	22	10
39	9.76 554	18	9.85 567	27	0.14 433	9.90 987	9	21	.1 1.0
40	9.76 572	18	9.85 594	26	0.14 406	9.90 978	9	20	.2 2.0
41	9.76 590	17	9.85 620	27	0.14 380	9.90 969	9	19	.3 3.0
42	9.76 607	18	9.85 647	27	0.14 353	9.90 960	9	18	.4 4.0
43	9.76 625	17	9.85 674	26	0.14 326	9.90 951	9	17	.5 5.0
44	9.76 642	18	9.85 700	27	0.14 300	9.90 942	9	16	.6 6.0
45	9.76 660	17	9.85 727	27	0.14 273	9.90 933	9	15	.7 7.0
46	9.76 677	18	9.85 754	26	0.14 246	9.90 924	9	14	.8 8.0
47	9.76 695	17	9.85 780	27	0.14 220	9.90 915	9	13	.9 9.0
48	9.76 712	18	9.85 807	27	0.14 193	9.90 906	9	12	
49	9.76 730	17	9.85 834	26	0.14 166	9.90 896	10	11	
50	9.76 747	18	9.85 860	27	0.14 140	9.90 887	9	10	9 8
51	9.76 765	17	9.85 887	26	0.14 113	9.90 878	9	9	.1 0.9 0.8
52	9.76 782	18	9 85 913	27	0.14 087	9.90 869	9	8	.2 1.8 1.6
53	9.76 800	17	9 85 940	27	0.14 060	9.90 860	9	7	.3 2.7 2.4
54	9.76 817	18	9.85 967	26	0.14 033	9.90 851	9	6	.4 3.6 3.2
55	9.76 835	17	9.85 993	27	0.14 007	9.90 842	9	5	.5 4.5 4.0
56	9.76 852	18	9.86 020	26	0.13 980	9.90 832	10	4	.6 5.4 4.8
57	9.76 870	17	9 86 046	27	0.13 954	9.90 823	9	3	.7 6.3 5.6
58	9.76 887	17	9.86 073	27	0.13 927	9.90 814	9	2	.8 7.2 6.4
59	9.76 904	18	9.86 100	27	0.13 900	9.90 805	9	1	.9 8.1 7 2
60	9.76 922		9.86 126	26	0.13 874	9.90 796	9	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

54°

36°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.76 922	17	9.86 126	27	0.13 874	9.90 796	9	60	
1	9.76 939	18	9.86 153	26	0.13 847	9.90 787	10	59	
2	9.76 957	17	9.86 179	27	0.13 821	9.90 777	9	58	
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57	27 26
4	9.76 991	18	9.86 232	27	0.13 768	9.90 759	9	56	.1 2.7 2.6
5	9.77 009	17	9.86 259	26	0.13 741	9.90 750	9	55	.2 5.4 5.2
6	9.77 026	17	9.86 285	27	0.13 715	9.90 741	10	54	.3 8.1 7.8
7	9.77 043	18	9.86 312	26	0.13 688	9.90 731	9	53	.4 10.8 10.4
8	9.77 061	17	9.86 338	27	0.13 662	9.90 722	9	52	.5 13.5 13.0
9	9.77 078	17	9.86 365	27	0.13 635	9.90 713	9	51	.6 16.2 15.6
10	9.77 095	17	9.86 392	26	0.13 608	9.90 704	9	50	.7 18.9 18.2
11	9.77 112	18	9.86 418	27	0.13 582	9.90 694	10	49	.8 21.6 20.8
12	9.77 130	17	9.86 445	26	0.13 555	9.90 685	9	48	.9 24.3 23.4
13	9.77 147	17	9.86 471	27	0.13 529	9.90 676	9	47	
14	9.77 164	17	9.86 498	26	0.13 502	9.90 667	9	46	18
15	9.77 181	18	9.86 524	27	0.13 476	9.90 657	10	45	.1 1.8
16	9.77 199	17	9.86 551	26	0.13 449	9.90 648	9	44	.2 3.6
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43	.3 5.4
18	9.77 233	17	9.86 603	27	0.13 397	9.90 630	9	42	.4 7.2
19	9.77 250	18	9.86 630	26	0.13 370	9.90 620	10	41	.5 9.0
20	9.77 268	17	9.86 656	27	0.13 344	9.90 611	9	40	.6 10.8
21	9.77 285	17	9.86 683	26	0.13 317	9.90 602	9	39	.7 12.6
22	9.77 302	17	9.86 709	27	0.13 291	9.90 592	10	38	.8 14.4
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37	.9 16.2
24	9.77 336	17	9.86 762	27	0.13 238	9.90 574	9	36	
25	9.77 353	17	9.86 789	26	0.13 211	9.90 565	9	35	17
26	9.77 370	17	9.86 815	27	0.13 185	9.90 555	10	34	.1 1.7
27	9.77 387	18	9.86 842	26	0.13 158	9.90 546	9	33	.2 3.4
28	9.77 405	17	9.86 868	26	0.13 132	9.90 537	9	32	.3 5.1
29	9.77 422	17	9.86 894	27	0.13 106	9.90 527	10	31	.4 6.8
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	30	.5 8.5
31	9.77 456	17	9.86 947	27	0.13 053	9.90 509	9	29	.6 10.2
32	9.77 473	17	9.86 974	26	0.13 026	9.90 499	10	28	.7 11.9
33	9.77 490	17	9.87 000	27	0.13 000	9.90 490	9	27	.8 13.6
34	9.77 507	17	9.87 027	26	0.12 973	9.90 480	10	26	.9 15.3
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25	
36	9.77 541	17	9.87 079	27	0.12 921	9.90 462	9	24	
37	9.77 558	17	9.87 106	26	0.12 894	9.90 452	10	23	
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22	16
39	9.77 592	17	9.87 158	27	0.12 842	9.90 434	9	21	.1 1.6
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	10	20	.2 3.2
41	9.77 626	17	9.87 211	27	0.12 789	9.90 415	9	19	.3 4.8
42	9.77 643	17	9.87 238	26	0.12 762	9.90 405	10	18	.4 6.4
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	9	17	.5 8.0
44	9.77 677	17	9.87 290	27	0.12 710	9.90 386	10	16	.6 9.6
45	9.77 694	17	9.87 317	26	0.12 683	9.90 377	9	15	.7 11.2
46	9.77 711	17	9.87 343	26	0.12 657	9.90 368	9	14	.8 12.8
47	9.77 728	16	9.87 369	27	0.12 631	9.90 358	10	13	.9 14.4
48	9.77 744	17	9.87 396	26	0.12 604	9.90 349	9	12	
49	9.77 761	17	9.87 422	26	0.12 578	9.90 339	10	11	
50	9.77 778	17	9.87 448	27	0.12 552	9.90 330	9	10	10 9
51	9.77 795	17	9.87 475	26	0.12 525	9.90 320	10	9	.1 1.0 0.9
52	9.77 812	17	9.87 501	26	0.12 499	9.90 311	9	8	.2 2.0 1.8
53	9.77 829	17	9.87 527	27	0.12 473	9.90 301	10	7	.3 3.0 2.7
54	9.77 846	16	9.87 554	26	0.12 446	9.90 292	9	6	.4 4.0 3.6
55	9.77 862	17	9.87 580	26	0.12 420	9.90 282	10	5	.5 5.0 4.5
56	9.77 879	17	9.87 606	26	0.12 394	9.90 273	9	4	.6 6.0 5.4
57	9.77 896	17	9.87 633	27	0.12 367	9.90 263	10	3	.7 7.0 6.3
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	9	2	.8 8.0 7.2
59	9.77 930	16	9.87 685	26	0.12 315	9.90 244	10	1	.9 9.0 8.1
60	9.77 946		9.87 711		0.12 289	9.90 235	9	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

53°

37°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.77 946	17	9.87 711	27	0.12 289	9.90 235	10	60	
1	9.77 963	17	9.87 738	26	0.12 262	9.90 225	9	59	
2	9.77 980	17	9.87 764	26	0.12 236	9.90 216	10	58	
3	9.77 997	16	9.87 790	27	0.12 210	9.90 206	9	57	1 | 27
4	9.78 013	17	9.87 817	26	0.12 183	9.90 197	10	56	.1 | 2.7
5	9.78 030	17	9.87 843	26	0.12 157	9.90 187	9	55	.2 | 5.4
6	9.78 047	16	9.87 869	26	0.12 131	9.90 178	10	54	.3 | 8.1
7	9.78 063	17	9.87 895	27	0.12 105	9.90 168	9	53	.4 | 10.8
8	9.78 080	17	9.87 922	26	0.12 078	9.90 159	10	52	.5 | 13.5
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	10	51	.6 | 16.2
10	9.78 113	17	9.87 974	26	0.12 026	9.90 139	9	50	.7 | 18.9
11	9.78 130	17	9.88 000	27	0.12 000	9.90 130	10	49	.8 | 21.6
12	9.78 147	16	9.88 027	26	0.11 973	9.90 120	9	48	.9 | 24.3
13	9.78 163	17	9.88 053	26	0.11 947	9.90 111	10	47	
14	9.78 180	17	9.88 079	26	0.11 921	9.90 101	10	46	
15	9.78 197	16	9.88 105	26	0.11 895	9.90 091	9	45	26
16	9.78 213	17	9.88 131	27	0.11 869	9.90 082	10	44	.1 | 2.6
17	9.78 230	16	9.88 158	26	0.11 842	9.90 072	10	43	.2 | 5.2
18	9.78 246	17	9.88 184	26	0.11 816	9.90 063	9	42	.3 | 7.8
19	9.78 263	17	9.88 210	26	0.11 790	9.90 053	10	41	.4 | 10.4
20	9.78 280	16	9.88 236	26	0.11 764	9.90 043	10	40	.5 | 13.0
21	9.78 296	17	9.88 262	27	0.11 738	9.90 034	9	39	.6 | 15.6
22	9.78 313	16	9.88 289	26	0.11 711	9.90 024	10	38	.7 | 18.2
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	10	37	.8 | 20.8
24	9.78 346	16	9.88 341	26	0.11 659	9.90 005	9	36	.9 | 23.4
25	9.78 362	17	9.88 367	26	0.11 633	9.89 995	10	35	
26	9.78 379	16	9.88 393	27	0.11 607	9.89 985	10	34	17
27	9.78 395	17	9.88 420	26	0.11 580	9.89 976	9	33	.1 | 1.7
28	9.78 412	16	9.88 446	26	0.11 554	9.89 966	10	32	.2 | 3.4
29	9.78 428	17	9.88 472	26	0.11 528	9.89 956	10	31	.3 | 5.1
30	9.78 445	16	9.88 498	26	0.11 502	9.89 947	9	30	.4 | 6.8
31	9.78 461	17	9.88 524	26	0.11 476	9.89 937	10	29	.5 | 8.5
32	9.78 478	16	9.88 550	27	0.11 450	9.89 927	10	28	.6 | 10.2
33	9.78 494	16	9.88 577	26	0.11 423	9.89 918	9	27	.7 | 11.9
34	9.78 510	17	9.88 603	26	0.11 397	9.89 908	10	26	.8 | 13.6
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	10	25	.9 | 15.3
36	9.78 543	17	9.88 655	26	0.11 345	9.89 888	10	24	
37	9.78 560	16	9.88 681	26	0.11 319	9.89 879	9	23	
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22	16
39	9.78 592	17	9.88 733	26	0.11 267	9.89 859	10	21	.1 | 1.6
40	9.78 609	16	9.88 759	27	0.11 241	9.89 849	10	20	.2 | 3.2
41	9.78 625	17	9.88 786	26	0.11 214	9.89 840	9	19	.3 | 4.8
42	9.78 642	16	9.88 812	26	0.11 188	9.89 830	10	18	.4 | 6.4
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17	.5 | 8.0
44	9.78 674	17	9.88 864	26	0.11 136	9.89 810	10	16	.6 | 9.6
45	9.78 691	16	9.88 890	26	0.11 110	9.89 801	9	15	.7 | 11.2
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14	.8 | 12.8
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13	.9 | 14.4
48	9.78 739	17	9.88 968	26	0.11 032	9.89 771	10	12	
49	9.78 756	16	9.88 994	26	0.11 006	9.89 761	10	11	
50	9.78 772	16	9.89 020	26	0.10 980	9.89 752	9	10	10 | 9
51	9.78 788	17	9.89 046	26	0.10 954	9.89 742	10	9	.1 | 1.0 | 0.9
52	9.78 805	16	9.89 073	27	0.10 927	9.89 732	10	8	.2 | 2.0 | 1.8
53	9.78 821	16	9.89 099	26	0.10 901	9.89 722	10	7	3 | 3.0 | 2.7
54	9.78 837	16	9.89 125	26	0.10 875	9.89 712	10	6	4 | 4.0 | 3.6
55	9.78 853	16	9.89 151	26	0.10 849	9.89 702	10	5	.5 | 5.0 | 4.5
56	9.78 869	17	9.89 177	26	0.10 823	9.89 693	9	4	.6 | 6.0 | 5.4
57	9.78 886	16	9.89 203	26	0.10 797	9.89 683	10	3	.7 | 7.0 | 6.3
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2	.8 | 8.0 | 7.2
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1	.9 | 9.0 | 8.1
60	9.78 934		9.89 281		0.10 719	9.89 653		0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

52°

38°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	60	
1	9.78 950	17	9.89 307	26	0.10 693	9.89 643	10	59	
2	9 78 967	16	9.89 333	26	0.10 667	9.89 633	10	58	
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	10	57	26 25
4	9.78 999	16	9.89 385	26	0.10 615	9.89 614	10	56	.1 2.6 2.5
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	55	.2 5.2 5.0
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54	.3 7.8 7.5
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53	.4 10.4 10.0
8	9.79 063	16	9 89 489	26	0.10 511	9.89 574	10	52	.5 13.0 12.5
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51	.6 15.6 15 0
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	10	50	.7 18.2 17.5
11	9.79 111	17	9.89 567	26	0.10 433	9.89 544	10	49	.8 20.8 20.0
12	9.79 128	16	9.89 593	26	0.10 407	9.89 534	10	48	.9 23.4 22.5
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47	
14	9.79 160	16	9.89 645	26	0.10 355	9.89 514	10	46	17
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	9	45	.1 1.7
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495	10	44	.2 3.4
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485	10	43	.3 5.1
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475	10	42	.4 6.8
19	9.79 240	16	9.89 775	26	0.10 225	9.89 465	10	41	.5 8.5
20	9.79 256	16	9.89 801	26	0.10 199	9.89 455	10	40	.6 10.2
21	9.79 272	16	9 89 827	26	0.10 173	9.89 445	10	39	.7 11.9
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435	10	38	.8 13.6
23	9.79 304	16	9.89 879	26	0.10 121	9.89 425	10	37	.9 15.3
24	9.79 319	15	9.89 905	26	0.10 095	9.89 415	10	36	
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405	10	35	16 15
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395	10	34	.1 1.6 1.5
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385	10	33	.2 3.2 3.0
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375	11	32	.3 4.8 4.5
29	9.79 399	16	9.90 035	26	0.09 965	9.89 364	10	31	.4 6 4 6.0
30	9.79 415	16	9.90 061	25	0.09 939	9.89 354	10	30	.5 8.0 7.5
31	9.79 431	16	9.90 086	26	0.09 914	9.89 344	10	29	.6 9.6 9.0
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28	.7 11.2 10.5
33	9.79 463	15	9.90 138	26	0.09 862	9.89 324	10	27	.8 12.8 12.0
34	9.79 478	16	9.90 164	26	0.09 836	9.89 314	10	26	.9 14.4 13.5
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25	
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24	
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23	
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22	11
39	9.79 558	16	9.90 294	26	0.09 706	9.89 264	10	21	.1 1.1
40	9.79 573	15	9.90 320	26	0.09 680	9.89 254	10	20	.2 2.2
41	9.79 589	16	9.90 346	26	0.09 654	9.89 244	10	19	.3 3.3
42	9.79 605	16	9.90 371	25	0.09 629	9.89 233	11	18	.4 4.4
43	9.79 621	16	9.90 397	26	0.09 603	9.89 223	10	17	.5 5.5
44	9.79 636	15	9.90 423	26	0.09 577	9.89 213	10	16	.6 6.6
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15	.7 7.7
46	9.79 668	16	9.90 475	26	0.09 525	9.89 193	10	14	.8 8.8
47	9.79 684	16	9.90 501	26	0.09 499	9.89 183	10	13	.9 9.9
48	9.79 699	15	9.90 527	26	0.09 473	9.89 173	10	12	
49	9.79 715	16	9.90 553	26	0.09 447	9.89 162	11	11	
50	9.79 731	16	9.90 578	25	0.09 422	9.89 152	10	10	10 9
51	9.79 746	15	9.90 604	26	0.09 396	9.89 142	10	9	1 1.0 0.9
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8	2 2.0 1.8
53	9.79 778	16	9.90 656	26	0.09 344	9.89 122	10	7	3 3.0 2.7
54	9.79 793	15	9.90 682	26	0.09 318	9.89 112	10	6	4 4.0 3.6
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	11	5	5 5.0 4.5
56	9.79 825	16	9.90 734	26	0.09 266	9.89 091	10	4	6 6.0 5.4
57	9.79 840	15	9.90 759	25	0.09 241	9.89 081	10	3	7 7.0 6 3
58	9.79 856	16	9.90 785	26	0.09 215	9.89 071	10	2	8 8.0 7.2
59	9.79 872	16	9.90 811	26	0.09 189	9.89 060	11	1	9 9.0 8.1
60	9.79 887	15	9.90 837	26	0.09 163	9.89 050	10	0	
′	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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39°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.79 887	16	9.90 837	26	0.09 163	9.89 050	10	60	
1	9.79 903	15	9.90 863	26	0.09 137	9.89 040	10	59	
2	9.79 918	16	9.90 889	25	0.09 111	9.89 030	10	58	
3	9.79 934	16	9.90 914	26	0.09 086	9.89 020	11	57	26
4	9.79 950	15	9.90 940	26	0.09 060	9.89 009	10	56	.1 2.6
5	9.79 965	16	9.90 966	26	0.09 034	9.88 999	10	55	.2 5.2
6	9.79 981	15	9.90 992	26	0.09 008	9.88 989	11	54	.3 7.8
7	9.79 996	16	9.91 018	25	0.08 982	9.88 978	10	53	.4 10.4
8	9.80 012	15	9.91 043	26	0.08 957	9.88 968	10	52	.5 13.0
9	9.80 027	16	9.91 069	26	0.08 931	9.88 958	10	51	.6 15.6
10	9.80 043	15	9.91 095	26	0.08 905	9.88 948	11	50	.7 18.2
11	9.80 058	16	9.91 121	26	0.08 879	9.88 937	10	49	.8 20.8
12	9.80 074	15	9.91 147	25	0.08 853	9.88 927	10	48	.9 23.4
13	9.80 089	16	9.91 172	26	0.08 828	9.88 917	11	47	
14	9.80 105	15	9.91 198	26	0.08 802	9.88 906	10	46	
15	9.80 120	16	9.91 224	26	0.08 776	9.88 896	10	45	25
16	9.80 136	15	9.91 250	26	0.08 750	9.88 886	11	44	.1 2.5
17	9.80 151	15	9.91 276	26	0.08 724	9.88 875	10	43	.2 5.0
18	9.80 166	16	9.91 301	25	0.08 699	9.88 865	10	42	.3 7.5
19	9.80 182	15	9.91 327	26	0.08 673	9.88 855	11	41	.4 10.0
20	9.80 197	16	9.91 353	26	0.08 647	9.88 844	10	40	.5 12.5
21	9.80 213	15	9.91 379	25	0.08 621	9.88 834	10	39	.6 15.0
22	9.80 228	16	9.91 404	26	0.08 596	9.88 824	11	38	.7 17.5
23	9.80 244	15	9.91 430	26	0.08 570	9.88 813	10	37	.8 20.0
24	9.80 259	15	9.91 456	26	0.08 544	9.88 803	10	36	.9 22.5
25	9.80 274	16	9.91 482	25	0.08 518	9.88 793	11	35	
26	9.80 290	15	9.91 507	26	0.08 493	9.88 782	10	34	16
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	11	33	.1 1.6
28	9.80 320	16	9.91 559	26	0.08 441	9.88 761	10	32	.2 3.2
29	9.80 336	15	9.91 585	25	0.08 415	9.88 751	10	31	.3 4.8
30	9.80 351	15	9.91 610	26	0.08 390	9.88 741	11	30	.4 6.4
31	9.80 366	16	9.91 636	26	0.08 364	9.88 730	10	29	.5 8.0
32	9.80 382	15	9.91 662	26	0.08 338	9.88 720	11	28	.6 9.6
33	9.80 397	15	9.91 688	25	0.08 312	9.88 709	10	27	.7 11.2
34	9.80 412	16	9.91 713	26	0.08 287	9.88 699	11	26	.8 12.8
35	9.80 428	15	9.91 739	26	0.08 261	9.88 688	10	25	.9 14.4
36	9.80 443	15	9.91 765	26	0.08 235	9.88 678	10	24	
37	9.80 458	15	9 91 791	26	0.08 209	9.88 668	11	23	
38	9.80 473	16	9.91 816	25	0.08 184	9.88 657	10	22	15
39	9.80 489	15	9.91 842	26	0.08 158	9.88 647	11	21	.1 1.5
40	9.80 504	15	9.91 868	26	0.08 132	9.88 636	10	20	.2 3.0
41	9.80 519	15	9.91 893	25	0.08 107	9.88 626	11	19	3 4.5
42	9.80 534	16	9.91 919	26	0.08 081	9.88 615	10	18	.4 6.0
43	9.80 550	15	9.91 945	26	0.08 055	9.88 605	11	17	.5 7.5
44	9.80 565	15	9.91 971	26	0.08 029	9.88 594	10	16	.6 9.0
45	9.80 580	15	9.91 996	25	0.08 004	9.88 584	11	15	7 10.5
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	11	14	.8 12 0
47	9.80 610	15	9.92 048	26	0.07 952	9.88 563	10	13	.9 13.5
48	9.80 625	15	9.92 073	25	0.07 927	9.88 552	11	12	
49	9.80 641	16	9.92 099	26	0.07 901	9.88 542	10	11	
50	9.80 656	15	9.92 125	26	0.07 875	9.88 531	11	10	11 10
51	9.80 671	15	9.92 150	25	0.07 850	9.88 521	10	9	.1 1.1 1.0
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	11	8	.2 2.2 2.0
53	9.80 701	15	9.92 202	26	0.07 798	9.88 499	11	7	.3 3.3 3.0
54	9.80 716	15	9.92 227	25	0.07 773	9.88 489	10	6	.4 4.4 4.0
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	11	5	.5 5.5 5.0
56	9.80 746	15	9.92 279	26	0.07 721	9.88 468	10	4	.6 6.6 6.0
57	9.80 762	16	9.92 304	25	0.07 696	9.88 457	11	3	.7 7.7 7.0
58	9.80 777	15	9.92 330	26	0.07 670	9.88 447	10	2	.8 8 8 8.0
59	9.80 792	15	9.92 356	26	0.07 644	9.88 436	11	1	.9 9.9 9.0
60	9.80 807	15	9.92 381	25	0.07 619	9.88 425	11	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

50°

40°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.80 807	15	9.92 381	26	0.07 619	9.88 425	10	60	
1	9.80 822	15	9.92 407	26	0.07 593	9.88 415	11	59	
2	9.80 837	15	9.92 433	25	0.07 567	9.88 404	10	58	
3	9.80 852	15	9.92 458	26	0.07 542	9.88 394	11	57	26
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56	.1 2.6
5	9.80 882	15	9.92 510	25	0.07 490	9.88 372	10	55	.2 5.2
6	9.80 897	15	9.92 535	26	0.07 465	9.88 362	11	54	.3 7.8
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53	.4 10 4
8	9.80 927	15	9.92 587	26	0.07 413	9.88 340	10	52	.5 13.0
9	9.80 942	15	9.92 612	25	0.07 388	9.88 330	11	51	.6 15.6
10	9.80 957	15	9 92 638	26	0.07 362	9 88 319	11	50	.7 18 2
11	9.80 972	15	9.92 663	25	0.07 337	9.88 308	10	49	.8 20.8
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	11	48	.9 23.4
13	9.81 002	15	9.92 715	26	0.07 285	9.88 287	11	47	
14	9 81 017	15	9.92 740	25	0.07 260	9.88 276	11	46	
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	10	45	25
16	9.81 047	15	9.92 792	26	0.07 208	9.88 255	11	44	.1 2.5
17	9 81 061	14	9.92 817	25	0.07 183	9.88 244	11	43	.2 5.0
18	9.81 076	15	9.92 843	26	0.07 157	9.88 234	10	42	.3 7.5
19	9.81 091	15	9.92 868	25	0.07 132	9.88 223	11	41	.4 10.0
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	40	.5 12.5
21	9.81 121	15	9.92 920	26	0.07 080	9.88 201	11	39	.6 15.0
22	9.81 136	15	9.92 945	25	0.07 055	9.88 191	10	38	.7 17.5
23	9.81 151	15	9.92 971	26	0.07 029	9.88 180	11	37	.8 20.0
24	9.81 166	15	9.92 996	25	0.07 004	9.88 169	11	36	.9 22.5
25	9 81 180	14	9.93 022	26	0.06 978	9.88 158	11	35	
26	9.81 195	15	9.93 048	26	0.06 952	9.88 148	10	34	15
27	9.81 210	15	9.93 073	25	0.06 927	9 88 137	11	33	.1 1.5
28	9.81 225	15	9.93 099	26	0.06 901	9 88 126	11	32	.2 3.0
29	9.81 240	15	9.93 124	25	0.06 876	9.88 115	11	31	.3 4.5
30	9.81 254	14	9.93 150	26	0.06 850	9.88 105	10	30	.4 6.0
31	9.81 269	15	9.93 175	25	0.06 825	9.88 094	11	29	.5 7.5
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28	.6 9 0
33	9.81 299	15	9.93 227	26	0.06 773	9.88 072	11	27	.7 10.5
34	9 81 314	15	9.93 252	25	0.06 748	9.88 061	11	26	.8 12.0
35	9 81 328	14	9.93 278	26	0.06 722	9.88 051	10	25	.9 13.5
36	9.81 343	15	9.93 303	25	0.06 697	9.88 040	11	24	
37	9.81 358	15	9.93 329	26	0.06 671	9.88 029	11	23	
38	9.81 372	14	9.93 354	25	0.06 646	9.88 018	11	22	14
39	9 81 387	15	9 93 380	26	0.06 620	9.88 007	11	21	.1 1.4
40	9 81 402	15	9.93 406	26	0.06 594	9 87 996	11	20	.2 2.8
41	9.81 417	15	9.93 431	25	0.06 569	9.87 985	11	19	.3 4.2
42	9 81 431	14	9 93 457	26	0.06 543	9.87 975	10	18	.4 5.6
43	9 81 446	15	9 93 482	25	0.06 518	9.87 964	11	17	.5 7.0
44	9 81 461	15	9 93 508	26	0.06 492	9.87 953	11	16	.6 8.4
45	9 81 475	14	9 93 533	25	0.06 467	9.87 942	11	15	.7 9.8
46	9 81 490	15	9 93 559	26	0.06 441	9.87 931	11	14	.8 11.2
47	9 81 505	15	9 93 584	25	0 06 416	9 87 920	11	13	.9 12.6
48	9 81 519	14	9 93 610	26	0.06 390	9.87 909	11	12	
49	9 81 534	15	9.93 636	26	0.06 364	9 87 898	11	11	
50	9 81 549	15	9 93 661	25	0.06 339	9 87 887	11	10	11 10
51	9 81 563	14	9 93 687	26	0.06 313	9 87 877	10	9	.1 1.1 1.0
52	9 81 578	15	9 93 712	25	0.06 288	9.87 866	11	8	.2 2.2 2 0
53	9 81 592	14	9 93 738	26	0.06 262	9 87 855	11	7	.3 3.3 3 0
54	9 81 607	15	9 93 763	25	0.06 237	9 87 844	11	6	.4 4.4 4.0
55	9 81 622	15	9 93 789	26	0.06 211	9 87 833	11	5	.5 5.5 5 0
56	9 81 636	14	9 93 814	25	0.06 186	9 87 822	11	4	.6 6.6 6.0
57	9 81 651	15	9 93 840	26	0.06 160	9 87 811	11	3	.7 7.7 7.0
58	9 81 665	14	9 93 865	25	0.06 135	9.87 800	11	2	.8 8.8 8.0
59	9 81 680	15	9 93 891	26	0.06 109	9 87 789	11	1	.9 9.9 9.0
60	9.81 694	14	9 93 916	25	0.06 084	9 87 778	11	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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41°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.81 694	15	9.93 916	26	0.06 084	9.87 778	11	60	
1	9.81 709	14	9.93 942	25	0.06 058	9.87 767	11	59	
2	9.81 723	15	9.93 967	26	0.06 033	9.87 756	11	58	
3	9.81 738	14	9.93 993	25	0.06 007	9.87 745	11	57	26
4	9.81 752	15	9.94 018	26	0.05 982	9.87 734	11	56	.1 2.6
5	9.81 767	14	9.94 044	25	0.05 956	9.87 723	11	55	.2 5.2
6	9.81 781	15	9.94 069	26	0.05 931	9.87 712	11	54	.3 7.8
7	9.81 796	14	9.94 095	25	0.05 905	9.87 701	11	53	.4 10.4
8	9.81 810	15	9.94 120	26	0.05 880	9.87 690	11	52	.5 13.0
9	9.81 825	14	9.94 146	25	0.05 854	9.87 679	11	51	.6 15.6
10	9.81 839	15	9.94 171	26	0.05 829	9.87 668	11	50	.7 18.2
11	9.81 854	14	9.94 197	25	0.05 803	9.87 657	11	49	.8 20.8
12	9.81 868	14	9.94 222	26	0.05 778	9.87 646	11	48	.9 23.4
13	9.81 882	15	9.94 248	25	0.05 752	9.87 635	11	47	
14	9.81 897	14	9.94 273	26	0.05 727	9.87 624	11	46	
15	9.81 911	15	9.94 299	25	0.05 701	9.87 613	12	45	25
16	9.81 926	14	9.94 324	26	0.05 676	9.87 601	11	44	.1 2.5
17	9.81 940	15	9.94 350	25	0.05 650	9.87 590	11	43	.2 5.0
18	9.81 955	14	9.94 375	26	0.05 625	9.87 579	11	42	.3 7.5
19	9.81 969	14	9.94 401	25	0.05 599	9.87 568	11	41	.4 10.0
20	9.81 983	15	9.94 426	26	0.05 574	9.87 557	11	40	.5 12.5
21	9.81 998	14	9.94 452	25	0.05 548	9.87 546	11	39	.6 15.0
22	9.82 012	14	9.94 477	26	0.05 523	9.87 535	11	38	.7 17.5
23	9.82 026	15	9.94 503	25	0.05 497	9.87 524	11	37	.8 20.0
24	9.82 041	14	9.94 528	26	0.05 472	9.87 513	12	36	.9 22.5
25	9.82 055	14	9.94 554	25	0.05 446	9.87 501	11	35	
26	9.82 069	15	9.94 579	25	0.05 421	9.87 490	11	34	15
27	9.82 084	14	9.94 604	26	0.05 396	9.87 479	11	33	1 1.5
28	9.82 098	14	9.94 630	25	0.05 370	9.87 468	11	32	.2 3.0
29	9.82 112	14	9.94 655	26	0.05 345	9.87 457	11	31	.3 4.5
30	9.82 126	15	9.94 681	25	0.05 319	9.87 446	12	30	.4 6.0
31	9.82 141	14	9.94 706	26	0.05 294	9.87 434	11	29	.5 7.5
32	9.82 155	14	9.94 732	25	0.05 268	9.87 423	11	28	.6 9.0
33	9.82 169	15	9.94 757	26	0.05 243	9.87 412	11	27	.7 10.5
34	9.82 184	14	9.94 783	25	0.05 217	9.87 401	11	26	.8 12.0
35	9.82 198	14	9.94 808	26	0.05 192	9.87 390	12	25	.9 13.5
36	9.82 212	14	9.94 834	25	0.05 166	9.87 378	11	24	
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23	14
38	9.82 240	15	9.94 884	26	0.05 116	9.87 356	11	22	.1 1.4
39	9.82 255	14	9.94 910	25	0.05 090	9.87 345	11	21	.2 2.8
40	9.82 269	14	9.94 935	26	0.05 065	9.87 334	12	20	.3 4.2
41	9.82 283	14	9.94 961	25	0 05 039	9.87 322	11	19	.4 5.6
42	9.82 297	14	9.94 986	26	0.05 014	9.87 311	11	18	.5 7.0
43	9.82 311	14	9.95 012	25	0.04 988	9.87 300	12	17	.6 8.4
44	9 82 326	15	9.95 037	25	0.04 963	9.87 288	11	16	.7 9.8
45	9.82 340	14	9.95 062	26	0.04 938	9.87 277	11	15	.8 11.2
46	9.82 354	14	9.95 088	25	0.04 912	9.87 266	11	14	.9 12.6
47	9.82 368	14	9.95 113	26	0.04 887	9.87 255	12	13	
48	9.82 382	14	9.95 139	25	0.04 861	9.87 243	11	12	
49	9.82 396	14	9.95 164	26	0.04 836	9.87 232	11	11	12 11
50	9.82 410	14	9.95 190	25	0.04 810	9.87 221	12	10	.1 1.2 1.1
51	9.82 424	15	9.95 215	25	0.04 785	9.87 209	11	9	.2 2.4 2.2
52	9.82 439	14	9.95 240	26	0.04 760	9.87 198	11	8	.3 3.6 3.3
53	9.82 453	14	9.95 266	25	0.04 734	9.87 187	12	7	.4 4.8 4 4
54	9.82 467	14	9.95 291	26	0.04 709	9.87 175	11	6	.5 6.0 5 5
55	9.82 481	14	9.95 317	25	0.04 683	9.87 164	11	5	.6 7.2 6.6
56	9.82 495	14	9.95 342	26	0.04 658	9.87 153	12	4	.7 8.4 7 7
57	9.82 509	14	9.95 368	26	0.04 632	9.87 141	11	3	.8 9.6 8.8
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2	.9 10.8 9.9
59	9.82 537	14	9.95 418	25	0.04 582	9.87 119	11	1	
60	9.82 551		9.95 444	26	0.04 556	9.87 107	12	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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42°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.82 551	14	9.95 444	25	0.04 556	9.87 107	11	60	
1	9.82 565	14	9 95 469	26	0.04 531	9.87 096	11	59	
2	9.82 579	14	9.95 495	25	0.04 505	9.87 085	12	58	
3	9.82 593	14	9 95 520	25	0.04 480	9.87 073	11	57	26
4	9 82 607	14	9 95 545	26	0.04 455	9 87 062	12	56	.1 2.6
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	55	.2 5 2
6	9.82 635	14	9.95 596	26	0.04 404	9.87 039	11	54	.3 7 8
7	9.82 649	14	9.95 622	25	0.04 378	9.87 028	12	53	.4 10.4
8	9.82 663	14	9 95 647	25	0.04 353	9.87 016	11	52	.5 13.0
9	9.82 677	14	9.95 672	26	0.04 328	9.87 005	12	51	.6 15.6
10	9.82 691	14	9.95 698	25	0.04 302	9.86 993	11	50	.7 18 2
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	12	49	.8 20.8
12	9.82 719	14	9.95 748	26	0.04 252	9.86 970	11	48	.9 23.4
13	9.82 733	14	9.95 774	25	0.04 226	9.86 959	12	47	
14	9.82 747	14	9.95 799	26	0.04 201	9.86 947	11	46	25
15	9.82 761	14	9.95 825	25	0.04 175	9.86 936	12	45	.1 2.5
16	9.82 775	13	9.95 850	25	0.04 150	9.86 924	11	44	.2 5.0
17	9.82 788	14	9.95 875	26	0.04 125	9.86 913	11	43	.3 7.5
18	9.82 802	14	9.95 901	25	0.04 099	9.86 902	12	42	.4 10.0
19	9.82 816	14	9.95 926	26	0.04 074	9.86 890	11	41	.5 12.5
20	9.82 830	14	9.95 952	25	0.04 048	9.86 879	12	40	.6 15.0
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	12	39	.7 17.5
22	9.82 858	14	9.96 002	26	0.03 998	9.86 855	11	38	.8 20.0
23	9.82 872	14	9.96 028	25	0.03 972	9.86 844	12	37	.9 22.5
24	9.82 885	13	9.96 053	25	0.03 947	9.86 832	11	36	
25	9.82 899	14	9.96 078	26	0.03 922	9.86 821	12	35	14
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34	.1 1.4
27	9.82 927	14	9.96 129	26	0.03 871	9.86 798	12	33	.2 2.8
28	9.82 941	14	9.96 155	25	0.03 845	9 86 786	11	32	.3 4.2
29	9.82 955	13	9.96 180	25	0.03 820	9.86 775	12	31	.4 5.6
30	9.82 968	14	9.96 205	26	0.03 795	9.86 763	11	30	.5 7.0
31	9.82 982	14	9.96 231	25	0.03 769	9.86 752	12	29	.6 8.4
32	9.82 996	14	9.96 256	25	0.03 744	9.86 740	12	28	.7 9.8
33	9.83 010	13	9.96 281	26	0.03 719	9.86 728	11	27	.8 11.2
34	9.83 023	14	9.96 307	25	0.03 693	9.86 717	12	26	.9 12.6
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	11	25	
36	9.83 051	14	9.96 357	26	0.03 643	9.86 694	12	24	
37	9.83 065	13	9.96 383	25	0.03 617	9.86 682	12	23	13
38	9.83 078	14	9.96 408	25	0.03 592	9.86 670	11	22	.1 1.3
39	9.83 092	14	9.96 433	26	0.03 567	9.86 659	12	21	.2 2.6
40	9.83 106	14	9.96 459	25	0.03 541	9.86 647	12	20	.3 3 9
41	9 83 120	13	9.96 484	26	0.03 516	9.86 635	11	19	.4 5.2
42	9 83 133	14	9.96 510	25	0.03 490	9.86 624	12	18	.5 6.5
43	9.83 147	14	9.96 535	25	0.03 465	9.86 612	12	17	.6 7.8
44	9.83 161	13	9.96 560	26	0.03 440	9.86 600	12	16	.7 9.1
45	9 83 174	14	9.96 586	25	0.03 414	9.86 589	11	15	.8 10.4
46	9 83 188	14	9.96 611	25	0.03 389	9.86 577	12	14	.9 11.7
47	9.83 202	13	9.96 636	26	0.03 364	9.86 565	12	13	
48	9.83 215	14	9.96 662	25	0.03 338	9.86 554	11	12	
49	9 83 229	13	9 96 687	25	0.03 313	9.86 542	12	11	12 11
50	9.83 242	14	9.96 712	26	0.03 288	9.86 530	12	10	.1 1.2 1.1
51	9.83 256	14	9 96 738	25	0.03 262	9.86 518	12	9	.2 2.4 2.2
52	9.83 270	13	9.96 763	25	0.03 237	9.86 507	11	8	.3 3.6 3.3
53	9.83 283	14	9 96 788	26	0.03 212	9.86 495	12	7	.4 4.8 4.4
54	9.83 297	13	9.96 814	25	0.03 186	9.86 483	12	6	.5 6.0 5 5
55	9 83 310	14	9.96 839	25	0.03 161	9.86 472	11	5	.6 7 2 6 6
56	9.83 324	14	9.96 864	26	0.03 136	9.86 460	12	4	.7 8.4 7 7
57	9 83 338	14	9 96 890	25	0.03 110	9.86 448	12	3	.8 9.6 8 8
58	9 83 351	13	9.96 915	25	0.03 085	9.86 436	12	2	.9 10.8 9.9
59	9 83 365	14	9 96 940	26	0.03 060	9.86 425	11	1	
60	9 83 378	13	9.96 966		0.03 034	9.86 413	12	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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43°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9 83 378	14	9 96 966	25	0.03 034	9 86 413	12	60	
1	9 83 392	13	9 96 991	25	0.03 009	9 86 401	12	59	
2	9 83 405	14	9 97 016	26	0.02 984	9 86 389	12	58	
3	9 83 419	13	9 97 042	25	0.02 958	9 86 377	11	57	26
4	9 83 432	14	9 97 067	25	0.02 933	9 86 366	12	56	.1 2.6
5	9 83 446	13	9 97 092	26	0.02 908	9 86 354	12	55	.2 5.2
6	9 83 459	14	9 97 118	25	0.02 882	9 86 342	12	54	.3 7.8
7	9 83 473	13	9 97 143	25	0.02 857	9 86 330	12	53	.4 10.4
8	9 83 486	14	9 97 168	25	0.02 832	9 86 318	12	52	.5 13.0
9	9 83 500	13	9 97 193	26	0.02 807	9 86 306	11	51	.6 15.6
10	9 83 513	14	9 97 219	25	0.02 781	9 86 295	12	50	.7 18.2
11	9 83 527	13	9 97 244	25	0.02 756	9 86 283	12	49	.8 20.8
12	9 83 540	14	9 97 269	26	0.02 731	9 86 271	12	48	.9 23.4
13	9 83 554	13	9 97 295	25	0.02 705	9 86 259	12	47	
14	9 83 567	14	9 97 320	25	0.02 680	9 86 247	12	46	25
15	9 83 581	13	9 97 345	26	0.02 655	9 86 235	12	45	.1 2.5
16	9 83 594	14	9 97 371	25	0.02 629	9 86 223	12	44	.2 5.0
17	9 83 608	13	9 97 396	25	0.02 604	9 86 211	11	43	.3 7.5
18	9 83 621	13	9 97 421	26	0.02 579	9 86 200	12	42	.4 10.0
19	9 83 634	14	9 97 447	25	0.02 553	9 86 188	12	41	.5 12.5
20	9 83 648	13	9 97 472	25	0.02 528	9 86 176	12	40	.6 15.0
21	9 83 661	13	9 97 497	26	0.02 503	9 86 164	12	39	.7 17.5
22	9 83 674	14	9 97 523	25	0.02 477	9 86 152	12	38	.8 20.0
23	9 83 688	13	9 97 548	25	0.02 452	9 86 140	12	37	.9 22.5
24	9 83 701	14	9 97 573	25	0.02 427	9 86 128	12	36	
25	9 83 715	13	9 97 598	26	0.02 402	9 86 116	12	35	14
26	9 83 728	13	9 97 624	25	0.02 376	9 86 104	12	34	.1 1.4
27	9 83 741	14	9 97 649	25	0.02 351	9 86 092	12	33	.2 2.8
28	9 83 755	13	9 97 674	26	0.02 326	9 86 080	12	32	.3 4.2
29	9 83 768	13	9 97 700	25	0.02 300	9 86 068	12	31	.4 5.6
30	9 83 781	14	9 97 725	25	0.02 275	9 86 056	12	30	.5 7.0
31	9 83 795	13	9 97 750	26	0.02 250	9 86 044	12	29	.6 8.4
32	9 83 808	13	9 97 776	25	0.02 224	9 86 032	12	28	.7 9.8
33	9 83 821	13	9 97 801	25	0.02 199	9 86 020	12	27	.8 11.2
34	9 83 834	14	9 97 826	25	0.02 174	9 86 008	12	26	.9 12.6
35	9 83 848	13	9 97 851	26	0.02 149	9 85 996	12	25	
36	9 83 861	13	9 97 877	25	0.02 123	9 85 984	12	24	
37	9 83 874	13	9 97 902	25	0.02 098	9 85 972	12	23	
38	9 83 887	14	9 97 927	26	0.02 073	9 85 960	12	22	13
39	9 83 901	13	9 97 953	25	0.02 047	9 85 948	12	21	.1 1.3
40	9 83 914	13	9 97 978	25	0.02 022	9 85 936	12	20	.2 2.6
41	9 83 927	13	9 98 003	26	0.01 997	9 85 924	12	19	.3 3.9
42	9 83 940	14	9 98 029	25	0.01 971	9 85 912	12	18	.4 5.2
43	9 83 954	13	9 98 054	25	0.01 946	9 85 900	12	17	.5 6.5
44	9 83 967	13	9 98 079	25	0.01 921	9 85 888	12	16	.6 7.8
45	9 83 980	13	9 98 104	26	0.01 896	9 85 876	12	15	.7 9.1
46	9 83 993	13	9 98 130	25	0.01 870	9 85 864	12	14	.8 10.4
47	9 84 006	14	9 98 155	25	0.01 845	9 85 851	13	13	.9 11.7
48	9 84 020	13	9 98 180	25	0.01 820	9 85 839	12	12	
49	9 84 033	13	9 98 206	26	0.01 794	9 85 827	12	11	12 11
50	9 84 046	13	9 98 231	25	0.01 769	9 85 815	12	10	.1 1.2 1.1
51	9 84 059	13	9 98 256	25	0.01 744	9 85 803	12	9	.2 2.4 2.2
52	9 84 072	13	9 98 281	25	0.01 719	9 85 791	12	8	.3 3.6 3.3
53	9 84 085	13	9 98 307	26	0.01 693	9 85 779	12	7	.4 4.8 4.4
54	9 84 098	14	9 98 332	25	0.01 668	9 85 766	13	6	.5 6.0 5.5
55	9 84 112	13	9 98 357	26	0.01 643	9 85 754	12	5	.6 7.2 6.6
56	9 84 125	13	9 98 383	25	0.01 617	9 85 742	12	4	.7 8.4 7.7
57	9 84 138	13	9 98 408	25	0.01 592	9 85 730	12	3	.8 9.6 8.8
58	9 84 151	13	9 98 433	25	0.01 567	9 85 718	12	2	.9 10.8 9.9
59	9 84 164	13	9 98 458	25	0.01 542	9 85 706	12	1	
60	9 84 177		9 98 484	26	0.01 516	9 85 693	13	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

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44°

′	L. Sin.	d.	L. Tang.	c. d.	L. Cotg.	L. Cos.	d.		Prop. Pts.
0	9.84 177		9.98 484		0.01 516	9.85 693		60	
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59	
2	9.84 203	13	9.98 534	25	0.01 466	9.85 669	12	58	
3	9.84 216	13	9.98 560	26	0.01 440	9.85 657	12	57	26
4	9.84 229	13	9.98 585	25	0.01 415	9.85 645	12	56	.1 2.6
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	13	55	.2 5.2
6	9.84 255	13	9.98 635	25	0.01 365	9.85 620	12	54	.3 7.8
7	9.84 269	14	9.98 661	26	0.01 339	9.85 608	12	53	.4 10.4
8	9.84 282	13	9.98 686	25	0.01 314	9.85 596	12	52	.5 13.0
9	9.84 295	13	9.98 711	25	0.01 289	9.85 583	13	51	.6 15.6
10	9.84 308	13	9.98 737	26	0.01 263	9.85 571	12	50	.7 18.2
11	9.84 321	13	9.98 762	25	0.01 238	9.85 559	12	49	.8 20.8
12	9.84 334	13	9.98 787	25	0.01 213	9.85 547	12	48	.9 23.4
13	9.84 347	13	9.98 812	25	0.01 188	9.85 534	13	47	
14	9.84 360	13	9.98 838	26	0.01 162	9.85 522	12	46	
15	9.84 373	13	9.98 863	25	0.01 137	9.85 510	12	45	25
16	9.84 385	12	9.98 888	25	0.01 112	9.85 497	13	44	.1 2.5
17	9.84 398	13	9.98 913	25	0.01 087	9.85 485	12	43	.2 5.0
18	9.84 411	13	9.98 939	26	0.01 061	9.85 473	12	42	.3 7.5
19	9.84 424	13	9.98 964	25	0.01 036	9.85 460	13	41	.4 10.0
20	9.84 437	13	9.98 989	25	0.01 011	9.85 448	12	40	.5 12.5
21	9.84 450	13	9.99 015	26	0.00 985	9.85 436	12	39	.6 15.0
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	13	38	.7 17.5
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37	.8 20.0
24	9.84 489	13	9.99 090	25	0.00 910	9 85 399	12	36	.9 22.5
25	9.84 502	13	9.99 116	26	0.00 884	9.85 386	13	35	
26	9.84 515	13	9.99 141	25	0.00 859	9.85 374	12	34	14
27	9.84 528	13	9.99 166	25	0.00 834	9.85 361	13	33	.1 1.4
28	9.84 540	12	9.99 191	25	0.00 809	9.85 349	12	32	.2 2.8
29	9.84 553	13	9.99 217	26	0.00 783	9.85 337	12	31	.3 4.2
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	13	30	.4 5.6
31	9.84 579	13	9.99 267	25	0.00 733	9.85 312	12	29	.5 7.0
32	9.84 592	13	9.99 293	26	0.00 707	9.85 299	13	28	.6 8.4
33	9.84 605	13	9.99 318	25	0.00 682	9.85 287	12	27	.7 9.8
34	9.84 618	13	9.99 343	25	0.00 657	9.85 274	13	26	.8 11.2
35	9.84 630	12	9.99 368	25	0.00 632	9.85 262	12	25	.9 12.6
36	9.84 643	13	9.99 394	26	0.00 606	9.85 250	12	24	
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	13	23	
38	9.84 669	13	9.99 444	25	0.00 556	9.85 225	12	22	13
39	9.84 682	13	9.99 469	25	0.00 531	9.85 212	13	21	.1 1.3
40	9.84 694	12	9.99 495	26	0.00 505	9.85 200	12	20	.2 2.6
41	9.84 707	13	9.99 520	25	0.00 480	9 85 187	13	19	.3 3.9
42	9.84 720	13	9.99 545	25	0.00 455	9.85 175	12	18	.4 5.2
43	9 84 733	13	9.99 570	25	0.00 430	9.85 162	13	17	.5 6.5
44	9.84 745	12	9.99 596	26	0.00 404	9.85 150	12	16	.6 7.8
45	9 84 758	13	9.99 621	25	0.00 379	9.85 137	13	15	.7 9.1
46	9.84 771	13	9.99 646	25	0.00 354	9.85 125	12	14	.8 10.4
47	9.84 784	13	9.99 672	26	0.00 328	9.85 112	13	13	.9 11.7
48	9.84 796	12	9.99 697	25	0.00 303	9.85 100	12	12	
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	13	11	
50	9.84 822	13	9.99 747	25	0.00 253	9.85 074	13	10	12
51	9.84 835	13	9.99 773	26	0.00 227	9.85 062	12	9	.1 1.2
52	9.84 847	12	9.99 798	25	0.00 202	9.85 049	13	8	.2 2.4
53	9.84 860	13	9.99 823	25	0.00 177	9.85 037	12	7	.3 3.6
54	9.84 873	13	9.99 848	25	0.00 152	9.85 024	13	6	.4 4.8
55	9.84 885	12	9.99 874	26	0.00 126	9.85 012	12	5	.5 6.0
56	9.84 898	13	9.99 899	25	0.00 101	9.84 999	13	4	.6 7.2
57	9.84 911	13	9.99 924	25	0.00 076	9.84 986	13	3	.7 8.4
58	9.84 923	12	9.99 949	25	0.00 051	9.84 974	12	2	.8 9.6
59	9.84 936	13	9.99 975	26	0.00 025	9.84 961	13	1	.9 10.8
60	9.84 949	13	0.00 000	25	0.00 000	9.84 949	12	0	
	L. Cos.	d.	L. Cotg.	c. d.	L. Tang.	L. Sin.	d.	′	Prop. Pts.

45°

TABLE III

NATURAL TRIGONOMETRIC FUNCTIONS

FOR

EACH MINUTE

	0°		1°		2°		3°		4°		
′	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	
0	.00000	1.00000	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	1.00000	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	1.00000	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	1.00000	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	1.00000	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	1.00000	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	1.00000	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	1.00000	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	1.00000	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	1.00000	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	1.00000	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	′
	89°		88°		87°		86°		85°		

	5°		6°		7°		8°		9°		
′	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	′
	84°		83°		82°		81°		80°		

′	10° N. sine	10° N. cos.	11° N. sine	11° N. cos.	12° N. sine	12° N. cos.	13° N. sine	13° N. cos.	14° N. sine	14° N. cos.	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	′
	79°		78°		77°		76°		75°		

	15°		16°		17°		18°		19°		
′	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
	N. cos.	N. sine	N. cos.	N sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	′
	74°		73°		72°		71°		70°		

′	20° N. sine	20° N. cos.	21° N. sine	21° N. cos.	22° N. sine	22° N. cos.	23° N. sine	23° N. cos.	24° N. sine	24° N. cos.	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	′
	69°		68°		67°		66°		65°		

	25°		26°		27°		28°		29°		
′	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	′
	64°		63°		62°		61°		60°		

′	30° N. sine	30° N. cos.	31° N. sine	31° N. cos.	32° N. sine	32° N. cos.	33° N. sine	33° N. cos.	34° N. sine	34° N. cos.	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
3.	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
	N cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	′
	59°		58°		57°		56°		55°		

	35°		36°		37°		38°		39°		
′	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	′
	54°		53°		52°		51°		50°		

′	40° N. sine	40° N. cos.	41° N. sine	41° N. cos.	42° N. sine	42° N. cos.	43° N. sine	43° N. cos.	44° N. sine	44° N. cos.	
0	.64279	.76604	.65606	.75471	66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	′
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		4°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	0000	Infinite	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3007	60
1	0003	3437.75	0177	56.3506	0352	28.3994	0527	18.9755	0702	14.2411	59
2	0006	1718.87	0180	55.4415	0355	28.1664	0530	18.8711	0705	14.1821	58
3	0009	1145.92	0183	54.5613	0358	27.9372	0533	18.7678	0708	14.1235	57
4	0012	859.436	0186	53.7086	0361	27.7117	0536	18.6656	0711	14.0655	56
5	0015	687.549	0189	52.8821	0364	27.4899	0539	18.5645	0714	14.0079	55
6	0017	572.957	0192	52.0807	0367	27.2715	0542	18.4645	0717	13.9507	54
7	0020	491.106	0195	51.3032	0370	27.0566	0544	18.3655	0720	13.8940	53
8	0023	429.718	0198	50.5485	0373	26.8450	0547	18.2677	0723	13.8378	52
9	0026	381.971	0201	49.8157	0375	26.6367	0550	18.1708	0726	13.7821	51
10	0029	343.774	0204	49.1039	0378	26.4316	0553	18.0750	0729	13.7267	50
11	0032	312.521	0207	48.4121	0381	26.2296	0556	17.9802	0731	13.6719	49
12	0035	286.478	0209	47.7395	0384	26.0307	0559	17.8863	0734	13.6174	48
13	0038	264.441	0212	47.0853	0387	25.8348	0562	17.7934	0737	13.5634	47
14	0041	245.552	0215	46.4489	0390	25.6418	0565	17.7015	0740	13.5098	46
15	0044	229.182	0218	45.8294	0393	25.4517	0568	17.6106	0743	13.4566	45
16	0047	214.858	0221	45.2261	0396	25.2644	0571	17.5205	0746	13.4039	44
17	0049	202.219	0224	44.6386	0399	25.0798	0574	17.4314	0749	13.3515	43
18	0052	190.984	0227	44.0661	0402	24.8978	0577	17.3432	0752	13.2996	42
19	0055	180.932	0230	43.5081	0405	24.7185	0580	17.2558	0755	13.2480	41
20	0058	171.885	0233	42.9641	0407	24.5418	0582	17.1693	0758	13.1969	40
21	0061	163.700	0236	42.4335	0410	23.3675	0585	17.0837	0761	13.1461	39
22	0064	156.259	0239	41.9158	0413	24.1957	0588	16.9990	0764	13.0958	38
23	0067	149.465	0241	41.4106	0416	24.0263	0591	16.9150	0767	13.0458	37
24	0070	143.237	0244	40.9174	0419	23.8593	0594	16.8319	0769	12.9962	36
25	0073	137.507	0247	40.4358	0422	23.6945	0597	16.7496	0772	12.9469	35
26	0076	132.219	0250	39.9655	0425	23.5321	0600	16.6681	0775	12.8981	34
27	0079	127.321	0253	39.5059	0428	23.3718	0603	16.5874	0778	12.8496	33
28	0081	122.774	0256	39.0568	0431	23.2137	0606	16.5075	0781	12.8014	32
29	0084	118.540	0259	38.6177	0434	23.0577	0609	16.4283	0784	12.7536	31
30	0087	114.589	0262	38.1885	0437	22.9038	0612	16.3499	0787	12.7062	30
31	0090	110.892	0265	37.7686	0440	22.7519	0615	16.2722	0790	12.6591	29
32	0093	107.426	0268	37.3579	0442	22.6020	0617	16.1952	0793	12.6124	28
33	0096	104.171	0271	36.9560	0445	22.4541	0620	16.1190	0796	12.5660	27
34	0099	101.107	0274	36.5627	0448	22.3081	0623	16.0435	0799	12.5199	26
35	0102	98.2179	0276	36.1776	0451	22.1640	0626	15.9687	0802	12.4742	25
36	0105	95.4895	0279	35.8006	0454	22.0217	0629	15.8945	0805	12.4288	24
37	0108	92.9085	0282	35.4313	0457	21.8813	0632	15.8211	0808	12.3838	23
38	0111	90.4633	0285	35.0695	0460	21.7426	0635	15.7483	0810	12.3390	22
39	0113	88.1436	0288	34.7151	0463	21.6056	0638	15.6762	0813	12.2946	21
40	0116	85.9398	0291	34.3678	0466	21.4704	0641	15.6048	0816	12.2505	20
41	0119	83.8435	0294	34.0273	0469	21.3369	0644	15.5340	0819	12.2067	19
42	0122	81.8470	0297	33.6935	0472	21.2049	0647	15.4638	0822	12.1632	18
43	0125	79.9434	0300	33.3662	0475	21.0747	0650	15.3943	0825	12.1201	17
44	0128	78.1263	0303	33.0452	0477	20.9460	0653	15.3254	0828	12.0772	16
45	0131	76.3900	0306	32.7303	0480	20.8188	0655	15.2571	0831	12.0346	15
46	0134	74.7292	0308	32.4213	0483	20.6932	0658	15.1893	0834	11.9923	14
47	0137	73.1390	0311	32.1181	0486	20.5691	0661	15.1222	0837	11.9504	13
48	0140	71.6151	0314	31.8205	0489	20.4465	0664	15.0557	0840	11.9087	12
49	0143	70.1533	0317	31.5284	0492	20.3253	0667	14.9898	0843	11.8673	11
50	0146	68.7501	0320	31.2416	0495	20.2056	0670	14.9244	0846	11.8262	10
51	0148	67.4019	0323	30.9599	0498	20.0872	0673	14.8596	0849	11.7853	9
52	0151	66.1055	0326	30.6833	0501	19.9702	0676	14.7954	0851	11.7448	8
53	0154	64.8580	0329	30.4116	0504	19.8546	0679	14.7317	0854	11.7045	7
54	0157	63.6567	0332	30.1446	0507	19.7403	0682	14.6685	0857	11.6645	6
55	0160	62.4992	0335	29.8823	0509	19.6273	0685	14.6059	0860	11.6248	5
56	0163	61.3829	0338	29.6245	0512	19.5156	0688	14.5438	0863	11.5853	4
57	0166	60.3058	0340	29.3711	0515	19.4051	0690	14.4823	0866	11.5461	3
58	0169	59.2659	0343	29.1220	0518	19.2959	0693	14.4212	0869	11.5072	2
59	0172	58.2612	0346	28.8771	0521	19.1879	0696	14.3607	0872	11.4685	1
60	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3007	0875	11.4301	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
′	89°		88°		87°		86°		85°		′

′	5°		6°		7°		8°		9°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	0875	11.4301	1051	9.5144	1228	8.1443	1405	7.1154	1584	6.3138	60
1	0878	11.3919	1054	9.4878	1231	8.1248	1408	7.1004	1587	6.3019	59
2	0881	11.3540	1057	9.4614	1234	8.1054	1411	7.0855	1590	6.2901	58
3	0884	11.3163	1060	9.4352	1237	8.0860	1414	7.0706	1593	6.2783	57
4	0887	11.2789	1063	9.4090	1240	8.0667	1417	7.0558	1596	6.2666	56
5	0890	11.2417	1066	9.3831	1243	8.0476	1420	7.0410	1599	6.2549	55
6	0892	11.2048	1069	9.3572	1246	8.0285	1423	7.0264	1602	6.2432	54
7	0895	11.1681	1072	9.3315	1249	8.0095	1426	7.0117	1605	6.2316	53
8	0898	11.1316	1075	9.3060	1251	7.9906	1429	6.9972	1608	6.2200	52
9	0901	11.0954	1078	9.2806	1254	7.9718	1432	6.9827	1611	6.2085	51
10	0904	11.0594	1080	9.2553	1257	7.9530	1435	6.9682	1614	6.1970	50
11	0907	11.0237	1083	9.2302	1260	7.9344	1438	6.9538	1617	6.1856	49
12	9910	10.9882	1086	9.2052	1263	7.9158	1441	6.9395	1620	6.1742	48
13	0913	10.9529	1089	9.1803	1266	7.8973	1444	6.9252	1623	6.1628	47
14	0916	10.9178	1092	9.1555	1269	7.8789	1447	6.9110	1626	6.1515	46
15	0919	10.8829	1095	9.1309	1272	7.8606	1450	6.8969	1629	6.1402	45
16	0922	10.8483	1098	9.1065	1275	7.8424	1453	6.8828	1632	6.1290	44
17	0925	10.8139	1101	9.0821	1278	7.8243	1456	6.8687	1635	6.1178	43
18	0928	10.7797	1104	9.0579	1281	7.8062	1459	6.8548	1638	6.1066	42
19	0931	10.7457	1107	9.0338	1284	7.7883	1462	6.8408	1641	6.0955	41
20	0934	10.7119	1110	9.0098	1287	7.7704	1465	6.8269	1644	6.0844	40
21	0936	10.6783	1113	8.9860	1290	7.7525	1468	6.8131	1647	6.0734	39
22	0939	10.6450	1116	8.9623	1293	7.7348	1471	6.7994	1650	6.0624	38
23	0942	10.6118	1119	8.9387	1296	7.7171	1474	6.7856	1653	6.0514	37
24	0945	10.5789	1122	8.9152	1299	7.6996	1477	6.7720	1655	6.0405	36
25	0948	10.5462	1125	8.8919	1302	7.6821	1480	6.7584	1658	6.0296	35
26	0951	10.5136	1128	8.8686	1305	7.6647	1483	6.7448	1661	6.0188	34
27	0954	10.4813	1131	8.8455	1308	7.6473	1486	6.7313	1664	6.0080	33
28	0957	10.4491	1134	8.8225	1311	7.6301	1489	6.7179	1667	5.9972	32
29	0960	10.4172	1136	8.7996	1314	7.6129	1492	6.7045	1670	5.9865	31
30	0963	10.3854	1139	8.7769	1317	7.5958	1495	6.6912	1673	5.9758	30
31	0966	10.3538	1142	8.7542	1319	7.5787	1497	6.6779	1676	5.9651	29
32	0969	10.3224	1145	8.7317	1322	7.5618	1500	6.6646	1679	5.9545	28
33	0972	10.2913	1148	8.7093	1325	7.5449	1503	6.6514	1682	5.9439	27
34	0975	10.2602	1151	8.6870	1328	7.5281	1506	6.6383	1685	5.9333	26
35	0978	10.2294	1154	8.6648	1331	7.5113	1509	6.6252	1688	5.9228	25
36	0981	10.1988	1157	8.6427	1334	7.4947	1512	6.6122	1691	5.9124	24
37	0983	10.1683	1160	8.6208	1337	7.4781	1515	6.5992	1694	5.9019	23
38	0986	10.1381	1163	8.5989	1340	7.4615	1518	6.5863	1697	5.8915	22
39	0989	10.1080	1166	8.5772	1343	7.4451	1521	6.5734	1700	5.8811	21
40	0992	10.0780	1169	8.5555	1346	7.4287	1524	6.5606	1703	5.8708	20
41	0995	10.0483	1172	8.5340	1349	7.4124	1527	6.5478	1706	5.8605	19
42	0998	10.0187	1175	8.5126	1352	7.3962	1530	6.5350	1709	5.8502	18
43	1001	9.9893	1178	8.4913	1355	7.3800	1533	6.5223	1712	5.8400	17
44	1004	9.9601	1181	8.4701	1358	7.3639	1536	6.5097	1715	6.8298	16
45	1007	9.9310	1184	8.4490	1361	7.3479	1539	6.4971	1718	5.8197	15
46	1010	9.9021	1187	8.4280	1364	7.3319	1542	6.4846	1721	5.8095	14
47	1013	9.8734	1189	8.4071	1367	7.3160	1545	6.4721	1724	5.7994	13
48	1016	9.8448	1192	8.3863	1370	7.3002	1548	6.4596	1727	5.7894	12
49	1019	9.8164	1195	8.3656	1373	7.2844	1551	6.4472	1730	5.7794	11
50	1022	9.7882	1198	8.3450	1376	7.2687	1554	6.4348	1733	5.7694	10
51	1025	9.7601	1201	8.3245	1379	7.2531	1557	6.4225	1736	5.7594	9
52	1028	9.7322	1204	8.3041	1382	7.2375	1560	6.4103	1739	5.7495	8
53	1030	9.7044	1207	8.2838	1385	7.2220	1563	6.3980	1742	5.7396	7
54	1033	9.6768	1210	8.2636	1388	7.2066	1566	6.3859	1745	5.7297	6
55	1036	9.6499	1213	8.2434	1391	7.1912	1569	6.3737	1748	5.7199	5
56	1039	9.6220	1216	8.2234	1394	7.1759	1572	6.3617	1751	5.7101	4
57	1042	9.5949	1219	8.2035	1397	7.1607	1575	6.3496	1754	5.7004	3
58	1045	9.5679	1222	8.1837	1399	7.1455	1578	6.3376	1757	5.6906	2
59	1048	9.5411	1225	8.1640	1402	7.1304	1581	6.3257	1760	5.6809	1
60	1051	9.5144	1228	8.1443	1405	7.1154	1584	6.3138	1763	5.6713	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
′	84°		83°		82°		81°		80°		′

′	10°		11°		12°		18°		14°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	1763	5.6713	1944	5.1446	2126	4.7046	2309	4.3315	2493	4.0108	60
1	1766	5.6617	1947	5.1366	2129	4.6979	2312	4.3257	2496	4.0058	59
2	1769	5.6521	1950	5.1286	2132	4.6912	2315	4.3200	2499	4.0009	58
3	1772	5.6425	1953	5.1207	2135	4.6845	2318	4.3143	2503	3.9959	57
4	1775	5.6330	1956	5.1128	2138	4.6779	2321	4.3086	2506	3.9910	56
5	1778	5.6234	1959	5.1049	2141	4.6712	2324	4.3029	2509	3.9861	55
6	1781	5.6140	1962	5.0970	2144	4.6646	2327	4.2972	2512	3.9812	54
7	1784	5.6045	1965	5.0892	2147	4.6580	2330	4.2916	2515	3.9763	53
8	1787	5.5951	1968	5.0814	2150	4.6514	2333	4.2859	2518	3.9714	52
9	1790	5.5857	1971	5.0736	2153	4.6448	2336	4.2803	2521	3.9665	51
10	1793	5.5764	1974	5.0658	2156	4.6382	2339	4.2747	2524	3.9617	50
11	1796	5.5671	1977	5.0581	2159	4.6317	2342	4.2691	2527	3.9568	49
12	1799	5.5578	1980	5.0504	2162	4.6252	2345	4.2635	2530	3.9520	48
13	1802	5.5485	1983	5.0427	2165	4.6187	2349	4.2580	2533	3.9471	47
14	1805	5.5393	1986	5.0350	2168	4.6122	2352	4.2524	2537	3.9423	46
15	1808	5.5301	1989	5.0273	2171	4.6057	2355	4.2468	2540	3.9375	45
16	1811	5.5209	1992	5.0197	2174	4.5993	2358	4.2413	2543	3.9327	44
17	1814	5.5118	1995	5.0121	2177	4.5928	2361	4.2358	2546	3.9279	43
18	1817	5.5026	1998	5.0045	2180	4.5864	2364	4.2303	2549	3.9232	42
19	1820	5.4936	2001	4.9969	2183	4.5800	2367	4.2248	2553	3.9184	41
20	1823	5.4845	2004	4.9894	2186	4.5736	2370	4.2193	2555	3.9136	40
21	1826	5.4755	2007	4.9819	2189	4.5673	2373	4.2139	2558	3.9089	39
22	1829	5.4665	2010	4.9744	2193	4.5609	2376	4.2084	2561	3.9042	38
23	1832	5.4575	2013	4.9669	2196	4.5546	2379	4.2030	2564	3.8995	37
24	1835	5.4486	2016	4.9594	2199	4.5483	2382	4.1976	2568	3.8947	36
25	1838	5.4397	2019	4.9520	2202	4.5420	2385	4.1922	2571	3.8900	35
26	1841	5.4308	2022	4.9446	2205	4.5357	2388	4.1868	2574	3.8854	34
27	1844	5.4219	2025	4.9372	2208	4.5294	2392	4.1814	2577	3.8807	33
28	1847	5.4131	2028	4.9298	2211	4.5232	2395	4.1760	2580	3.8760	32
29	1850	5.4043	2031	4.9225	2214	4.5169	2398	4.1706	2583	3.8714	31
30	1853	5.3955	2035	4.9152	2217	4.5107	2401	4.1653	2586	3.8667	30
31	1856	5.3868	2038	4.9078	2220	4.5045	2404	4.1600	2589	3.8621	29
32	1859	5.3781	2041	4.9006	2223	4.4983	2407	4.1547	2592	3.8575	28
33	1862	5.3694	2044	4.8933	2226	4.4922	2410	4.1493	2595	3.8528	27
34	1865	5.3607	2047	4.8860	2229	4.4860	2413	4.1441	2599	3.8482	26
35	1868	5.3521	2050	4.8788	2232	4.4799	2416	4.1388	2602	3.8436	25
36	1871	5.3435	2053	4.8716	2235	4.4737	2419	4.1335	2605	3.8391	24
37	1874	5.3349	2056	4.8644	2238	4.4676	2422	4.1282	2608	3.8345	23
38	1877	5.3263	2059	4.8573	2241	4.4615	2425	4.1230	2611	3.8299	22
39	1880	5.3178	2062	4.8501	2244	4.4555	2428	4.1178	2614	3.8254	21
40	1883	5.3093	2065	4.8430	2247	4.4494	2432	4.1126	2617	3.8208	20
41	1887	5.3008	2068	4.8359	2251	4.4434	2435	4.1074	2620	3.8163	19
42	1890	5.2924	2071	4.8288	2254	4.4374	2438	4.1022	2623	3.8118	18
43	1893	5.2839	2074	4.8218	2257	4.4313	2441	4.0970	2627	3.8073	17
44	1896	5.2755	2077	4.8147	2260	4.4253	2444	4.0918	2630	3.8028	16
45	1899	5.2672	2080	4.8077	2263	4.4194	2447	4.0867	2633	3.7983	15
46	1902	5.2588	2083	4.8007	2266	4.4134	2450	4.0815	2636	3.7938	14
47	1905	5.2505	2086	4.7937	2269	4.4075	2453	4.0764	2639	3.7893	13
48	1908	5.2422	2089	4.7867	2272	4.4015	2456	4.0713	2642	3.7848	12
49	1911	5.2339	2092	4.7798	2275	4.3956	2459	4.0662	2645	3.7804	11
50	1914	5.2257	2095	4.7729	2278	4.3897	2462	4.0611	2648	3.7760	10
51	1917	5.2174	2098	4.7659	2281	4.3838	2465	4.0560	2651	3.7715	9
52	1920	5.2092	2101	4.7591	2284	4.3779	2469	4.0509	2655	3.7671	8
53	1923	5.2011	2104	4.7522	2287	4.3721	2472	4.0459	2658	3.7627	7
54	1926	5.1929	2107	4.7453	2290	4.3662	2475	4.0408	2661	3.7583	6
55	1929	5.1848	2110	4.7385	2293	4.3604	2478	4.0358	2664	3.7539	5
56	1932	5.1767	2113	4.7317	2296	4.3546	2481	4.0308	2667	3.7495	4
57	1935	5.1686	2116	4.7249	2299	4.3488	2484	4.0257	2670	3.7451	3
58	1938	5.1606	2119	4.7181	2303	4.3430	2487	4.0207	2673	3.7408	2
59	1941	5.1526	2123	4.7114	2306	4.3372	2490	4.0158	2676	3.7364	1
60	1944	5.1446	2126	4.7046	2309	4.3315	2493	4.0108	2679	3.7321	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	2679	3.7321	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	60
1	2683	3.7277	2871	3.4836	3060	3.2675	3252	3.0746	3447	2.9015	59
2	2686	3.7234	2874	3.4798	3064	3.2641	3256	3.0716	3450	2.8987	58
3	2689	3.7191	2877	3.4760	3067	3.2607	3259	3.0686	3453	2.8960	57
4	2692	3.7148	2880	3.4722	3070	3.2573	3262	3.0655	3456	2.8933	56
5	2695	3.7105	2883	3.4684	3073	3.2539	3265	3.0625	3460	2.8905	55
6	2698	3.7062	2886	3.4646	3076	3.2506	3269	3.0595	3463	2.8878	54
7	2701	3.7019	2890	3.4608	3080	3.2472	3272	3.0565	3466	2.8851	53
8	2704	3.6976	2893	3.4570	3083	3.2438	3275	3.0535	3469	2.8824	52
9	2708	3.6933	2896	3.4533	3086	3.2405	3278	3.0505	3473	2.8797	51
10	2711	3.6891	2899	3.4495	3089	3.2371	3281	3.0475	3476	2.8770	50
11	2714	3.6848	2902	3.4458	3092	3.2338	3285	3.0445	3479	2.8743	49
12	2717	3.6806	2905	3.4420	3096	3.2305	3288	3.0415	3482	2.8716	48
13	2720	3.6764	2908	3.4383	3099	3.2272	3291	3.0385	3486	2.8689	47
14	2723	3.6722	2912	3.4346	3102	3.2238	3294	3.0356	3489	2.8662	46
15	2726	3.6680	2915	3.4308	3105	3.2205	3298	3.0326	3492	2.8636	45
16	2729	3.6638	2918	3.4271	3108	3.2172	3301	3.0296	3495	2.8609	44
17	2733	3.6596	2921	3.4234	3111	3.2139	3304	3.0267	3499	2.8582	43
18	2736	3.6554	2924	3.4197	3115	3.2106	3307	3.0237	3502	2.8556	42
19	2739	3.6512	2927	3.4160	3118	3.2073	3310	3.0208	3505	2.8529	41
20	2742	3.6470	2931	3.4124	3121	3.2041	3314	3.0178	3508	2.8502	40
21	2745	3.6429	2934	3.4087	3124	3.2008	3317	3.0149	3512	2.8476	39
22	2748	3.6387	2937	3.4050	3127	3.1975	3320	3.0120	3515	2.8449	38
23	2751	3.6346	2940	3.4014	3131	3.1943	3323	3.0090	3518	2.8423	37
24	2754	3.6305	2943	3.3977	3134	3.1910	3327	3.0061	3522	2.8397	36
25	2758	3.6264	2946	3.3941	3137	3.1878	3330	3.0032	3525	2.8370	35
26	2761	3.6222	2949	3.3904	3140	3.1845	3333	3.0003	3528	2.8344	34
27	2764	3.6181	2953	3.3868	3143	3.1813	3336	2.9974	3531	2.8318	33
28	2767	3.6140	2956	3.3832	3147	3.1780	3339	2.9945	3535	2.8291	32
29	2770	3.6100	2959	3.3796	3150	3.1748	3343	2.9916	3538	2.8265	31
30	2773	3.6059	2962	3.3759	3153	3.1716	3346	2.9887	3541	2.8239	30
31	2776	3.6018	2965	3.3723	3156	3.1684	3349	2.9858	3544	2.8213	29
32	2780	3.5978	2968	3.3687	3159	3.1652	3352	2.9829	3548	2.8187	28
33	2783	3.5937	2972	3.3652	3163	3.1620	3356	2.9800	3551	2.8161	27
34	2786	3.5897	2975	3.3616	3166	3.1588	3359	2.9772	3554	2.8135	26
35	2789	3.5856	2978	3.3580	3169	3.1556	3362	2.9743	3558	2.8109	25
36	2792	3.5816	2981	3.3544	3172	3.1524	3365	2.9714	3561	2.8083	24
37	2795	3.5776	2984	3.3509	3175	3.1492	3369	2.9686	3564	2.8057	23
38	2798	3.5736	2987	3.3473	3179	3.1460	3372	2.9657	3567	2.8032	22
39	2801	3.5696	2991	3.3438	3182	3.1429	3375	2.9629	3571	2.8006	21
40	2805	3.5656	2994	3.3402	3185	3.1397	3378	2.9600	3574	2.7980	20
41	2808	3.5616	2997	3.3367	3188	3.1366	3382	2.9572	3577	2.7955	19
42	2811	3.5576	3000	3.3332	3191	3.1334	3385	2.9544	3581	2.7929	18
43	2814	3.5536	3003	3.3297	3195	3.1303	3388	2.9515	3584	2.7903	17
44	2817	3.5497	3006	3.3261	3198	3.1271	3391	2.9487	3587	2.7878	16
45	2820	3.5457	3010	3.3226	3201	3.1240	3395	2.9459	3590	2.7852	15
46	2823	3.5418	3013	3.3191	3204	3.1209	3398	2.9431	3594	2.7827	14
47	2827	3.5379	3016	3.3156	3207	3.1178	3401	2.9403	3597	2.7801	13
48	2830	3.5339	3019	3.3122	3211	3.1146	3404	2.9375	3600	2.7776	12
49	2833	3.5300	3022	3.3087	3214	3.1115	3408	2.9347	3604	2.7751	11
50	2836	3.5261	3026	3.3052	3217	3.1084	3411	2.9319	3607	2.7725	10
51	2839	3.5222	3029	3.3017	3220	3.1053	3414	2.9291	3610	2.7700	9
52	2842	3.5183	3032	3.2983	3223	3.1022	3417	2.9263	3613	2.7675	8
53	2845	3.5144	3035	3.2948	3227	3.0991	3421	2.9235	3617	2.7650	7
54	2849	3.5105	3038	3.2914	3230	3.0961	3424	2.9208	3620	2.7625	6
55	2852	3.5067	3041	3.2880	3233	3.0930	3427	2.9180	3623	2.7600	5
56	2855	3.5028	3045	3.2845	3236	3.0899	3430	2.9152	3627	2.7575	4
57	2858	3.4989	3048	3.2811	3240	3.0868	3434	2.9125	3630	2.7550	3
58	2861	3.4951	3051	3.2777	3243	3.0838	3437	2.9097	3633	2.7525	2
59	2864	3.4912	3054	3.2743	3246	3.0807	3440	2.9070	3636	2.7500	1
60	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	3640	2.7475	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	3640	2.7475	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	60
1	3643	2.7450	3842	2.6028	4044	2.4730	4248	2.3539	4456	2.2443	59
2	3646	2.7425	3845	2.6006	4047	2.4709	4252	2.3520	4459	2.2425	58
3	3650	2.7400	3849	2.5983	4050	2.4689	4255	2.3501	4463	2.2408	57
4	3653	2.7376	3852	2.5961	4054	2.4668	4258	2.3483	4466	2.2390	56
5	3656	2.7351	3855	2.5938	4057	2.4648	4262	2.3464	4470	2.2373	55
6	3659	2.7326	3859	2.5916	4061	2.4627	4265	2.3445	4473	2.2355	54
7	3663	2.7302	3862	2.5893	4064	2.4606	4269	2.3426	4477	2.2338	53
8	3666	2.7277	3865	2.5871	4067	2.4586	4272	2.3407	4480	2.2320	52
9	3669	2.7253	3869	2.5848	4071	2.4566	4276	2.3388	4484	2.2303	51
10	3673	2.7228	3872	2.5826	4074	2.4545	4279	2.3369	4487	2.2286	50
11	3676	2.7204	3875	2.5804	4078	2.4525	4283	2.3351	4491	2.2268	49
12	3679	2.7179	3879	2.5782	4081	2.4504	4286	2.3332	4494	2.2251	48
13	3683	2.7155	3882	2.5759	4084	2.4484	4289	2.3313	4498	2.2234	47
14	3686	2.7130	3885	2.5737	4088	2.4464	4293	2.3294	4501	2.2216	46
15	3689	2.7106	3889	2.5715	4091	2.4443	4296	2.3276	4505	2.2199	45
16	3693	2.7082	3892	2.5693	4095	2.4423	4300	2.3257	4508	2.2182	44
17	3696	2.7058	3895	2.5671	4098	2.4403	4303	2.3238	4512	2.2165	43
18	3699	2.7034	3899	2.5649	4101	2.4383	4307	2.3220	4515	2.2148	42
19	3702	2.7009	3902	2.5627	4105	2.4362	4310	2.3201	4519	2.2130	41
20	3706	2.6985	3906	2.5605	4108	2.4342	4314	2.3183	4522	2.2113	40
21	3709	2.6961	3909	2.5533	4111	2.4322	4317	2.3164	4526	2.2096	39
22	3712	2.6937	3912	2.5561	4115	2.4302	4320	2.3146	4529	2.2079	38
23	3716	2.6913	3916	2.5539	4118	2.4282	4324	2.3127	4533	2.2062	37
24	3719	2.6889	3919	2.5517	4122	2.4262	4327	2.3109	4536	2.2045	36
25	3722	2.6865	3922	2.5495	4125	2.4242	4331	2.3090	4540	2.2028	35
26	3726	2.6841	3926	2.5473	4129	2.4222	4334	2.3072	4543	2.2011	34
27	3729	2.6818	3929	2.5452	4132	2.4202	4338	2.3053	4547	2.1994	33
28	3732	2.6794	3932	2.5430	4135	2.4182	4341	2.3035	4550	2.1977	32
29	3736	2.6770	3936	2.5408	4139	2.4162	4345	2.3017	4554	2.1960	31
30	3739	2.6746	3939	2.5386	4142	2.4142	4348	2.2998	4557	2.1943	30
31	3742	2.6723	3942	2.5365	4146	2.4122	4352	2.2980	4561	2.1926	29
32	3745	2.6699	3946	2.5343	4149	2.4102	4355	2.2962	4564	2.1909	28
33	3749	2.6675	3949	2.5322	4152	2.4083	4359	2.2944	4568	2.1892	27
34	3752	2.6652	3953	2.5300	4156	2.4063	4362	2.2925	4571	2.1876	26
35	3755	2.6628	3956	2.5279	4159	2.4043	4365	2.2907	4575	2.1859	25
36	3759	2.6605	3959	2.5257	4163	2.4023	4369	2.2889	4578	2.1842	24
37	3762	2.6581	3963	2.5236	4166	2.4004	4372	2.2871	4582	2.1825	23
38	3765	2.6558	3966	2.5214	4169	2.3984	4376	2.2853	4585	2.1808	22
39	3769	2.6534	3969	2.5193	4173	2.3964	4379	2.2835	4589	2.1792	21
40	3772	2.6511	3973	2.5172	4176	2.3945	4383	2.2817	4592	2.1775	20
41	3775	2.6488	3976	2.5150	4180	2.3925	4386	2.2799	4596	2.1758	19
42	3779	2.6464	3979	2.5129	4183	2.3906	4390	2.2781	4599	2.1742	18
43	3782	2.6441	3983	2.5108	4187	2.3886	4393	2.2763	4603	2.1725	17
44	3785	2.6418	3986	2.5086	4190	2.3867	4397	2.2745	4607	2.1708	16
45	3789	2.6395	3990	2.5065	4193	2.3847	4400	2.2727	4610	2.1692	15
46	3792	2.6371	3993	2.5044	4197	2.3828	4404	2.2709	4614	2.1675	14
47	3795	2.6348	3996	2.5023	4200	2.3808	4407	2.2691	4617	2.1659	13
48	3799	2.6325	4000	2.5002	4204	2.3789	4411	2.2673	4621	2.1642	12
49	3802	2.6302	4003	2.4981	4207	2.3770	4414	2.2655	4624	2.1625	11
50	3805	2.6279	4006	2.4960	4210	2.3750	4417	2.2637	4628	2.1609	10
51	3809	2.6256	4010	2.4939	4214	2.3731	4421	2.2620	4631	2.1592	9
52	3812	2.6233	4013	2.4918	4217	2.3712	4424	2.2602	4635	2.1576	8
53	3815	2.6210	4017	2.4897	4221	2.3693	4428	2.2584	4638	2.1560	7
54	3819	2.6187	4020	2.4876	4224	2.3673	4431	2.2566	4642	2.1543	6
55	3822	2.6165	4023	2.4855	4228	2.3654	4435	2.2549	4645	2.1527	5
56	3825	2.6142	4027	2.4834	4231	2.3635	4438	2.2531	4649	2.1510	4
57	3829	2.6119	4030	2.4813	4234	2.3616	4442	2.2513	4652	2.1494	3
58	3832	2.6096	4033	2.4792	4238	2.3597	4445	2.2496	4656	2.1478	2
59	3835	2.6074	4037	2.4772	4241	2.3578	4449	2.2478	4660	2.1461	1
60	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	4663	2.1445	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
′	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	4663	2.1445	4877	2.0503	5095	1.9626	5317	1.8807	5543	1.8040	60
1	4667	2.1429	4881	2.0488	5099	1.9612	5321	1.8794	5547	1.8028	59
2	4670	2.1413	4885	2.0473	5103	1.9598	5325	1.8781	5551	1.8016	58
3	4674	2.1396	4888	2.0458	5106	1.9584	5328	1.8768	5555	1.8003	57
4	4677	2.1380	4892	2.0443	5110	1.9570	5332	1.8755	5558	1.7991	56
5	4681	2.1364	4895	2.0428	5114	1.9556	5336	1.8741	5562	1.7979	55
6	4684	2.1348	4899	2.0413	5117	1.9542	5340	1.8728	5566	1.7966	54
7	4688	2.1332	4903	2.0398	5121	1.9528	5343	1.8715	5570	1.7954	53
8	4691	2.1315	4906	2.0383	5125	1.9514	5347	1.8702	5574	1.7942	52
9	4695	2.1299	4910	2.0368	5128	1.9500	5351	1.8689	5577	1.7930	51
10	4699	2.1283	4913	2.0353	5132	1.9486	5354	1.8676	5581	1.7917	50
11	4702	2.1267	4917	2.0338	5136	1.9472	5358	1.8663	5585	1.7905	49
12	4706	2.1251	4921	2.0323	5139	1.9458	5362	1.8650	5589	1.7893	48
13	4709	2.1235	4924	2.0308	5143	1.9444	5366	1.8637	5593	1.7881	47
14	4713	2.1219	4928	2.0293	5147	1.9430	5369	1.8624	5596	1.7868	46
15	4716	2.1203	4931	2.0278	5150	1.9416	5373	1.8611	5600	1.7856	45
16	4720	2.1187	4935	2.0263	5154	1.9402	5377	1.8598	5604	1.7844	44
17	4723	2.1171	4939	2.0248	5158	1.9388	5381	1.8585	5608	1.7832	43
18	4727	2.1155	4942	2.0233	5161	1.9375	5384	1.8572	5612	1.7820	42
19	4731	2.1139	4946	2.0219	5165	1.9361	5388	1.8559	5616	1.7808	41
20	4734	2.1123	4950	2.0204	5169	1.9347	5392	1.8546	5619	1.7796	40
21	4738	2.1107	4953	2.0189	5172	1.9333	5396	1.8533	5623	1.7783	39
22	4741	2.1092	4957	2.0174	5176	1.9319	5399	1.8520	5627	1.7771	38
23	4745	2.1076	4960	2.0160	5180	1.9306	5403	1.8507	5631	1.7759	37
24	4748	2.1060	4964	2.0145	5184	1.9292	5407	1.8495	5635	1.7747	36
25	4752	2.1044	4968	2.0130	5187	1.9278	5411	1.8482	5639	1.7735	35
26	4755	2.1028	4971	2.0115	5191	1.9265	5415	1.8469	5642	1.7723	34
27	4759	2.1013	4975	2.0101	5195	1.9251	5418	1.8456	5646	1.7711	33
28	4763	2.0997	4979	2.0086	5198	1.9237	5422	1.8443	5650	1.7699	32
29	4766	2.0981	4982	2.0072	5202	1.9223	5426	1.8430	5654	1.7687	31
30	4770	2.0965	4986	2.0057	5206	1.9210	5430	1.8418	5658	1.7675	30
31	4773	2.0950	4989	2.0042	5209	1.9196	5433	1.8405	5662	1.7663	29
32	4777	2.0934	4993	2.0028	5213	1.9183	5437	1.8392	5665	1.7651	28
33	4780	2.0918	4997	2.0013	5217	1.9169	5441	1.8379	5669	1.7639	27
34	4784	2.0903	5000	1.9999	5220	1.9155	5445	1.8367	5673	1.7627	26
35	4788	2.0887	5004	1.9984	5224	1.9142	5448	1.8354	5677	1.7615	25
36	4791	2.0872	5008	1.9970	5228	1.9128	5452	1.8341	5681	1.7603	24
37	4795	2.0856	5011	1.9955	5232	1.9115	5456	1.8329	5685	1.7591	23
38	4798	2.0840	5015	1.9941	5235	1.9101	5460	1.8316	5688	1.7579	22
39	4802	2.0825	5019	1.9926	5239	1.9088	5464	1.8303	5692	1.7567	21
40	4806	2.0809	5022	1.9912	5243	1.9074	5467	1.8291	5696	1.7556	20
41	4809	2.0794	5026	1.9897	5426	1.9061	5471	1.8278	5700	1.7544	19
42	4813	2.0778	5029	1.9883	5250	1.9047	5475	1.8265	5704	1.7532	18
43	4816	2.0763	5033	1.9868	5254	1.9034	5479	1.8253	5708	1.7520	17
44	4820	2.0748	5037	1.9854	5258	1.9020	5482	1.8240	5712	1.7508	16
45	4823	2.0732	5040	1.9840	5261	1.9007	5486	1.8228	5715	1.7496	15
46	4827	2.0717	5044	1.9825	5265	1.8993	5490	1.8215	5719	1.7485	14
47	4831	2.0701	5048	1.9811	5269	1.8980	5494	1.8202	5723	1.7473	13
48	4834	2.0686	5051	1.9797	5272	1.8967	5498	1.8190	5727	1.7461	12
49	4838	2.0671	5055	1.9782	5276	1.8953	5501	1.8177	5731	1.7449	11
50	4841	2.0655	5059	1.9768	5280	1.8940	5505	1.8165	5735	1.7437	10
51	4845	2.0640	5062	1.9754	5284	1.8927	5509	1.8152	5739	1.7426	9
52	4849	2.0625	5066	1.9740	5287	1.8913	5513	1.8140	5743	1.7414	8
53	4852	2.0609	5070	1.9725	5291	1.8900	5517	1.8127	5746	1.7402	7
54	4856	2.0594	5073	1.9711	5295	1.8887	5520	1.8115	5750	1.7391	6
55	4859	2.0579	5077	1.9697	5298	1.8873	5524	1.8103	5754	1.7379	5
56	4863	2.0564	5081	1.9683	5302	1.8860	5528	1.8090	5758	1.7367	4
57	4867	2.0549	5084	1.9669	5306	1.8847	5532	1.8078	5762	1.7355	3
58	4870	2.0533	5088	1.9654	5310	1.8834	5535	1.8065	5766	1.7344	2
59	4874	2.0518	5092	1.9640	5313	1.8820	5539	1.8053	5770	1.7332	1
60	4877	2.0503	5095	1.9626	5317	1.8807	5543	1.8040	5774	1.7321	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
′	64°		63°		62°		61°		60°		′

′	80°		81°		82°		83°		84°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	5774	1.7321	6009	1.6643	6249	1.6003	6494	1.5399	6745	1.4826	60
1	5777	1.7309	6013	1.6632	6253	1.5993	6498	1.5389	6749	1.4816	59
2	5781	1.7297	6017	1.6621	6257	1.5983	6502	1.5379	6754	1.4807	58
3	5785	1.7286	6020	1.6610	6261	1.5972	6506	1.5369	6758	1.4798	57
4	5789	1.7274	6024	1.6599	6265	1.5962	6511	1.5359	6762	1.4788	56
5	5793	1.7262	6028	1.6588	6269	1.5952	6515	1.5350	6766	1.4779	55
6	5797	1.7251	6032	1.6577	6273	1.5941	6519	1.5340	6771	1.4770	54
7	5801	1.7239	6036	1.6566	6277	1.5931	6523	1.5330	6775	1.4761	53
8	5805	1.7228	6040	1.6555	6281	1.5921	6527	1.5320	6779	1.4751	52
9	5808	1.7216	6044	1.6545	6285	1.5911	6531	1.5311	6783	1.4742	51
10	5812	1.7205	6048	1.6534	6289	1.5900	6536	1.5301	6787	1.4733	50
11	5816	1.7193	6052	1.6523	6293	1.5890	6540	1.5291	6792	1.4724	49
12	5820	1.7182	6056	1.6512	6297	1.5880	6544	1.5282	6796	1.4715	48
13	5824	1.7170	6060	1.6501	6301	1.5869	6548	1.5272	6800	1.4705	47
14	5828	1.7159	6064	1.6490	6305	1.5859	6552	1.5262	6805	1.4696	46
15	5832	1.7147	6068	1.6479	6310	1.5849	6556	1.5253	6809	1.4687	45
16	5836	1.7136	6072	1.6469	6314	1.5839	6560	1.5243	6813	1.4678	44
17	5840	1.7124	6076	1.6458	6318	1.5829	6565	1.5233	6817	1.4669	43
18	5844	1.7113	6080	1.6447	6322	1.5818	6569	1.5224	6822	1.4659	42
19	5847	1.7102	6084	1.6436	6326	1.5808	6573	1.5214	6826	1.4650	41
20	5851	1.7090	6088	1.6426	6330	1.5798	6577	1.5204	6830	1.4641	40
21	5855	1.7079	6092	1.6415	6334	1.5788	6581	1.5195	6834	1.4632	39
22	5859	1.7067	6096	1.6404	6338	1.5778	6585	1.5185	6839	1.4623	38
23	5863	1.7056	6100	1.6393	6342	1.5768	6590	1.5175	6843	1.4614	37
24	5867	1.7045	6104	1.6383	6346	1.5757	6594	1.5166	6847	1.4605	36
25	5871	1.7033	6108	1.6372	6350	1.5747	6598	1.5156	6851	1.4596	35
26	5875	1.7022	6112	1.6361	6354	1.5737	6602	1.5147	6856	1.4586	34
27	5879	1.7011	6116	1.6351	6358	1.5727	6606	1.5137	6860	1.4577	33
28	5883	1.6999	6120	1.6340	6363	1.5717	6610	1.5127	6864	1.4568	32
29	5887	1.6988	6124	1.6329	6367	1.5707	6615	1.5118	6869	1.4559	31
30	5890	1.6977	6128	1.6319	6371	1.5697	6619	1.5108	6873	1.4550	30
31	5894	1.6965	6132	1.6308	6375	1.5687	6623	1.5099	6877	1.4541	29
32	5898	1.6954	6136	1.6297	6379	1.5677	6627	1.5089	6881	1.4532	28
33	5902	1.6943	6140	1.6287	6383	1.5667	6631	1.5080	6886	1.4523	27
34	5906	1.6932	6144	1.6276	6387	1.5657	6636	1.5070	6890	1.4514	26
35	5910	1.6920	6148	1.6265	6391	1.5647	6640	1.5061	6894	1.4505	25
36	5914	1.6909	6152	1.6255	6395	1.5637	6644	1.5051	6899	1.4496	24
37	5918	1.6898	6156	1.6244	6399	1.5627	6648	1.5042	6903	1.4487	23
38	5922	1.6887	6160	1.6234	6403	1.5617	6652	1.5032	6907	1.4478	22
39	5926	1.6875	6164	1.6223	6408	1.5607	6657	1.5023	6911	1.4469	21
40	5930	1.6864	6168	1.6212	6412	1.5597	6661	1.5013	6916	1.4460	20
41	5934	1.6853	6172	1.6202	6416	1.5587	6665	1.5004	6920	1.4451	19
42	5938	1.6842	6176	1.6191	6420	1.5577	6669	1.4994	6924	1.4442	18
43	5942	1.6831	6180	1.6181	6424	1.5567	6673	1.4985	6929	1.4433	17
44	5945	1.6820	6184	1.6170	6428	1.5557	6678	1.4975	6933	1.4424	16
45	5949	1.6808	6188	1.6160	6432	1.5547	6682	1.4966	6937	1.4415	15
46	5953	1.6797	6192	1.6149	6436	1.5537	6686	1.4957	6942	1.4406	14
47	5957	1.6786	6196	1.6139	6440	1.5527	6690	1.4947	6946	1.4397	13
48	5961	1.6775	6200	1.6128	6445	1.5517	6694	1.4938	6950	1.4388	12
49	5965	1.6764	6204	1.6118	6449	1.5507	6699	1.4928	6954	1.4379	11
50	5969	1.6753	6208	1.6107	6453	1.5497	6703	1.4919	6959	1.4370	10
51	5973	1.6742	6212	1.6097	6457	1.5487	6707	1.4910	6963	1.4361	9
52	5977	1.6731	6216	1.6087	6461	1.5477	6711	1.4900	6967	1.4352	8
53	5981	1.6720	6220	1.6076	6465	1.5468	6716	1.4891	6972	1.4344	7
54	5985	1.6709	6224	1.6066	6469	1.5458	6720	1.4882	6976	1.4335	6
55	5989	1.6698	6228	1.6055	6473	1.5448	6724	1.4872	6980	1.4326	5
56	5993	1.6687	6233	1.6045	6478	1.5438	6728	1.4863	6985	1.4317	4
57	5997	1.6676	6237	1.6034	6482	1.5428	6732	1.4854	6989	1.4308	3
58	6001	1.6665	6241	1.6024	6486	1.5418	6737	1.4844	6993	1.4299	2
59	6005	1.6654	6245	1.6014	6490	1.5408	6741	1.4835	6998	1.4290	1
60	6009	1.6643	6249	1.6003	6494	1.5399	6745	1.4826	7002	1.4281	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
′	59°		58°		57°		56°		55°		′

′	85°		86°		87°		88°		89°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	7002	1.4281	7265	1.3764	7536	1.3270	7813	1.2799	8098	1.2349	60
1	7006	1.4273	7270	1.3755	7540	1.3262	7818	1.2792	8103	1.2342	59
2	7011	1.4264	7274	1.3747	7545	1.3254	7822	1.2784	8107	1.2334	58
3	7015	1.4255	7279	1.3739	7549	1.3246	7827	1.2776	8112	1.2327	57
4	7019	1.4246	7283	1.3730	7554	1.3238	7832	1.2769	8117	1.2320	56
5	7024	1.4237	7288	1.3722	7558	1.3230	7836	1.2761	8122	1.2312	55
6	7028	1.4229	7292	1.3713	7563	1.3222	7841	1.2753	8127	1.2305	54
7	7032	1.4220	7297	1.3705	7568	1.3214	7846	1.2746	8132	1.2298	53
8	7037	1.4211	7301	1.3697	7572	1.3206	7850	1.2738	8136	1.2290	52
9	7041	1.4202	7306	1.3688	7577	1.3198	7855	1.2731	8141	1.2283	51
10	7046	1.4193	7310	1.3680	7581	1.3190	7860	1.2723	8146	1.2276	50
11	7050	1.4185	7314	1.3672	7586	1.3182	7865	1.2715	8151	1.2268	49
12	7054	1.4176	7319	1.3663	7590	1.3175	7869	1.2708	8156	1.2261	48
13	7059	1.4167	7323	1.3655	7595	1.3167	7874	1.2700	8161	1.2254	47
14	7063	1.4158	7328	1.3647	7600	1.3159	7879	1.2693	8165	1.2247	46
15	7067	1.4150	7332	1.3638	7604	1.3151	7883	1.2685	8170	1.2239	45
16	7072	1.4141	7337	1.3630	7609	1.3143	7888	1.2677	8175	1.2232	44
17	7076	1.4132	7341	1.3622	7613	1.3135	7893	1.2670	8180	1.2225	43
18	7080	1.4124	7346	1.3613	7618	1.3127	7898	1.2662	8185	1.2218	42
19	7085	1.4115	7350	1.3605	7623	1.3119	7902	1.2655	8190	1.2210	41
20	7089	1.4106	7355	1.3597	7627	1.3111	7907	1.2647	8195	1.2203	40
21	7094	1.4097	7359	1.3588	7632	1.3103	7912	1.2640	8199	1.2196	39
22	7098	1.4089	7364	1.3580	7636	1.3095	7916	1.2632	8204	1.2189	38
23	7102	1.4080	7368	1.3572	7641	1.3087	7921	1.2624	8209	1.2181	37
24	7107	1.4071	7373	1.3564	7646	1.3079	7926	1.2617	8214	1.2174	36
25	7111	1.4063	7377	1.3555	7650	1.3072	7931	1.2609	8219	1.2167	35
26	7115	1.4054	7382	1.3547	7655	1.3064	7935	1.2602	8224	1.2160	34
27	7120	1.4045	7386	1.3539	7659	1.3056	7940	1.2594	8229	1.2153	33
28	7124	1.4037	7391	1.3531	7664	1.3048	7945	1.2587	8234	1.2145	32
29	7129	1.4028	7395	1.3522	7669	1.3040	7950	1.2579	8238	1.2138	31
30	7133	1.4019	7400	1.3514	7673	1.3032	7954	1.2572	8243	1.2131	30
31	7137	1.4011	7404	1.3506	7678	1.3024	7959	1.2564	8248	1.2124	29
32	7142	1.4002	7409	1.3498	7683	1.3017	7964	1.2557	8253	1.2117	28
33	7146	1.3994	7413	1.3490	7687	1.3009	7969	1.2549	8258	1.2109	27
34	7151	1.3985	7418	1.3481	7692	1.3001	7973	1.2542	8263	1.2102	26
35	7155	1.3976	7422	1.3473	7696	1.2993	7978	1.2534	8268	1.2095	25
36	7159	1.3968	7427	1.3465	7701	1.2985	7983	1.2527	8273	1.2088	24
37	7164	1.3959	7431	1.3457	7706	1.2977	7988	1.2519	8278	1.2081	23
38	7168	1.3951	7436	1.3449	7710	1.2970	7992	1.2512	8283	1.2074	22
39	7173	1.3942	7440	1.3440	7715	1.2962	7997	1.2504	8287	1.2066	21
40	7177	1.3934	7445	1.3432	7720	1.2954	8002	1.2497	8292	1.2059	20
41	7181	1.3925	7449	1.3424	7724	1.2946	8007	1.2489	8297	1.2052	19
42	7186	1.3916	7454	1.3416	7729	1.2938	8012	1.2482	8302	1.2045	18
43	7190	1.3908	7458	1.3408	7734	1.2931	8016	1.2475	8307	1.2038	17
44	7195	1.3899	7463	1.3400	7738	1.2923	8021	1.2467	8312	1.2031	16
45	7199	1.3891	7467	1.3392	7743	1.2915	8026	1.2460	8317	1.2024	15
46	7203	1.3882	7472	1.3384	7747	1.2907	8031	1.2452	8322	1.2017	14
47	7208	1.3874	7476	1.3375	7752	1.2900	8035	1.2445	8327	1.2009	13
48	7212	1.3865	7481	1.3367	7757	1.2892	8040	1.2437	8332	1.2002	12
49	7217	1.3857	7485	1.3359	7761	1.2884	8045	1.2430	8337	1.1995	11
50	7221	1.3848	7490	1.3351	7766	1.2876	8050	1.2423	8342	1.1988	10
51	7226	1.3840	7495	1.3343	7771	1.2869	8055	1.2415	8346	1.1981	9
52	7230	1.3831	7499	1.3335	7775	1.2861	8059	1.2408	8351	1.1974	8
53	7234	1.3823	7504	1.3327	7780	1.2853	8064	1.2401	8356	1.1967	7
54	7239	1.3814	7508	1.3319	7785	1.2846	8069	1.2393	8361	1.1960	6
55	7243	1.3806	7513	1.3311	7789	1.2838	8074	1.2386	8366	1.1953	5
56	7248	1.3798	7517	1.3303	7794	1.2830	8079	1.2378	8371	1.1946	4
57	7252	1.3789	7522	1.3295	7799	1.2822	8083	1.2371	8376	1.1939	3
58	7257	1.3781	7526	1.3287	7803	1.2815	8088	1.2364	8381	1.1932	2
59	7261	1.3772	7531	1.3278	7808	1.2807	8093	1.2356	8386	1.1925	1
60	7265	1.3764	7536	1.3270	7813	1.2799	8098	1.2349	8391	1.1918	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
′	54°		53°		52°		51°		50°		′

′	40°		41°		42°		43°		44°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	8391	1.1918	8693	1.1504	9004	1.1106	9325	1.0724	9657	1.0355	60
1	8396	1.1910	8698	1.1497	9009	1.1100	9331	1.0717	9663	1.0349	59
2	8401	1.1903	8703	1.1490	9015	1.1093	9336	1.0711	9668	1.0343	58
3	8406	1.1896	8708	1.1483	9020	1.1087	9341	1.0705	9674	1.0337	57
4	8411	1.1889	8713	1.1477	9025	1.1080	9347	1.0699	9679	1.0331	56
5	8416	1.1882	8718	1.1470	9030	1.1074	9352	1.0692	9685	1.0325	55
6	8421	1.1875	8724	1.1463	9036	1.1067	9358	1.0686	9691	1.0319	54
7	8426	1.1868	8729	1.1456	9041	1.1061	9363	1.0680	9696	1.0313	53
8	8431	1.1861	8734	1.1450	9046	1.1054	9369	1.0674	9702	1.0307	52
9	8436	1.1854	8739	1.1443	9052	1.1048	9374	1.0668	9708	1.0301	51
10	8441	1.1847	8744	1.1436	9057	1.1041	9380	1.0661	9713	1.0295	50
11	8446	1.1840	8749	1.1430	9062	1.1035	9385	1.0655	9719	1.0289	49
12	8451	1.1833	8754	1.1423	9067	1.1028	9391	1.0649	9725	1.0283	48
13	8456	1.1826	8759	1.1416	9073	1.1022	9396	1.0643	9730	1.0277	47
14	8461	1.1819	8765	1.1410	9078	1.1016	9402	1.0637	9736	1.0271	46
15	8466	1.1812	8770	1.1403	9083	1.1009	9407	1.0630	9742	1.0265	45
16	8471	1.1806	8775	1.1396	9089	1.1003	9413	1.0624	9747	1.0259	44
17	8476	1.1799	8780	1.1389	9094	1.0996	9418	1.0618	9753	1.0253	43
18	8481	1.1792	8785	1.1383	9099	1.0990	9424	1.0612	9759	1.0247	42
19	8486	1.1785	8790	1.1376	9105	1.0983	9429	1.0606	9764	1.0241	41
20	8491	1.1778	8796	1.1369	9110	1.0977	9435	1.0599	9770	1.0235	40
21	8496	1.1771	8801	1.1363	9115	1.0971	9440	1.0593	9776	1.0230	39
22	8501	1.1764	8806	1.1356	9121	1.0964	9446	1.0587	9781	1.0224	38
23	8506	1.1757	8811	1.1349	9126	1.0958	9451	1.0581	9787	1.0218	37
24	8511	1.1750	8816	1.1343	9131	1.0951	9457	1.0575	9793	1.0212	36
25	8516	1.1743	8821	1.1336	9137	1.0945	9462	1.0569	9798	1.0206	35
26	8521	1.1736	8827	1.1329	9142	1.0939	9468	1.0562	9804	1.0200	34
27	8526	1.1729	8832	1.1323	9147	1.0932	9473	1.0556	9810	1.0194	33
28	8531	1.1722	8837	1.1316	9153	1.0926	9479	1.0550	9816	1.0188	32
29	8536	1.1715	8842	1.1310	9158	1.0919	9484	1.0544	9821	1.0182	31
30	8541	1.1708	8847	1.1303	9163	1.0913	9490	1.0538	9827	1.0176	30
31	8546	1.1702	8852	1.1296	9169	1.0907	9495	1.0532	9833	1.0170	29
32	8551	1.1695	8858	1.1290	9174	1.0900	9501	1.0526	9838	1.0164	28
33	8556	1.1688	8863	1.1283	9179	1.0894	9506	1.0519	9844	1.0158	27
34	8561	1.1681	8868	1.1276	9185	1.0888	9512	1.0513	9850	1.0152	26
35	8566	1.1674	8873	1.1270	9190	1.0881	9517	1.0507	9856	1.0147	25
36	8571	1.1667	8878	1.1263	9195	1.0875	9523	1.0501	9861	1.0141	24
37	8576	1.1660	8884	1.1257	9201	1.0869	9528	1.0495	9867	1.0135	23
38	8581	1.1653	8889	1.1250	9206	1.0862	9534	1.0489	9873	1.0129	22
39	8586	1.1647	8894	1.1243	9212	1.0856	9540	1.0483	9879	1.0123	21
40	8591	1.1640	8899	1.1237	9217	1.0850	9545	1.0477	9884	1.0117	20
41	8596	1.1633	8904	1.1230	9222	1.0843	9551	1.0470	9890	1.0111	19
42	8601	1.1626	8910	1.1224	9228	1.0837	9556	1.0464	9896	1.0105	18
43	8606	1.1619	8915	1.1217	9233	1.0831	9562	1.0458	9902	1.0099	17
44	8611	1.1612	8920	1.1211	9239	1.0824	9567	1.0452	9907	1.0094	16
45	8617	1.1606	8925	1.1204	9244	1.0818	9573	1.0446	9913	1.0088	15
46	8622	1.1599	8931	1.1197	9249	1.0812	9578	1.0440	9919	1.0082	14
47	8627	1.1592	8936	1.1191	9255	1.0805	9584	1.0434	9925	1.0076	13
48	8632	1.1585	8941	1.1184	9260	1.0799	9590	1.0428	9930	1.0070	12
49	8637	1.1578	8946	1.1178	9266	1.0793	9595	1.0422	9936	1.0064	11
50	8642	1.1571	8952	1.1171	9271	1.0786	9601	1.0416	9942	1.0058	10
51	8647	1.1565	8957	1.1165	9276	1.0780	9606	1.0410	9948	1.0052	9
52	8652	1.1558	8962	1.1158	9282	1.0774	9612	1.0404	9954	1.0047	8
53	8657	1.1551	8967	1.1152	9287	1.0768	9618	1.0398	9959	1.0041	7
54	8662	1.1544	8972	1.1145	9293	1.0761	9623	1.0392	9965	1.0035	6
55	8667	1.1538	8978	1.1139	9298	1.0755	9629	1.0385	9971	1.0029	5
56	8672	1.1531	8983	1.1132	9303	1.0749	9634	1.0379	9977	1.0023	4
57	8678	1.1524	8988	1.1126	9309	1.0742	9640	1.0373	9983	1.0017	3
58	8683	1.1517	8994	1.1119	9314	1.0736	9646	1.0367	9988	1.0012	2
59	8688	1.1510	8999	1.1113	9320	1.0730	9651	1.0361	9994	1.0006	1
60	8693	1.1504	9004	1.1106	9325	1.0724	9657	1.0355	1000	1.0000	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
′	49°		48°		47°		46°		45°		′

TABLE IV

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SQUARES OF NUMBERS

No.	Square.	No.	Square.	No.	Square.	No.	Square.	No.	Square.
0	0	20	400	40	1600	60	3600	80	6400
1	1	21	441	41	1681	61	3721	81	6561
2	4	22	484	42	1764	62	3844	82	6724
3	9	23	529	43	1849	63	3969	83	6889
4	16	24	576	44	1936	64	4096	84	7056
5	25	25	625	45	2025	65	4225	85	7225
6	36	26	676	46	2116	66	4356	86	7396
7	49	27	729	47	2209	67	4489	87	7569
8	64	28	784	48	2304	68	4624	88	7744
9	81	29	841	49	2401	69	4761	89	7921
10	100	30	900	50	2500	70	4900	90	8100
11	121	31	961	51	2601	71	5041	91	8281
12	144	32	1024	52	2704	72	5184	92	8464
13	169	33	1089	53	2809	73	5329	93	8649
14	196	34	1156	54	2916	74	5476	94	8836
15	225	35	1225	55	3025	75	5625	95	9025
16	256	36	1296	56	3136	76	5776	96	9216
17	289	37	1369	57	3249	77	5929	97	9409
18	324	38	1444	58	3364	78	6084	98	9604
19	361	39	1521	59	3481	79	6241	99	9801
20	400	40	1600	60	3600	80	6400	100	10000

	100	200	300	400	500	600	700	800	900	U	Diff.
00	100	400	900	1600	2500	3600	4900	6400	8100	00	1
01	102	404	906	1608	2510	3612	4914	6416	8118	01	3
02	104	408	912	1616	2520	3624	4928	6432	8136	04	5
03	106	412	918	1624	2530	3636	4942	6448	8154	09	7
04	108	416	924	1632	2540	3648	4956	6464	8172	16	9
05	110	420	930	1640	2550	3660	4970	6480	8190	25	11
06	112	424	936	1648	2560	3672	4984	6496	8208	36	13
07	114	428	942	1656	2570	3684	4998	6512	8226	49	15
08	116	432	948	1664	2580	3696	5012	6528	8244	64	17
09	118	436	954	1672	2590	3708	5026	6544	8262	81	19*
10	121	441	961	1681	2601	3721	5041	6561	8281	00	21
11	123	445	967	1689	2611	3733	5055	6577	8299	21	23
12	125	449	973	1697	2621	3745	5069	6593	8317	44	25
13	127	453	979	1705	2631	3757	5083	6609	8335	69	27
14	129	457	985	1713	2641	3769	5097	6625	8353	96	29*
15	132	462	992	1722	2652	3782	5112	6642	8372	25	31
16	134	466	998	1730	2662	3794	5126	6658	8390	56	33
17	136	470	1004	1738	2672	3806	5140	6674	8408	89	35*
18	139	475	1011	1747	2683	3819	5155	6691	8427	24	37
19	141	479	1017	1755	2693	3831	5169	6707	8445	61	39*
20	144	484	1024	1764	2704	3844	5184	6724	8464	00	41
21	146	488	1030	1772	2714	3856	5198	6740	8482	41	43
22	148	492	1036	1780	2724	3868	5212	6756	8500	84	45*
23	151	497	1043	1789	2735	3881	5227	6773	8519	29	47
24	153	501	1049	1797	2745	3893	5241	6789	8537	76	49*
25	156	506	1056	1806	2756	3906	5256	6806	8556	25	51
26	158	510	1062	1814	2766	3918	5270	6822	8574	76	53*
27	161	515	1069	1823	2777	3931	5285	6839	8593	29	55
28	163	519	1075	1831	2787	3943	5299	6855	8611	84	57*
29	166	524	1082	1840	2798	3956	5314	6872	8630	41	59*
30	169	529	1089	1849	2809	3969	5329	6889	8649	00	61
31	171	533	1095	1857	2819	3981	5343	6905	8667	61	63*
32	174	538	1102	1866	2830	3994	5358	6922	8686	24	65
33	176	542	1108	1874	2840	4006	5372	6938	8704	89	67*
34	179	547	1115	1883	2851	4019	5387	6955	8723	56	69*
35	182	552	1122	1892	2862	4032	5402	6972	8742	25	71
36	184	556	1128	1900	2872	4044	5416	6988	8760	96	73*
37	187	561	1135	1909	2883	4057	5431	7005	8779	69	75*
38	190	566	1142	1918	2894	4070	5446	7022	8798	44	77*
39	193	571	1149	1927	2905	4083	5461	7039	8817	21	79*
40	196	576	1156	1936	2916	4096	5476	7056	8836	00	81
41	198	580	1162	1944	2926	4108	5490	7072	8854	81	83*
42	201	585	1169	1953	2937	4121	5505	7089	8873	64	85*
43	204	590	1176	1962	2948	4134	5520	7106	8892	49	87*
44	207	595	1183	1971	2959	4147	5535	7123	8911	36	89*
45	210	600	1190	1980	2970	4160	5550	7140	8930	25	91*
46	213	605	1197	1989	2981	4173	5565	7157	8949	16	93*
47	216	610	1204	1998	2992	4186	5580	7174	8968	09	95*
48	219	615	1211	2007	3003	4199	5595	7191	8987	04	97*
49	222	620	1218	2016	3014	4212	5610	7208	9006	01	99*
50	225	625	1225	2025	3025	4225	5625	7225	9025	00	

	100	200	300	400	500	600	700	800	900	U	Diff.
50	225	625	1225	2025	3025	4225	5625	7225	9025	00	1
51	228	630	1232	2034	3036	4238	5640	7242	9044	01	
52	231	635	1239	2043	3047	4251	5655	7259	9063	04	3
53	234	640	1246	2052	3058	4264	5670	7276	9082	09	5
											7
54	237	645	1253	2061	3069	4277	5685	7293	9101	16	
55	240	650	1260	2070	3080	4290	5700	7310	9120	25	9
56	243	655	1267	2079	3091	4303	5715	7327	9139	36	11
											13
57	246	660	1274	2088	3102	4316	5730	7344	9158	49	
58	249	665	1281	2097	3113	4329	5745	7361	9177	64	15
59	252	670	1288	2106	3124	4342	5760	7378	9196	81	17
											19
60	256	676	1296	2116	3136	4356	5776	7396	9216	00	21
61	259	681	1303	2125	3147	4369	5791	7413	9235	21	
62	262	686	1310	2134	3158	4382	5806	7430	9254	44	23
63	265	691	1317	2143	3169	4395	5821	7447	9273	69	25
											27
64	268	696	1324	2152	3180	4408	5836	7464	9292	96	
65	272	702	1332	2162	3192	4422	5852	7482	9312	25	29
66	275	707	1339	2171	3203	4435	5867	7499	9331	56	31
											33
67	278	712	1346	2180	3214	4448	5882	7516	9350	89	
68	282	718	1354	2190	3226	4462	5898	7534	9370	24	35
69	285	723	1361	2199	3237	4475	5913	7551	9389	61	37
											39
70	289	729	1369	2209	3249	4489	5929	7569	9409	00	41
71	292	734	1376	2218	3260	4502	5944	7586	9428	41	
72	295	739	1383	2227	3271	4515	5959	7603	9447	84	43
73	299	745	1391	2237	3283	4529	5975	7621	9467	29	45
											47
74	302	750	1398	2246	3294	4542	5990	7638	9486	76	
75	306	756	1406	2256	3306	4556	6006	7656	9506	25	49
76	309	761	1413	2265	3317	4569	6021	7673	9525	76	51
											53
77	313	767	1421	2275	3329	4583	6037	7691	9545	29	
78	316	772	1428	2284	3340	4596	6052	7708	9564	84	55
79	320	778	1436	2294	3352	4610	6068	7726	9584	41	57
											59
80	324	784	1444	2304	3364	4624	6084	7744	9604	00	61
81	327	789	1451	2313	3375	4637	6099	7761	9623	61	
82	331	795	1459	2323	3387	4651	6115	7779	9643	24	63
83	334	800	1466	2332	3398	4664	6130	7796	9662	89	65
											67
84	338	806	1474	2342	3410	4678	6146	7814	9682	56	
85	342	812	1482	2352	3422	4692	6162	7832	9702	25	69
86	345	817	1489	2361	3433	4705	6177	7849	9721	96	71
											73
87	349	823	1497	2371	3445	4719	6193	7867	9741	69	
88	353	829	1505	2381	3457	4733	6209	7885	9761	44	75
89	357	835	1513	2391	3469	4747	6225	7903	9781	21	77
											79
90	361	841	1521	2401	3481	4761	6241	7921	9801	00	81
91	364	846	1528	2410	3492	4774	6256	7938	9820	81	
92	368	852	1536	2420	3504	4788	6272	7956	9840	64	83
93	372	858	1544	2430	3516	4802	6288	7974	9860	49	85
											87
94	376	864	1552	2440	3528	4816	6304	7992	9880	36	
95	380	870	1560	2450	3540	4830	6320	8010	9900	25	89
96	384	876	1568	2460	3552	4844	6336	8028	9920	16	91
											93
97	388	882	1576	2470	3564	4858	6352	8046	9940	09	
98	392	888	1584	2480	3576	4872	6368	8064	9960	04	95
99	396	894	1592	2490	3588	4886	6384	8082	9980	01	97
											99
100	400	900	1600	2500	3600	4900	6400	8100	10000	00	

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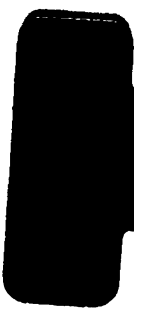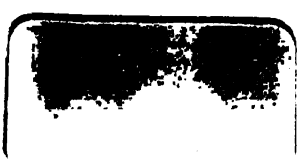
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